

A SURFACE HEATED GREENHOUSE MODEL FOR WASTE HEAT UTILIZATION ASSESSMENT*

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ABSTRACT

This is the fifth in a series of articles on the development of a methodology for assessing waste heat utilization technologies and optimizing the mix of technologies used on a site-specific basis. As part of this effort, a surface heated greenhouse model has been developed and is described. The model uses readily obtainable data regarding plant kinetics, climate conditions, and the inlet water temperature. The outputs include the time to harvest, crop yield, outlet water temperature, and mass flow rates of heated water in and out of the greenhouse required to maintain the desired temperature. This model has been used in simulations in order to assess the economic feasibility of this method of utilizing waste heat.

* This is the fifth in a series of articles on the development of a methodology for assessing waste heat utilization technologies and optimizing the mix of technologies used on a site-specific basis. The first article provided an overview of the methodology. The second and third dealt with the aquaculture model and the evaporative pad greenhouse model, and the fourth with livestock. Later contributions will describe models for simulating the crop drying and wastewater treatment components of an integrated waste heat utilization complex.

This article presents a model for use in simulating the behavior of a surface heated greenhouse. The model can be used to predict the mass flow rate of heated water required to maintain the greenhouse at a particular set temperature given ambient weather conditions and the water inlet temperature. This model may be used for stand alone simulations; however, it was actually developed as part of an extensive series of models used to simulate an integrated waste heat utilization complex [1, 2]. Using this set of models, it has been possible to optimize the mix of waste heat utilization technologies at a particular site.

The strategy used in developing this model has been to implement both materials and heat balances. From the materials balance, we derive relationships for computing the time from planting to harvest for a specified yield, under set temperature conditions. The heat balance leads to techniques for determining the mass flow rate of heated water needed to maintain the greenhouse at the desired temperature under ambient weather conditions.

SURFACE HEATED GREENHOUSE CONFIGURATION

The floor plan of the surface heated greenhouse modular unit used in the development of this model was similar to that described in an earlier article [3]. Iverson et al. give dimensions of a standard greenhouse with approximately an acre of growing area [4]. This provides a pattern for the greenhouse floor plan which we will use as a single module (Figure 1). Any greenhouse complex will consist of groups of these greenhouse modules. The greenhouse module consists of twenty bays of dimensions 24 ft \times 94 ft (7.3 m \times 28.65 m), connected by a main access aisle measuring 12 ft \times 240 ft (3.7 m \times 73 m).

The method of heat transfer in a surface heated greenhouse is quite distinctive. The heated water flows in a layer over the roof of the greenhouse and its heat energy is transferred by conduction through the glass (see Figure 2). The glazing used is glass rather than alternatives such as double polyethylene due to the need for structural rigidity, good thermal conductivity, and low thermal radiation transmittance.

MATERIALS BALANCE

In an earlier article, a materials balance was performed for an evaporative pad heated greenhouse [3]. The growth of the vegetables and flowers in the greenhouse was represented by the logistic equation. That effort leads to equations which are identical for the surface heated greenhouse, and so the derivation will not be duplicated here. Only the resulting mathematical relationships will be presented.

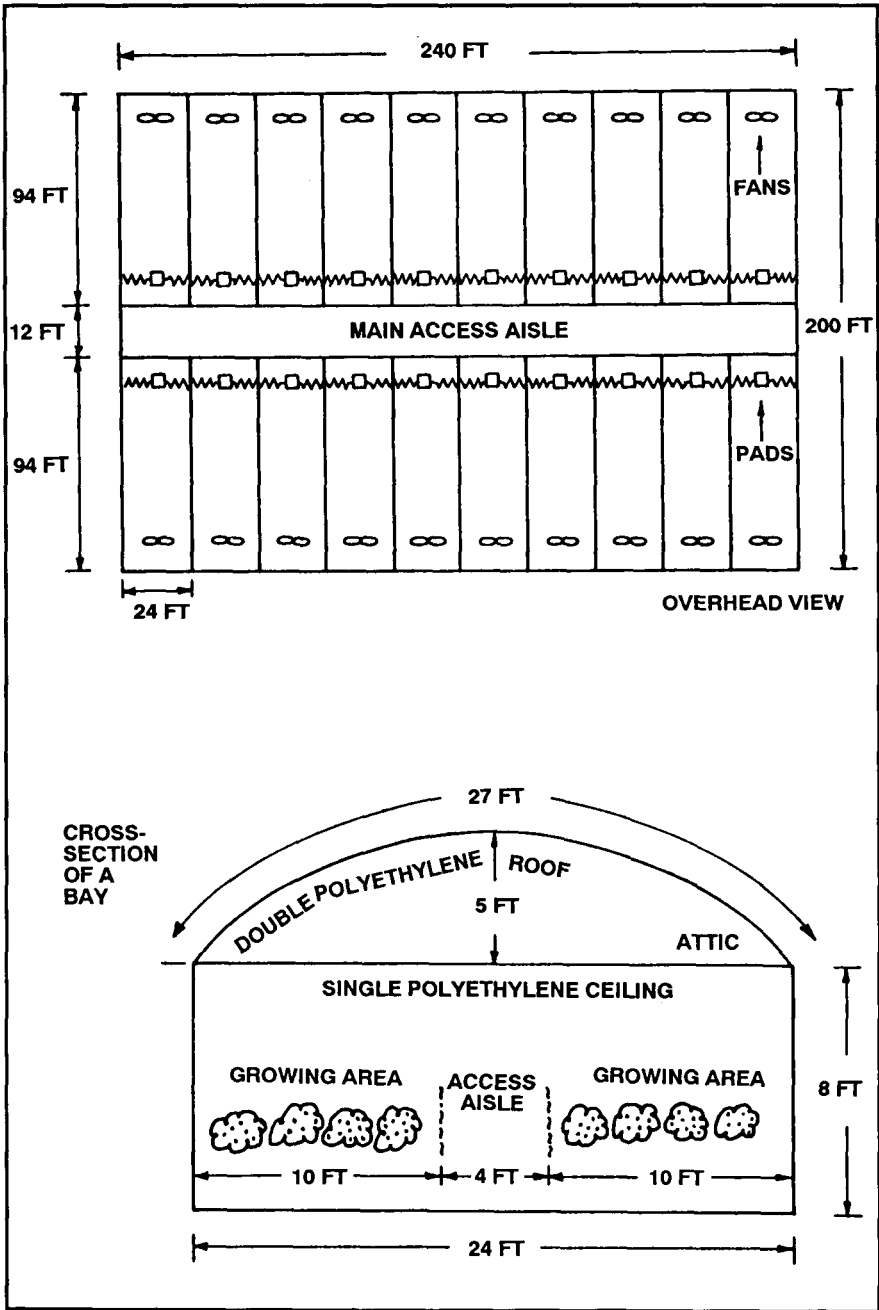


Figure 1. Pattern for floor plan used in greenhouse module.

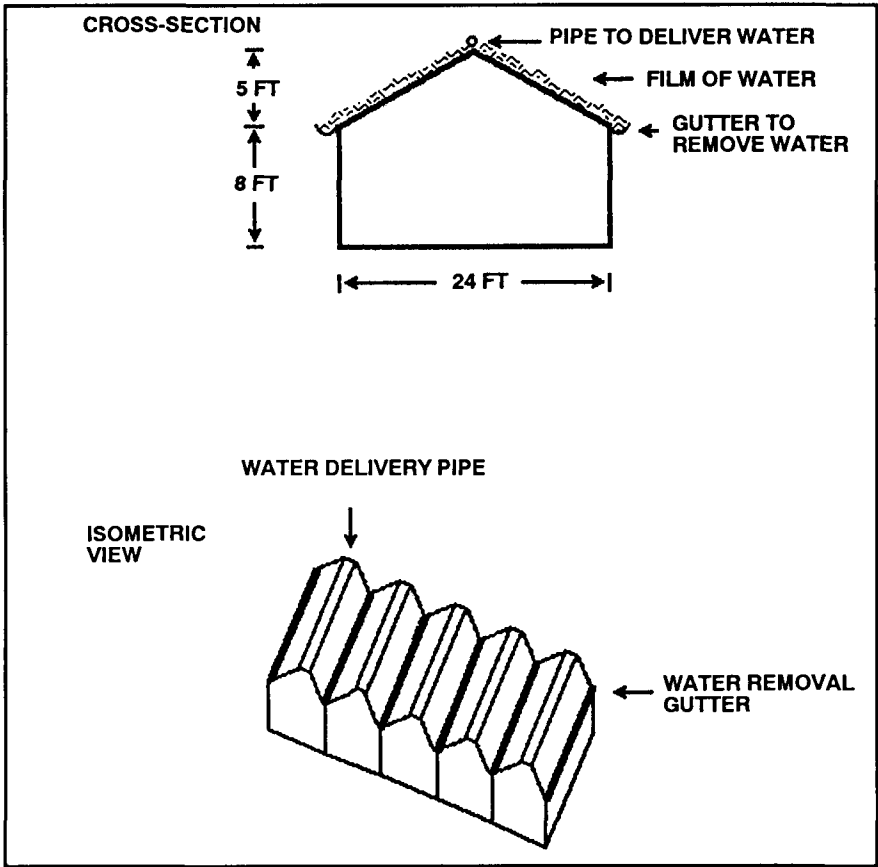


Figure 2. Heat transfer in surface heated greenhouse.

The time-to-harvest can be calculated as:

$$t_{\text{harv}} = \frac{\ln \left[\frac{KX_{\text{harv}}}{K - X_{\text{harv}}} \right] - \ln \left[\frac{KX_o}{K - X_o} \right]}{r} \tag{1}$$

where

- t_{harv} = time of harvest, hr
- r = intrinsic growth rate, hr^{-1}
- X_{harv} = weight at time of harvest, lb
- X_o = weight at time of planting, lb
- K = carrying capacity, lb

This can be simplified as:

$$t_{\text{harv}} = \frac{1}{r} \ln \left[\frac{X_{\text{harv}} (K - X_0)}{X_0 (K - X_{\text{harv}})} \right] \quad (2)$$

The growth rate of the plants is dependent on both temperature and light intensity. The dependence on temperature can be approximated by:

$$r = r_{\text{max}} \sin \left[\pi \left(\frac{T - T_L}{T_H - T_L} \right) \right] \quad (3)$$

where

$$\begin{aligned} r_{\text{max}} &= \text{maximum growth rate, hr}^{-1} \\ T &= \text{temperature at which the growth occurs, } ^\circ\text{F} \\ T_H &= \text{upper zero-growth temperature, } ^\circ\text{F} \\ T_L &= \text{lower zero-growth temperature, } ^\circ\text{F} \end{aligned}$$

Note that π should be replaced by 180 if the sine is taken using degrees instead of radians.

The dependence on sunlight may be expressed as:

$$r_s = 0.325r_o \left[\ln \left(\frac{2.71\tau S}{400} \right) + 1 \right] \quad (4)$$

where

$$\begin{aligned} r_s &= \text{growth rate under } \tau S \text{ light conditions, hr}^{-1} \\ \tau &= \text{solar radiation transmittance, decimal.} \\ S &= \text{total insolation, Btu-ft}^{-2}\text{-d}^{-1}. \end{aligned}$$

The τ factor is used to account for the glazing on the outer surface of the greenhouse. For single pane glass, $\tau = 0.89$ [5]. The 400 factor represents the typical saturation light intensity for the United States. In practice, we take the value obtained for r in equation (3) and use this as r_o in equation (4) to obtain a corrected value of r_s to use as the intrinsic growth rate in the model described by equation (2). The data and parameters needed to apply this growth model to tomatoes and hybrid tea roses are presented by Keenan and Amundsen [3]. The only other required input is the site-specific value, S .

HEAT BALANCE

A heat balance on the greenhouse [6] yields the following:

$$H_f + H_s + H_e + H_r = H_c + H_t + H_{PH} + H_g + H_v \quad (5)$$

where

$$\begin{aligned} H_f &= \text{heat supplied by circulating water, Btu/hr} \\ H_s &= \text{solar heat gain, Btu/hr} \end{aligned}$$

- H_e = heat released by equipment, Btu/hr
 H_r = heat from plant respiration, Btu/hr
 H_c = heat lost by conduction, Btu/hr
 H_t = thermal radiation heat loss, Btu/hr
 H_{PH} = solar energy used for photosynthesis, Btu/hr
 H_g = heat lost to the ground, Btu/hr
 H_v = heat lost or gained in the ventilating air, Btu/hr

The H_r , H_{PH} , and H_e terms are negligible and will be ignored in this derivation.

Heat Loss by Conduction

The conductive heat loss is found using:

$$H_c = UA (T - T_a) \quad (6)$$

where

- U = heat transfer coefficient, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot\text{°F}^{-1}$
 T_a = temperature of the ambient air, °F
 A = area exposed to the outside, ft^2

In the case of a surface heated greenhouse, we have a single pane of glass as the glazing surface, and $U = 1.1 \text{ Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot\text{°F}^{-1}$ [7].

The heat loss or gain by conduction through the roof will be treated separately as part of the analytic determination of the water-to-roof heat transfer (see p. 309). The wall area is

$$\begin{aligned}
 2 \text{ sides} \times 8 \text{ ft} \times 200 \text{ ft} &= 3200 \text{ ft}^2 \\
 20 \text{ ends} \times 252 \text{ ft}^2 &= 5040 \text{ ft}^2 \\
 \text{Total wall area} &= 8240 \text{ ft}^2
 \end{aligned}$$

Since $UA = 1.1 \times 8240 = 9064$, we can rewrite equation (6) as:

$$H_c = 9064 (T - T_a) \quad (7)$$

Heat Loss to the Ground

The heat lost to the ground, H_g , can be found using:

$$H_g = 0.1 A_g (T - T_g) \quad (8)$$

where

- A_g = area of the greenhouse floor, ft^2
 T_g = ground temperature, °F
 0.1 = heat transfer coefficient of ground, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot\text{°F}^{-1}$

The upper layer of soil will be in equilibrium with the greenhouse temperature, T . Although the soil surface will experience fluctuations due to evaporation and radiative heat transfer, the thermal conductivity of the soil is low enough to damp

these fluctuations considerably. The ground temperature is that of the ground water which is approximated by the mean annual temperature [8]. Here, $A_g = 240$ ft wide \times 200 ft long = 48,000 ft², thus,

$$H_g = 4800 (T - T_g) \quad (9)$$

Heat Loss by Radiation

The thermal radiation heat loss is the difference between the thermal radiation from the surface and the thermal radiation from the atmosphere. It may be determined as [6]:

$$H_t = \sigma A_t \tau_t (\epsilon_s T_s^4 - \epsilon_a T_{aa}^4) \quad (10)$$

where

- σ = Stefan-Boltzman constant = 1.714×10^{-9} Btu-hr⁻¹-ft⁻²-°F⁻⁴
- τ_t = thermal radiation transmittance, decimal
- A_t = area of the surface radiating thermally, ft²
- ϵ_s = emissivity of the surface, decimal
- T_s = absolute temperature of the surface, °R
- ϵ_a = apparent emissivity of the atmosphere, decimal
- T_{aa} = absolute atmospheric temperature near the ground, °R

The area of the surface radiating thermally, A_t , is essentially the area of the ground, unless there is considerable foliage. The thermal transmittance of single pane glass is 0.03 [5]. The emissivity of the surface is 0.90, and that of the atmosphere is 0.82. The temperature of the surface is close to the average temperature inside of the greenhouse, T . Converting the temperature values to °R and substituting the appropriate numerical values, we obtain:

$$H_t = 2.468 \times 10^{-6} [(0.9(T + 460))^4 - 0.82(T_a + 460)^4] \quad (11)$$

Solar Heat Gain

The solar gain is given by [6]:

$$H_s = a_s \tau I A_s \quad (12)$$

where

- a_s = absorptivity of the surface for solar radiation, decimal
- I = solar intensity on a horizontal surface, Btu-hr⁻¹-ft⁻²
- A_s = area of surface receiving solar radiation, ft²

The area receiving the solar radiation is essentially the area of the ground, $A_g = 48,000$ ft². For plants and soil, $a_s \approx 0.77$. The solar radiation transmittance for glass is 0.89 [5]. Glass has an extremely low thermal transmittance (0.03-0.04). To correct for the layer of water flowing over the glass, we have used the lower value

of thermal transmittance (0.03, see p. 307) and treated the layer of water as having a solar radiation transmittance of 0.90, giving $0.89 \times 0.9 \approx 0.8$ as the overall value of τ . The solar gain becomes:

$$\begin{aligned} H_s &= (0.77) (0.80) (48000)I \\ &= (29568)I \end{aligned} \quad (13)$$

Heat Loss by Ventilation

The ventilation heat losses are nearly the same as they are for the evaporative pad greenhouse [3], except that glass houses have a higher infiltration rate. The total ventilation heat loss is the sum of the sensible and latent heat losses. Ventilation sensible heat losses, H_{vs} , are given by:

$$H_{vs} = \frac{60C_p F(T - T_a)}{V_s} \quad (14)$$

where

- C_p = specific heat of air, $0.24 \text{ Btu}\cdot\text{lb}^{-1}\cdot\text{°F}^{-1}$
- V_s = specific volume of air, $13.35 \text{ ft}^3/\text{lb}$
- 60 = conversion factor, min/hr
- F = air flow rate into greenhouse, cfm

The air flow rate, F , is the sum of ventilation (F_{vent}) and infiltration (F_{inf}). The number of air changes per hour for a well constructed, new glass greenhouse is 1.1 [7], which can be converted to cfm by:

$$F_{inf} = \frac{nV}{60} \quad (15)$$

where

- n = number of air changes per hour, hr^{-1}
- 60 = conversion factor, min/hr
- V = volume of the greenhouse, ft^3
- F_{inf} = infiltration, cfm

In order to calculate V , we will consider the central access aisle volume as part of the total volume by ignoring the partitions. The volume for a glass greenhouse module is $504,000 \text{ ft}^3$. Using this value for V , equation (14) becomes:

$$H_{vs} = 1.08 (9240 + F_{vent}) (T - T_a) \quad (16)$$

To the ventilation sensible heat losses we must add the ventilation latent heat losses. These losses are caused by water, which has been evaporated by heat within the greenhouse, escaping as water vapor to the atmosphere and carrying the latent heat of vaporization with it. In the surface heated greenhouse, the growing area air is not completely saturated with water vapor. Rather, the latent heat loss is

determined by the amount of evaporation which occurs from the moist soil. Instead of building a highly complex model, we can simply base our estimates on measurements taken in existing greenhouses. The latent heat losses by ventilation may be found using [6]:

$$H_{vL} = 5436 A_g E_t \tag{17}$$

where

$$E_t = \text{evapotranspiration rate, in/hr}$$

$$5436 = \text{conversion factor, Btu-ft}^{-2}\text{-in}^{-1}$$

The conversion factor is obtained as the product of the latent heat of vaporization of water times the specific weight of water (1048.3 Btu/lb × 62.22 lb/ft² × 1 ft/12 in). This conversion factor does not vary by more than a few percent over the range of 32°F to 100°F, although this value was calculated for water at 80°F.

Evapotranspiration in greenhouses is highly correlated with solar radiation. Studies have shown that the maximum evapotranspiration rate for clear days at high insolation under typical greenhouse conditions is 0.025 in/hr; nighttime rates are nearly zero [6]. A reasonable approximation is therefore:

$$E_t = 0.025 \frac{I}{I_{\max}} \tag{18}$$

where

$$I_{\max} = \text{maximum solar intensity, Btu-hr}^{-1}\text{-ft}^{-2}$$

The maximum solar intensity for Lexington, Kentucky, is 306 Btu-hr⁻¹-ft⁻². We use the value for Lexington because this is where the original measurements were taken. This yields

$$H_{vL} = 5436 \times 48000 \times 0.025 \times \frac{I}{306} = 21317.65 I \tag{19}$$

The H_v term of equation (5) is determined by summing H_{vL} (equation (19)) and H_{vS} (equation (16)). Equation (5) is thus used to determine H_f, the heat supplied by the circulating water. This will be used later as the flow requirements are determined in the next section. The evaluation of H_f requires the following inputs: T, T_a, T_g, I, and F_{vent}. These are all readily available as design parameters or site-specific weather data.

FLOW REQUIREMENTS

Walker has presented an analytic treatment of the heat transfer characteristics of a surface heated greenhouse, and has provided experimentally determined values of the necessary coefficients [9]. Figure 3 identifies the differential segment (shaded rectangle) which is used in the integrations needed to derive this model.

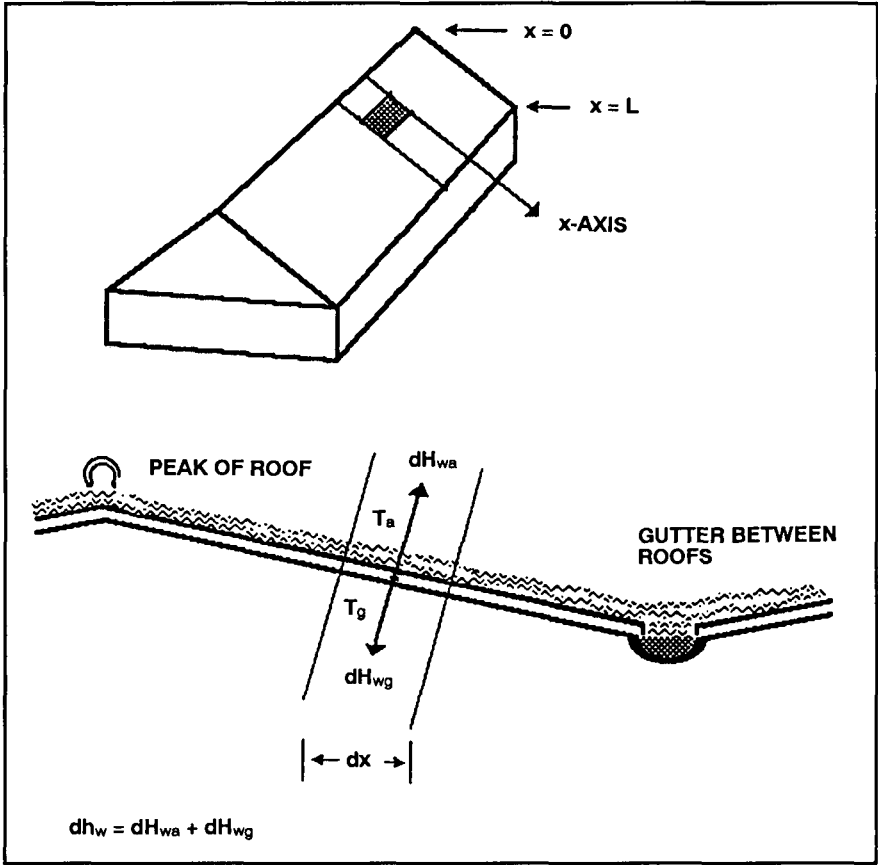


Figure 3. Definition of differential segment used in derivation of model.

Temperature of Circulating Water

The heat transferred from the water flowing over a differential unit of greenhouse area with dx parallel to the direction of water flow is:

$$dH_w = [U_{wg}(t_w - T) + U_{wa}(t_w - T_a)] dx \tag{20}$$

where

- H_w = rate of heat flow from water per unit width, Btu/ft
- U_{wg} = heat transfer coefficient between water and greenhouse air, $Btu \cdot hr^{-1} \cdot ft^{-2} \cdot ^\circ F^{-1}$
- t_w = variable temperature of the water, $^\circ F$
- x = distance along water flow path, ft

U_{wa} = heat transfer coefficient between water and the atmosphere,
 $\text{Btu-hr}^{-1}\text{-ft}^{-2}\text{-}^\circ\text{F}^{-1}$

The heat lost by the water, assuming a negligible change in flow rate due to evaporation, is:

$$dH_w = -F_w C dt_w \tag{21}$$

where

F_w = water flow rate per unit width of water flow path, $\text{ft}^3\text{-hr}^{-1}\text{-ft}^{-1}$
 C = specific heat of water, $\text{Btu-lb}^{-1}\text{-}^\circ\text{F}^{-1}$

Combining equations (20) and (21), we get:

$$-F_w C dt_w = [U_{wg}(t_w - T) + U_{wa}(t_w - T_a)] dx \tag{22}$$

The right hand side of equation (22) may be expanded and recombined to yield:

$$= [(U_{wg} + U_{wa})t_w - (U_{wg}T + U_{wa}T_a)] dx \tag{23}$$

or,

$$= (U_{wg}+U_{wa}) t_w - (U_{wg}+U_{wa}) \left(\frac{U_{wg}T+U_{wa}T_a}{U_{wg}+U_{wa}} \right) dx \tag{24}$$

Now, define K' as:

$$K' = \frac{U_{wg}T + U_{wa}T_a}{U_{wg} + U_{wa}} \tag{25}$$

Then:

$$-F_w C dt_w = [(U_{wg} + U_{wa}) (t_w - K')] dx \tag{26}$$

Rearrange and solve by separation of variables:

$$-\left(\frac{U_{wg} + U_{wa}}{F_w C} \right) dx = \frac{dt_w}{t_w - K'} \tag{27}$$

$$-\int_0^{x_0} \left(\frac{U_{wg} + U_{wa}}{F_w C} \right) dx = \int_{T_{w_0}}^{t_w} \frac{dt_w}{t_w - K'} \tag{28}$$

where

T_{w_0} = temperature of the water at the point $x = 0$ where first applied to the greenhouse (at peak of roof), $^\circ\text{F}$.

Now integrate:

$$-x \left(\frac{U_{wg} + U_{wa}}{F_w C} \right) = \ln \left[\frac{t_w - K'}{T_{w_0} - K'} \right] \tag{29}$$

Exponentiate and solve for t_w :

$$t_w = (T_{w0} - K') \exp \left[\frac{-x(U_{wg} + U_{wa})}{F_w C} \right] + K' \tag{30}$$

We now have an expression for the temperature of the water at every point x along the slope of the greenhouse roof.

Required Flow of Water

The heat transferred from the water into the greenhouse through the differential area is:

$$dH_{wg} = U_{wg}(t_w - T) dx \tag{31}$$

where

H_{wg} = rate of heat flow from water into greenhouse per unit width of water flow, Btu-hr⁻¹-ft⁻¹.

We substitute equation (30) into equation (31) and integrate to find the total heat transferred through a unit width of greenhouse roof:

$$dH_{wg} = U_{wg} \left[(T_{w0} - K') \exp \left[\frac{-x(U_{wg} + U_{wa})}{F_w C} \right] + K' - T \right] dx \tag{32}$$

$$\int_0^{W_{wg}} dH_{wg} = U_{wg} \int_0^L \left[(T_{w0} - K') \exp \left[\frac{-x(U_{wg} + U_{wa})}{F_w C} \right] + K' - T \right] dx \tag{33}$$

$$H_{wg} = \left(\frac{-F_w C U_{wg} (T_{w0} - K')}{U_{wg} + U_{wa}} \right) \left[\exp \left[-L \left(\frac{U_{wg} + U_{wa}}{F_w C} \right) \right] - 1 \right] - U_{wg} L (T - K') \tag{34}$$

We now have an expression for heat transfer per unit width. This can be expressed as heat transfer per unit area by dividing through by the length L :

$$\frac{H_{wg}}{L} = \left(\frac{(C(F_w/L)U_{wg}(T_{w0} - K'))}{U_{wg} + U_{wa}} \right) \left(1 - \exp \left[- \left(\frac{U_{wg} + U_{wa}}{C(F_w/L)} \right) L \right] \right) - U_{wg}(T - K') \tag{35}$$

Define:

$$q_{wg} = \frac{H_{wg}}{L}$$

$$f = \frac{F_w}{L}$$

where

q_{wg} = rate of heat flow from water into greenhouse per unit area of roof, Btu-hr⁻¹-ft⁻².

f = water flow rate per unit area, ft³-hr⁻¹-ft⁻²

Substituting q_{wg} and f into equation (35) gives our final version of this relationship:

$$q_{wg} = U_{wg} \left(\frac{fC(T_{w_o} - K')}{U_{wg} + U_{wa}} \right) \left(1 - \exp \left[- \left(\frac{U_{wg} + U_{wa}}{fC} \right) \right] \right) - U_{wg}(T - K') \quad (36)$$

Through experimentation, Walker determined the heat transfer coefficients for water on glass [9]:

$$U_{wg} = 10.51 \text{ Btu-hr}^{-1}\text{-ft}^{-2}\text{-}^{\circ}\text{F}^{-1} \quad (37)$$

$$U_{wa} = 39.96 \text{ Btu-hr}^{-1}\text{-ft}^{-2}\text{-}^{\circ}\text{F}^{-1} \quad (38)$$

From equation (25), we have:

$$K' = \frac{10.51T_i + 39.96T_a}{50.47} \quad (39)$$

Unfortunately, equation (36) is a transcendental function and can not be solved by ordinary algebraic means. An iterative solution based on Newton's method was used instead. Equation (36) may be considered to be a function of f . We define:

$$g(f) = (q_{wg}(f) - q_{req})^2 \quad (40)$$

where

q_{req} = heat lost by greenhouse per unit area, Btu-hr⁻¹-ft⁻²

There is a critical value of f which corresponds to $q_{wg} = q_{req}$. This value of f is the one which minimizes $g(f)$. We can find q_{req} quite easily using the results of the heat balance equation. The net heat loss (H_f in equation (5)) must be matched by the heat gain from the film of water into the greenhouse (q_{wg} from equation (36)). We divide H_f by the area of the greenhouse roofs; this gives us q_{req} for use in an

iterative procedure for determining f from the minimum of $g(f)$. The essentials of the method are as follows:

1. Select an initial estimate for f .
2. Determine:

$$g(f) = (q_{wg}(f) - q_{req})^2$$

3. Evaluate:

$$g'(f) = \frac{[(q_{wg}(f) - q_{req})^2 - (q_{wg}(f + \Delta f) - q_{req})^2]}{\Delta f}$$

4. Then evaluate:

$$g'(f + \Delta f) = \frac{[(q_{wg}(f + \Delta f) - q_{req})^2 - (q_{wg}(f + 2\Delta f) - q_{req})^2]}{\Delta f}$$

5. Then,

$$g''(f) = \frac{(g'(f) - g'(f + \Delta f))}{\Delta f}$$

6. Set

$$f_{old} = f$$

7. Determine a new value of f :

$$f_{new} = f - \left(\frac{g'(f)}{g''(f)} \right)$$

8. Convergence criterion:

If $|f_{new} - f_{old}| > \epsilon$, then set $f = f_{new}$ and return to step 2. The variables q_{wg} and f are expressed on a per-unit-area basis. Since we know the area of the greenhouse roof, we can convert to total heat loss and water flow:

$$H_{tot} = q_{wg} A_r N_{sg} \quad (41)$$

$$f_{tot} = f A_r N_{sg} \quad (42)$$

where

- A_r = area of greenhouse roof, ft^2
- N_{sg} = number of surface heated greenhouse modules
- H_{tot} = total heat loss from greenhouses, Btu/hr
- f_{tot} = total flow of water over roofs, ft^3/hr

Temperature of Exiting Water

We earlier derived an expression for the temperature of the water at every point x along the slope of the roof. At $x = 0$, $t_w = T_{w0}$. In order to find the temperature of the water as it leaves the roof, T_{wf} , we must substitute $x = L$ into equation (30):

$$T_{wf} = (T_{w0} - K') \exp \left[\frac{-L(U_{wg} + U_{wa})}{F_w C} \right] + K' \quad (43)$$

Note that $F_w = fL$:

$$T_{wf} = (T_{w0} - K') \exp \left[\frac{-(U_{wg} + U_{wa})}{fC} \right] + K' \quad (44)$$

Evaporative Losses

Evaporative losses from the film of water are:

$$F_e = 0.0161 \left(\frac{H_e}{h_{fg}} \right) \quad (45)$$

where

- h_{fg} = latent heat of vaporization of water, Btu/lb
- H_e = heat lost through evaporation, Btu/hr
- F_e = flow rate of evaporation loss, ft³/hr
- the 0.0161 is adapted from Wark [10]

and

$$h_{fg} = 1093.9 - 0.57T_{w_{av}} \quad (46)$$

$$T_{w_{av}} = \frac{T_{w0} + T_{wf}}{2} \quad (47)$$

H_e may be evaluated according to the relation [11]:

$$H_e = 200h_{hl}(p_w - p_a)A_r \quad (48)$$

where

- h_{hl} = heat loss coefficient, Btu-hr⁻¹-ft⁻²-°F⁻¹
- = 0.32v + 0.42 [11]
- v = wind speed, mph
- p_w = water vapor pressure of the air at the water surface, psi
- p_a = water vapor pressure of the surrounding air, psi

Empirical formulae are available for determining the vapor pressures [12]:

$$p_w = 0.491 \exp \left(17.62 - \left(\frac{9501}{T_{w_{av}} + 460} \right) \right) \quad (49)$$

$$p_a = 0.491 \exp \left(17.62 - \left(\frac{9501}{T_{wb} + 460} \right) \right) \quad (50)$$

$$T_{wb} = (0.655 + 0.36R)T_a \quad (51)$$

where

R = relative humidity, decimal

T_a = air temperature, °F

T_{wb} = wet bulb temperature, °F

The resulting model may be used to characterize and simulate a surface-heated greenhouse. Equation (4) provides the plant growth rate under the prevailing temperature and light conditions. Equation (2) is used to relate the harvest yield and the time of harvest to the growth rate. We use equation (5) to provide the heat energy required to maintain the greenhouse at the desired temperature. The flow rate of heated water needed to provide this heat energy is developed using the iterative solution to equation (36). The temperature of the exiting water is determined with equation (44). And, finally, equation (45) provides the means of determining evaporation losses, and consequently, the return flow. Data requirements are limited to design parameters, site-specific weather information, and various constants.

SUMMARY AND CONCLUSIONS

Surface heated greenhouses represent one element of an array of potential technologies for using waste heat. A methodology for assessing waste heat utilization options and for optimizing the site-specific mix of such technologies has been developed. An important component of that methodology is the model for simulating the behavior of surface heated greenhouses. The purpose of this article has been to present that model. The model was derived so as to minimize computing time and input data requirements. The finished product requires only well known constants, design parameters, and basic site-specific climatological data. The outputs of the model include, for a set greenhouse operating temperature, the time to harvest; the flow of heated water required to maintain the internal greenhouse temperature, and the characteristics of water exiting the system.

NOMENCLATURE

- A = area exposed to the outside, ft²
- A_g = area of the greenhouse floor, ft²
- A_s = area of surface receiving solar radiation, ft²
- A_r = area of greenhouse roof, ft²
- A_t = area of the surface radiating thermally, ft²
- C_p = specific heat of air, 0.24 Btu-lb⁻¹·°F⁻¹

- C = specific heat of water, $\text{Btu}\cdot\text{lb}^{-1}\cdot\text{°F}^{-1}$
 E_t = evapotranspiration rate, in/hr
 F = air flow rate into greenhouse, cfm
 F_{inf} = infiltration air flow rate, cfm
 F_{vent} = ventilation air flow rate, cfm
 F_w = water flow rate per unit width of water flow path, $\text{ft}^3\cdot\text{hr}^{-1}\cdot\text{ft}^{-1}$
 F_e = flow rate of evaporation loss, ft^3/hr
 H_e = heat lost through evaporation, Btu/hr
 HPH = solar energy used for photosynthesis, Btu/hr
 H_v = heat lost or gained in the ventilating air, Btu/hr
 H_c = heat lost by conduction, Btu/hr
 H_e = heat released by equipment, Btu/hr
 H_f = heat output of the evaporative pad, Btu/hr
 H_g = heat lost to the ground, Btu/hr
 H_r = heat from plant respiration, Btu/hr
 H_s = solar heat gain, Btu/hr
 H_t = thermal radiation heat loss, Btu/hr
 H_{tot} = total heat loss from greenhouses, Btu/hr
 H_w = rate of heat flow from water per unit width, Btu/ft
 H_{wa} = rate of heat flow from water into air per unit width, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-1}$
 H_{wg} = rate of heat flow from water into greenhouse per unit width, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-1}$
 I = solar intensity on a horizontal surface, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}$
 I_{max} = maximum solar intensity, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}$
 K = carrying capacity, lb
 $K' = \frac{U_{\text{wg}}T_i + U_{\text{wa}}T_a}{U_{\text{wg}} + U_{\text{wa}}}$
 N_{sg} = number of surface heated greenhouse modules
 S = total insolation, $\text{Btu}\cdot\text{ft}^{-2}\cdot\text{d}^{-1}$
 T = temperature at which growth occurs, $°\text{F}$
 T_{H} = upper zero-growth temperature, $°\text{F}$
 T_{L} = lower zero-growth temperature, $°\text{F}$
 T_a = temperature of the ambient air, $°\text{F}$
 T_{aa} = absolute atmospheric temperature near the ground, $°\text{R}$
 T_g = ground temperature, $°\text{F}$
 T_s = absolute temperature of the surface, $°\text{R}$
 T_{wo} = temperature of the water at the point $x = 0$, $°\text{F}$
 T_{wf} = temperature of the water as it leaves the roof, $°\text{F}$
 U = heat transfer coefficient, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot\text{°F}^{-1}$
 U_{wa} = heat transfer coefficient between water and the atmosphere, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot\text{°F}^{-1}$

- U_{wg} = heat transfer coefficient between water and greenhouse air, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot^{\circ}\text{F}^{-1}$
 V = volume of the greenhouse, ft^3
 V_s = specific volume of air, $13.35 \text{ ft}^3/\text{lb}$
 X_o = weight at time of planting, lb
 X_{harv} = weight at time of harvest, lb
 a_s = absorptivity of the surface for solar radiation, decimal
 f = water flow rate per unit area, $\text{ft}^3\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}$
 f_{tot} = total flow of water over roofs, ft^3/hr
 h_{hl} = heat loss coefficient, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot^{\circ}\text{F}^{-1}$
 h_{fg} = latent heat of vaporization of water, Btu/lb
 n = number of air changes per hour, hr^{-1}
 p_w = water vapor pressure of the air at the water surface, psi
 p_a = water vapor pressure of the surrounding air, psi
 q_{wg} = rate of heat flow from water into greenhouse per unit area of roof, $\text{Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}$
 r = intrinsic growth rate, hr^{-1}
 r_{max} = maximum growth rate, hr^{-1}
 r_s = growth rate under τS light conditions, hr^{-1}
 t_{harv} = time of harvest, hr
 t_w = variable temperature of the water, $^{\circ}\text{F}$
 v = wind speed, mph
 x = distance along water flow path, ft
 ϵ_a = apparent emissivity of the atmosphere, decimal
 ϵ_s = emissivity of the surface, decimal
 σ = Stefan-Boltzman constant, $1.714 \times 10^{-9} \text{ Btu}\cdot\text{hr}^{-1}\cdot\text{ft}^{-2}\cdot^{\circ}\text{F}^{-4}$
 τ = solar radiation transmittance, decimal
 τ_t = thermal radiation transmittance, decimal

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