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A Note On Decision Problems

DR. JOHN G. TRUXAL
Vice President for Academic Affairs
Polytechnic Institute of Brooklyn

We are used to decision problems with a solution. Often, we have just a few possible strategies. Further, the criterion is given—that is, we know what is to be maximized. Then, we simply evaluate the criterion function for each strategy. We pick the strategy which gives the best value. There is a single, best solution.

Problems in societal engineering often involve systems so complex that we can not determine the effect of a particular action (or strategy). In these circumstances the decision problem is not so clear. Consider the following problem.

You are the mayor of a large city. You sincerely want to improve the quality of life in your city, but you are limited by a lack of money. After essential services are paid for next year, you will have money for only *one* new program. Three possibilities have been presented to you by your advisors:

(A) Speed up traffic flow
(B) Reduce air pollution
(C) Improve rubbish collection

In each category, your staff has presented a detailed program for spending the available funds. Each advisor agrees his program is minimum cost; spending 1/3 of the money on each would result in no effect at all. You must select one of the three strategies.

You ask your system analysts to evaluate each program. If A is adopted, what will be the effect on the quality of life?

The system analysts are surprisingly honest. They report to you that the city is such a complex system, they really can't predict the effect of any

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one program. To be able to predict, they would require a complicated computer simulation of the city—a study that would take a year. By that time, it would be too late to start any program.

They are somewhat helpful, though. They say that if A is adopted (traffic flow speeded), three different results are *equally probable:*

a_1: transportation much better, with an improvement in the quality of life measured as +4.

a_2: no change (0).

a_3: more cars, more air pollution, more parking problems, poorer quality of life (−1).

Similarly, B and C give the three equally probable results:

$b_1 = 1 \qquad c_1 = 3$
$b_2 = 1 \qquad c_2 = 2$
$b_3 = 1 \qquad c_3 = -2$

You now have to compare the strategies A, B, and C. First, you compare A and B. Since each has three equally probable results, there are nine possible combinations:

$a_1 = 4$	$b_1 = 1$	$a_2 = 0$	$b_1 = 1$	$a_3 = -1$	$b_1 = 1$
$a_1 = 4$	$b_2 = 1$	$a_2 = 0$	$b_2 = 1$	$a_3 = -1$	$b_2 = 1$
$a_1 = 4$	$b_3 = 1$	$a_2 = 0$	$b_3 = 1$	$a_3 = -1$	$b_3 = 1$

In six cases (those underlined) B is better than A. Hence, you decide that 6/9 of the time B is better than A.

You now compare B to C. The same listing of combinations shows that C is better than B 6/9 of the time. Clearly, C should be chosen in preference to B; we earlier found B is preferred to A.

At this point, you should stop if your goal as mayor is happiness. C is preferable to B by 2:1; B is preferable to A by 2:1. If you should be inquisitive, you might compare C to A. Unfortunately, you find that A is better than C 5/9 of the time!

Summary of the "Solution"

We can summarize the solution graphically. You are in a vicious, closed cycle: A is inferior to B, B to C, and C to A. Regardless of the choice you make, you could have done better. (See Figure 1.)

Above, we found B better than A, C better than B, A better than C, and so forth. Every step things seem better and we can go on forever improving our decision. If we were pessimists we could go around the circle in the reverse direction, with a worse decision each change.

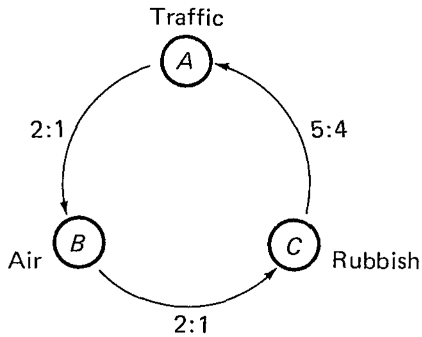

Figure 1.

Comment

A fascinating question immediately follows: Are there truly important social and political decision problems which have this disturbing feature that there is no best decision? What elements of the above example lead to this enigma?

The example is directly analogous to the nontransitive dice problem.[1] We have four dice (the normal, six-sided cubes) with the numbers on the faces as follows:

Die A:	2	3	3	9	10	11
Die B:	0	1	7	8	8	8
Die C:	5	5	6	6	6	6
Die D:	4	4	4	4	12	12

If you and I are playing a game, you select any one die. I'll then select a die which, when we roll against one another, will result in my winning 2/3 of the time (with 2:1 odds). Again, direct enumeration of the possible combinations shows A beats B, B beats C, C beats D, and D beats A. If you choose your die first, you are doomed.

We can find other illustrations of this nontransitive property in social problems. In politics, for example, a candidate may have to select one of three possible campaign strategies, with his opponent subsequently making his own choice. In an analogy with our original example we might have nine swing states, with the candidate who selects his strategy first doomed to lose a majority of these states if his opponent acts intelligently and selects a better strategy. For the first candidate, there is no optimum decision.

What elements of the problem lead to this enigma? First, we have a competitive situation in the sense of a two-person game (although as our first example of the mayor's decision illustrates, there need not be two distinct people or parties involved; rather, we are comparing two strategies). Second, the system is probabilistic: the result of a particular strategy can not be determined ahead of time. We can specify only what may happen (with estimates of the probability of each outcome).

If these are the only essential ingredients of decision problems which have *no* solution, we have reason to be alarmed as we try to apply scientific decision techniques to social problems, since most social problems seem to possess these characteristics. Perhaps to Forrester's emphasis on counterintuitive behavior, we should add the possibility of unsolvable systems.

REFERENCES

1. *Scientific American*, Vol. 233, No. 6, p. 110, Dec., 1970.