J. ENVIRONMENTAL SYSTEMS, Vol. 24(3) 275-298, 1995-96

A FUZZY LOGIC EVALUATION METHOD FOR ENVIRONMENTAL ASSESSMENT

P. N. SMITH
The University of Queensland, Australia

ABSTRACT

This article illustrates the potential of fuzzy logic and approximate reasoning in the context of environmental evaluation; that is to the evaluation of a set of development projects characterized along multiple environmental factors or dimensions. A simple example is given.

INTRODUCTION

There are many situations where a decision-maker is required to discriminate between a set of alternative projects characterized in terms of a common set of environmental impacts or factors. Such *evaluation* methods assist in environmental impact assessment in appraising the "aggregate worth" of projects in human and environmental terms [1]. Clearly, *evaluation* problems are sufficiently complex to require the use of formal methods. Determining the worth of complex alternatives varying on multiple dimensions presents formidable cognitive difficulties. Often task complexity is reduced by various heuristics. It has been observed that decision-makers ignore many significant factors in order to simplify the problem to a scale consistent with their cognitive limitations [2, 3]. Such simplification facilitates the discrimination and choice process, but clearly results in sub-optimal behavior.

Formal methods have been developed for impact evaluation purposes (for example, [4-7]). Recently, however, methods based on fuzzy sets and fuzzy logic have been proposed to assist the discrimination between projects characterized in terms of environmental factors. These methods explicitly acknowledge the uncertainty and imprecision common in environmental evaluation. Smith presented one such approach based on fuzzy numbers [8]. This article presents a method for

doi: 10.2190/YLNJ-XQ4J-NEYQ-EXFD
http://baywood.com

environmental evaluation involving fragments of imprecise information (conditional propositions, implications) where antecedents are environmental impacts or factors and the consequent is a measure of satisfaction associated with those factors. The method is based on aspects of fuzzy logic and approximate reasoning both of which are based on fuzzy sets.

FUZZY SETS

Formally a **fuzzy set A** in a set X (a collection of objects, universe of discourse or base set, denoted generically by x) is a set of ordered pairs $\mathbf{A} = \{(A(x), x|x \varepsilon X)$ where A(x) is called the **grade of membership** of x in **A** which maps X into a membership space, usually the [0,1] interval [9,10]. A simplified representation of a fuzzy set **A** when X is finite is $\mathbf{A} = \Sigma A(x)|x$ where the sigma notation indicates union rather than sum. Kaufmann [11] uses the term "fuzzy subset" rather than "fuzzy set" as the reference set, X, will not be fuzzy, though the terms *fuzzy set* and *fuzzy subset* are often used interchangeably [11]. For classical or **crisp** sets, the membership space is {0,1} consisting of only two possible degrees of membership, namely, complete membership (1) and complete non-membership (0).

A primary application of fuzzy subsets is in representing **linguistic variables**. Given a variable V, such as income, let X be the set of values that V can assume (universe of discourse). Often, only an imprecise value for V is available such as, for example, "low" income, or "about $25,000." For example, in a universe of discourse X = {$10000, $15000, $20000, $25000, $30000, $35000, $40000}, the linguistic variable "about $25000" may be represented by the fuzzy subset {0.1|10000, 0.5|15000, 0.8|20000, 1.0|25000, 0.8|30000, 0.5|35000, 0.1|40000}.

Certain **operations** may be carried out to aggregate fuzzy subsets [11]. If **A** and **B** are two fuzzy subsets defined on base set X, then we may define **C** as the **intersection (conjunction)** of **A** and **B**, or the largest fuzzy subset contained in both **A** and **B**. We write $\mathbf{C} = \mathbf{A} \cap \mathbf{B}$. The membership function of $x \varepsilon \mathbf{C}$ is given as $C(x) = A(x) \wedge B(x)$ where $a \wedge b = \min[a, b]$. The **union (disjunction)** of two fuzzy subsets **A** and **B** may be defined as the fuzzy subset $\mathbf{D} = \mathbf{A} \cup \mathbf{B}$ containing both **A** and **C**. The membership function of $x \varepsilon \mathbf{D}$ is given as $D(x) = A(x) \vee B(x)$ where $a \vee b = \max[a, b]$. The **complement** or **negation** of a fuzzy subset **A** (denoted $\mathbf{A}^c$ or $\neg\mathbf{A}$) is a set with membership values 1 - A(x).

The intersection operation above assumes that **A** and **B** are defined on the same base set. Given that X and Y are two base sets and let **A** be a fuzzy subset of X, then the **cylindrical extension** of **A** to X × Y (denoted $\hat{\mathbf{A}}$) is defined as a fuzzy subset of X × Y such that A(x,y) = A(x). If **B** is a fuzzy subset defined on Y, then the intersection of **A** and **B** is $\mathbf{C} = \hat{\mathbf{A}} \cap \hat{\mathbf{B}}$ and $C(x,y) = A(x) \wedge B(y)$. For example, if $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2\}$ and $\mathbf{A} = \{1.0|x_1, 0.3|x_2\}$, then $\hat{\mathbf{A}} = \{1.0|(x_1, y_1), 1.0|(x_1,y_2\}, 0.3|(x_2,y_1), 0.3|(x_2,y_2)\}$. If $\mathbf{B} = \{0.5|y_1, 0.9|y_2\}$, $\hat{\mathbf{B}} = \{0.5|(x_1,y_1), 0.9|(x_1,y_2\}, 0.5|(x_2,y_1), 0.9|(x_2,y_2)\}$ and $\mathbf{C} = \hat{\mathbf{A}} \cap \hat{\mathbf{B}} = \{0.5|(x_1,y_1), 0.9|(x_1,y_2\}, 0.3|(x_2,y_1), 0.3|(x_2,y_2)\}$.

A **fuzzy relation** is a fuzzy subset defined on the Cartesian product of base sets. For example, the fuzzy relation, **R**, defined on X × Y has membership function, R(x,y), representing the degree to which xεX, yεY belong to **R**. The cylindrical extension, $\hat{\mathbf{A}}$ above, is a relation defined on X × Y.

The fuzzy **implication** operation, '**If A then B**' or **A→B** is defined in a variety of ways [12, 13], for example, $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B} = \mathbf{R} = \hat{\mathbf{A}} \cap \hat{\mathbf{B}} = \mathbf{A} \times \mathbf{B}$, $R(x,y) = A(x) \wedge B(y)$ [14]; $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B} = \neg\hat{\mathbf{A}} \oplus \hat{\mathbf{B}} = \neg\mathbf{A} \times Y \oplus X \times \mathbf{B}$, where $\neg\hat{\mathbf{A}} = \neg\mathbf{A} \times Y$ and $\hat{\mathbf{B}} = X \times \mathbf{B}$, $\oplus$ denotes the bounded sum ($a \oplus b = 1 \wedge (a + b)$) and $R(x,y) = 1 \wedge (1 - A(x) + B(y))$; and $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B} = \neg\hat{\mathbf{A}} \cup \hat{\mathbf{B}} = (\neg\mathbf{A} \times Y) \cup (X \times \mathbf{B})$ and $R(x,y) = (1 - A(x)) \vee B(y)$.

The **point value** [15, 16] of a fuzzy subset $\mathbf{A} = \Sigma\{A(x)|x\}$ is given by

$$F(A) = (1/\alpha_{max}) \int_0^{\alpha_{max}} M(A_\alpha)\, d\alpha$$

where α_{max} is the maximum grade of membership of **A** and A_α is the alpha level set of **A**. An alpha level set is a crisp set $A_\alpha = \{x|A(x) \geq \alpha\}$. $M(A_\alpha)$ is the mean value of A_α. The point value "defuzzifies" the fuzzy subset **A**. For example, let X = {1, 2, 3} and let **A** = {1.0|1, 0.7|2, 0.1|3} be a fuzzy subset of X. Then for $0 < \alpha \leq 0.1$, $A_\alpha = \{1, 2, 3\}$, $M(A_\alpha) = 6/3 = 2$. For $0.1 < \alpha \leq 0.7$, $A_\alpha = \{1, 2\}$, $M(A_\alpha) = 3/2 = 1.5$ and for $0.7 < \alpha \leq 1.0$, $A_\alpha = \{1\}$ and $M(A_\alpha) = 1/1 = 1$. Then, since $\alpha_{max} = 1$,

$$F(A) = \int_0^{0.1} 2\, d\alpha + \int_{0.1}^{0.7} 3/2\, d\alpha + \int_{0.7}^{1.0} 1\, d\alpha$$

$$= 2(0.1) + (3/2)(0.6) + 1(0.3) = 1.4$$

Thus F(**A**) = 1.4 is the point value of fuzzy subset **A**.

FUZZY LOGIC AND APPROXIMATE REASONING

Fuzzy logic extends classical or two-valued logic relaxing the requirement for propositions to be absolutely "true" or absolutely "false" [17-19]. Truth values are expressed as the values of a linguistic variable "truth" which may assume linguistic values such as "true," "false," "not true," "very true," etc. The base set of the linguistic variable "truth" is the unit interval. Thus in classical logic, "truth" is single valued and unique whereas in fuzzy logic, "truth" is many-valued.

Classical two-valued logic (represented as $T_2 = \{0, 1\}$) can be extended to three-valued logic ($T_3 = \{0, 1/2, 1\}$) in various ways. Such logics denote truth and falsity as 1 and 0 and indeterminacy by 1/2. Generalizations of three-valued logics are n-valued logics. For any given $n \geq 2$, truth values are labeled by rational numbers in the unit interval [0,1] obtained by evenly dividing the interval. The set

$T_n = \{0, 1/(n-1), 2/(n-1), \ldots, (n-2)/(n-1), 1\}$ of truth values of an n-valued logic are interpreted as degrees of truth. For $n \geq 2$, the n-valued logic of Łukasiewicz [20] is denote $Ł_n$ ($n = 2,3, \ldots, \infty$). $Ł_2$ is two-valued logic and L_∞ is an infinite valued logic whose truth values are taken T_∞ from all rational numbers in the interval [0,1]. When truth values may be any real number in the [0,1] interval, the logic is referred to as standard Łukasiewicz logic and denoted $Ł_{\aleph_1}$ where $\aleph_1$ ('aleph 1') is used to represent the cardinality (number of elements) of the continuum. In this sense, the base logic for fuzzy logic is Łukasiewicz's $Ł_{\aleph_1}$ logic.

Fuzzy logic is the logic of **approximate reasoning** and bears the same relationship to approximate reasoning as does two-valued logic to precise reasoning. Fuzzy logic allows inferences even though the predicates that are supposed to be satisfied are only approximately satisfied. Approximate reasoning is the process of process or processes by which a possible imprecise conclusion is deduced from a collection of imprecise premises [18]. The constituents of approximate reasoning are a st of **translation rules** and a set of **rules of inference**. Translation rules consist of a set of procedures for forming composite propositions from basic (canonical) propositions "V is **A**" (also represented as "V = **A**") where V is a variable and **A** is a fuzzy subset of a base set X. Rules of inference are procedures for making logical deductions from fuzzy propositions. A commonly used approach to inference in approximate reasoning is compositional inference [21].

In traditional logic, one of the most important inference rules is **modus ponens**, that is

PREMISE	A is true
IMPLICATION	If A then B

CONCLUSION	B is true

Here A and B are crisply defined propositions. Fuzzy propositions may be constructed using fuzzy subsets. Introducing fuzzy propositions into modus ponens yields **generalized modus ponens**. Let **A, A*, B, B*** be fuzzy subsets. Then generalized modus ponens is

PREMISE	V is **A***
IMPLICATION	If V is **A** then U is **B**

CONCLUSION	U is **B***

In order to perform the above generalized modus ponens, inference is based on a fuzzy implication and a compositional rule of inference. A fuzzy implication or conditional proposition ("If V is **A** then U is **B**") is represented as **A→B**, where **A** is a fuzzy subset of X and B is a fuzzy subset of Y, and defined by a fuzzy relation **R**, a fuzzy subset of the Cartesian product X × Y. A fuzzy relation may be represented by a matrix. One of the most common forms of implication is based on the minimum operator, $R(x,y) = A(x) \wedge B(y)$, $x \varepsilon \mathbf{X}$, $y \varepsilon \mathbf{Y}$.

If **R** is a fuzzy relation from X to Y, and **A*** is a fuzzy subset of X and **B*** is a fuzzy subset of Y, then $\mathbf{B}^* = \mathbf{A}^* \circ \mathbf{R}$. In order to interpret the above expression, a compositional rule of inference is used. The most commonly used method is the max-min composition in which **B*** is computed by the max-min product of **A*** and **R.** The operation is similar to that of vector-matrix multiplication where multiplication is replaced by the min ($\wedge$) operator and addition is replaced by the max ($\vee$) operator $B^*(y) = \max_{x \in X} \min (A^*(x), R(x,y))$. Note that when $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B}$ is represented as $\mathbf{R} = \hat{\mathbf{A}} \cap \hat{\mathbf{B}} = \mathbf{A} \times \mathbf{B}$, $R(x,y) = A(x) \wedge B(y)$ and $\mathbf{A}^* = \mathbf{A}$, then $\mathbf{B}^* = \mathbf{A}^* \circ (\mathbf{A} \rightarrow \mathbf{B}) = \mathbf{B}$ as an exact identity. However, if other forms of implication are used (e.g., the arithmetic **rule,** $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B} = \neg\hat{\mathbf{A}} \oplus \hat{\mathbf{B}}$) then it is often the case that $\mathbf{B}^* = \mathbf{A}^* \circ (\mathbf{A} \rightarrow \mathbf{B}) \neq \mathbf{B}$; that is, the resultant fuzzy subset, **B***, is not exactly B [22].

Consider the simple example below. Let $X = \{x_1, x_2, x_3\} = \{1, 2, 3\}$, $Y = \{y_1, y_2, y_3\} = \{7, 8, 9\}$ and let fuzzy subsets be defined as **A** = **small** = $\{1.0|1, 0.6|2, 0.1|3\}$ and **B** = **large** = $\{0.1|7, 0.6|8, 1.0|9\}$. Then the implication "If V is **A** then U is **B**" or "If V is **small** then U is **large**" may be expressed as

$$\mathbf{R} = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ x_1 & 0.1 & 0.6 & 1.0 \\ x_2 & 0.5 & 1.0 & 1.0 \\ x_3 & 1.0 & 1.0 & 1.0 \end{array} \quad = \quad \begin{array}{c|ccc} & 7 & 8 & 9 \\ 1 & 0.1 & 0.6 & 1.0 \\ 2 & 0.5 & 1.0 & 1.0 \\ 3 & 1.0 & 1.0 & 1.0 \end{array}$$

where $\mathbf{R} = (\mathbf{A} \rightarrow \mathbf{B})$ is defined as $R(x,y) = 1 \wedge (1 - A(x) + B(y))$. Given a premise V is **very small** defined as **very small** = $\mathbf{A}^* = (\textbf{small})^2 = \{1.0|1, 0.36|2, 0.01|3\}$, then the conclusion is $\mathbf{B}^* = \{0.36|y_1, 0.6|y_2, 1.0|y_3\}$ where, for y_1, y_2, y_3

$$\vee\,[1.0 \wedge 0.1, 0.36 \wedge 0.5, 0.01 \wedge 1.0] = 0.36,$$
$$\vee\,[1.0 \wedge 0.6, 0.36 \wedge 1.0, 0.01 \wedge 1.0] = 0.6,$$
$$\vee\,[1.0 \wedge 1.0, 0.36 \wedge 1.0, 0.01 \wedge 1.0] = 1.0$$

respectively. Note that this is not identical to the fuzzy subset **very large** defined as **very large** = $(\textbf{large})^2 = \{0.01|7, 0.36|8, 1.0|9\}$.

A more general situation involving two antecedents is "If V_1 is $\mathbf{A_1}$ and V_2 is $\mathbf{A_2}$ then U is **B**" where $\mathbf{A_1}$, $\mathbf{A_2}$, and **B** are fuzzy subsets of X_1, X_2, and Y, respectively. Then $\mathbf{A} = \hat{\mathbf{A}}_1 \cap \hat{\mathbf{A}}_2$ is defined on $X = X_1 \times X_2$ and the implication $\mathbf{R} = \mathbf{A} \rightarrow \mathbf{B}$ is the fuzzy relation from X to Y. $\hat{\mathbf{A}}_1$, $\hat{\mathbf{A}}_2$ are the cylindrical extensions of $\mathbf{A_1}$, $\mathbf{A_2}$, respectively. Given V_1 is $\mathbf{A_1}^*$ and V_2 is $\mathbf{A_2}^*$, where $\mathbf{A_1}^*$ and $\mathbf{A_2}^*$ are fuzzy subsets of X_1 and X_2, respectively, then the conclusion is $\mathbf{B}^* = \mathbf{A}^* \circ \mathbf{R}$, where **B*** is a fuzzy subset of Y and $\mathbf{A}^* = \hat{\mathbf{A}}_1^* \cap \hat{\mathbf{A}}_2^*$. For example, let $X_1 = \{1, 2, 3\}$, $X_2 = \{4, 5, 6\}$, and $Y = \{7, 8, 9\}$ and let fuzzy subsets be defined as $\mathbf{A_1}$ = **small** = $\{1.0|1, 0.6|2, 0.1|3\}$, $\mathbf{A_2}$ = **large** = $\{0.1|4, 0.6|5, 1.0|6\}$, and **B** = **large** = $\{0.1|7, 0.6|8, 1.0|9\}$. Thus the implication is "If V_1 is **small** and V_2 is **large** then U is **large.**" $\hat{\mathbf{A}}_1$, $\hat{\mathbf{A}}_2$, $\mathbf{A} = \hat{\mathbf{A}}_1 \cap \hat{\mathbf{A}}_2$ and **R** are as follows

		4	5	6
	1	1.0	1.0	1.0
$\hat{A}_1$ =	2	0.6	0.6	0.6
	3	0.1	0.1	0.1

		4	5	6
	1	0.1	0.6	1.0
$\hat{A}_2$ =	2	0.1	0.6	1.0
	3	0.1	0.6	1.0

X_1	X_2	$\hat{A}_1$	$\hat{A}_2$	$A=\hat{A}_1 \cap \hat{A}_2$
1	4	1.0	0.1	0.1
2	4	0.6	0.1	0.1
3	4	0.1	0.1	0.1
1	5	1.0	0.6	0.6
2	5	0.6	0.6	0.6
3	5	0.1	0.6	0.1
1	6	1.0	1.0	1.0
2	6	0.6	1.0	0.6
3	6	0.1	1.0	0.1

			7	8	9
	1	4	1.0	1.0	1.0
	2	4	1.0	1.0	1.0
	3	4	1.0	1.0	1.0
	1	5	0.5	1.0	1.0
R =	2	5	0.5	1.0	1.0
	3	5	1.0	1.0	1.0
	1	6	0.1	0.6	1.0
	2	6	0.5	1.0	1.0
	3	6	1.0	1.0	1.0

Now assume that V is $\mathbf{A}_1$* = **very small** and V is $\mathbf{A}_2$* = **very large**, where **very small** = {1.0|1, 0.36|2, 0.01|3}, and **very large** = {0.01|4, 0.36|5, 1.0|6}. Then **A*** = $\hat{\mathbf{A}}_1^* \cap \hat{\mathbf{A}}_2^*$ is as follows

X_1	X_2	$\hat{A}_1$*	$\hat{A}_2$*	$A^*=\hat{A}_1^* \cap \hat{A}_2^*$
1	4	1.0	0.01	0.01
2	4	0.36	0.01	0.01
3	4	0.01	0.01	0.01
1	5	1.0	0.36	0.36
2	5	0.36	0.36	0.36
3	5	0.01	0.36	0.01
1	6	1.0	1.0	1.0
2	6	0.36	1.0	0.36
3	6	0.01	1.0	0.01

Then $\mathbf{B}^* = \mathbf{A}^* \circ \mathbf{R}$ = {0.3617, 0.618, 1.019}. Again, this is not identical to **very large** = {0.0117, 0.3618, 1.019}.

It is also possible to combine propositions disjunctively as, for example, in "If V_1 is $\mathbf{A_1}$ or V_2 is $\mathbf{A_2}$ then U is **B**" where $\mathbf{A_1}$, $\mathbf{A_2}$, and B are fuzzy subsets of X_1, X_2, and Y, respectively. Then $\mathbf{A} = \hat{\mathbf{A}}_1 \cup \hat{\mathbf{A}}_2$ is defined on $X = X_1 \times X_2$.

Weights may be placed on the propositions of generic form "V is **A**." Sanchez proposes that for conjunctions, $\mathbf{A}^w = (1\text{-}w) \vee \mathbf{A}$, $\mathbf{A}^w(x) = (1\text{-}w) \vee A(x)$, and for disjunctions $\mathbf{A}_w = w \wedge \mathbf{A}$, $A_w(x) = \mathbf{w} \wedge A(x)$ [23]. Thus for conjunctions, w = 0, $A^0(x) = 1$ and **A** is neutral and w = 1, $A^1(x) = A(x)$ and the weight has no effect. For disjunctions, w = 0, $A_0(x) = 0$ and **A** is neutral and w = 1, $A_1(x) = A(x)$ and the weight has no effect.

In general, m multiple implications or conditional propositions each with n antecedents, may be expressed as follows

If $V_1 = \mathbf{A_{11}}$ and $V_2 = \mathbf{A_{12}} \ldots V_n = \mathbf{A_{1n}}$ then $U = \mathbf{B_1}$ else
If $V_1 = \mathbf{A_{21}}$ and $V_2 = \mathbf{A_{22}} \ldots V_n = \mathbf{A_{2n}}$ then $U = \mathbf{B_2}$ else

.

.

.

If $V_1 = \mathbf{A_{m1}}$ and $V_2 = \mathbf{A_{m2}} \ldots V_n = \mathbf{A_{mn}}$ then $U = \mathbf{B_m}$

where $\mathbf{A_{i1}}$ is a fuzzy subset of X_1, $\mathbf{A_{i2}}$ is a fuzzy subset of X_2, etc., and $\mathbf{B_i}$ is a fuzzy subset of Y, and where "else" is interpreted as "and" or "or" [24, 25]. However, if Y and each X_j (j=1, . . . , n) have cardinality g, then $X = X_1 \times X_2 \times \ldots \times X_n$ has cardinality g^n and R has cardinality g^{n+1}. For example if Y and each base set X_j have 10 elements, then for n = 5, X has $g^n = 10^5$ elements and R defined on $X \times Y$ has $g^{n+1} = 10^6$ elements.

ENVIRONMENTAL EVALUATION BASED ON FUZZY LOGIC

One way of overcoming the dimensionality of the resulting set X has been suggested [25, 26]. Another approach is to assume a known set of projects $P = \{P_1, P_2, \ldots, P_I\}$ and to assume that each of the linguistic variables $V_1, V_2, \ldots, V_n$ have base set P; that is, $X_1 = X_2 = \ldots = X_n = P$ [16]. In this case, linguistic values, $\mathbf{A_{ij}}$, are evaluated by exemplification with P. Then d_i: If G is $\mathbf{A_i}$ then S is $\mathbf{B_i}$, is translated to a fuzzy relation $\mathbf{D_i}$ defined on $P \times Y$, since $\mathbf{A}_i = \mathbf{A_{i1}} \cap \mathbf{A_{12}} \cap \ldots \cap \mathbf{A_{in}}$ is defined on P.

Conditional propositions or implications (that is, fragments of imprecise information) consist of a measure of the satisfaction of some or all of the factors on which evaluation is to be based. Satisfaction is measured on a base set Y = [0.0, 0.1, . . . , 1.0]. The fragments of information, $d_1, d_2, \ldots, d_m$, may involve different sets of factors, Let,

d_i: If $G_1 = \mathbf{A_{i1}}$ and $G_2 = \mathbf{A_{i2}}$ and . . . and $G_n = \mathbf{A_{in}}$ then $S = \mathbf{B_i}$

where $\mathbf{A}_{i1}, \mathbf{A}_{i2}, \ldots, \mathbf{A}_{in}$ are fuzzy subsets of P, and $\mathbf{B}_i$ is a fuzzy subset of Y. The expression "$G_1 = \mathbf{A}_{i1}$ and $G_2 = \mathbf{A}_{i2}$ and . . . and $G_n = \mathbf{A}_{in}$" might be more simply represented $G = \mathbf{A}_i$ where $\mathbf{A}_i = \mathbf{A}_{i1} \cap \mathbf{A}_{i2} \cap \ldots \cap \mathbf{A}_{in}$ and $A_i(p) = A_{i1}(p) \wedge A_{i2}(p) \wedge \ldots \wedge A_{in}(p)$, $p \varepsilon P$. If a fragment, d_i, excludes a particular factor, G_k, then the fuzzy subset of the base set of projects, P, is $\mathbf{A}_{ik} = \{A_{ik}(P_1)|P_1, A_{ik}(P_2)|P_2, \ldots, A_{ik}(P_I)|P_I\} = \{1.0|P_1, 1.0|P_2, \ldots, 1.0|P_I\}$ which effectively means that G_k can be anything. Otherwise, if G_j is included in fragment d_i, $\mathbf{A}_{ij} = \mathbf{G}_j = \{G_j(P_1)|P_1, G_j(P_2)|P_2, \ldots, G_j(P_I)|P_I\}$. $\mathbf{G}_j$ is a fuzzy subset of projects with grades of membership $G_j(p)$ indicating the degree to which $p\varepsilon P$ achieves $\mathbf{G}_j$. Then, d_i: if $G = \mathbf{A}_i$, then $S = \mathbf{B}_i$ is a fuzzy implicational proposition. This is translated into a fuzzy subset (relation) $\mathbf{D}_i$ of $P \times Y$ where $D_i(p,y) = 1 \wedge (1 - A_i(p) + B_i(y))$. Thus for fragment d_i ($i=1, \ldots, m$), $\mathbf{D}_i$ is a fuzzy subset of $P \times Y$ and the overall evaluation function for fragments is given as $\mathbf{D} = \mathbf{D}_1 \cap \mathbf{D}_2 \cap \ldots \cap \mathbf{D}_m$ where $D(p,y) = \wedge_i D_i(p,y)$. To calculate the satisfaction associated with each project, the max-min rule of compositional inference, $\mathbf{H}_k = \mathbf{C}_k \text{ o } \mathbf{D}$ is applied, where $\mathbf{H}_k$ is the satisfaction associated with project P_k, $\mathbf{C}_k$ is the description of project P_k as a fuzzy subset of P, and $\mathbf{D}$ is the evaluation function. Thus, $H_k(y) = \max_{p\varepsilon P}[C_k(p) \wedge D(p,y)]$. In this case, $C_k(P_q) = 1$ for $k=q$ ($k, q=1, \ldots, I$) and $C_k(P_q) = 0$ for $k \neq q$. Thus, $H_k(y)$ and $D(P_k,y)$. The fuzzy subsets of the unit interval, $\mathbf{H}_k$, are then ranked according to their point value.

As an example, consider four transportation projects (alternative route alignments) assessed against six factors G_1 (travel-time savings), G_2 (social impact), G_3 (noise impact), G_4 (flora/fauna impact), G_5 (water quality impact), and, G_6 (capital cost). Let the fragments of information be as follows

d_1: If G_1 = **very high** and G_2 = **low** and G_3 = **low** and G_6 = **not low** then S = **fairly satisfactory**

d_2: If G_1 = **high** and G_2 = **low** and G_3 = **low** and G_4 = **low** and G_5 = **low** then S = **more than satisfactory**

d_3: If G_1 = **very high** and G_2 = **very low** and G_3 = **low** and G_4 = **low** and G_5 = **low** and G_6 = **low** then S = **perfect**

d_4: If G_1 = **high** and G_2 = **low** and G_3 = **low** and G_6 = **low** then S = **satisfactory**

d_5: If G_1 = **very high** and G_2 = **low** and G_3 = **low** and G_4 = **low** and G_6 = **low** then S = **very satisfactory**

d_6: If G_1 = **not high** and G_2 = **not low** then S = **fairly unsatisfactory**

Let Y = {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0} and let **satisfactory** be defined as $S(y) = y$, $y\varepsilon Y$. That is, fuzzy subset **S** = (0.0|0.0, 0.1|0.1, 0.2|0.2, 0.3|0.3, 0.4|0.4, 0.5|0.5, 0.6|0.6, 0.7|0.7, 0.8|0.8, 0.9|0.9, 1.0|1.0}. Then **fairly satisfactory** is defined as fuzzy subset $\mathbf{FS} = \mathbf{S}^{1/2}$ = {0.0|0.0, 0.32|0.1, 0.45|0.2, 0.55|0.3, 0.63|0.4, 0.71|0.5, 0.77|0.6, 0.84|0.7, 0.89|0.8, 0.95|0.9, 1.0|1.0}, **more than satisfactory** as fuzzy subset $\mathbf{MS} = \mathbf{S}^{3/2}$ = {0.0|0.0, 0.03|0.1, 0.09|0.2, 0.16|0.3, 0.25|0.4, 0.35|0.5, 0.46|0.6, 0.59|0.7, 0.72|0.8, 0.85|0.9, 1.0|1.0}, **very satisfactory** as fuzzy subset $\mathbf{VS} = \mathbf{S}^2$ = {0.0|0.0, 0.01|0.1, 0.04|0.2, 0.09|0.3, 0.16|0.4, 0.25|0.5, 0.36|0.6, 0.49|0.7, 0.64|0.8, 0.81|0.9, 1.0|1.0} and **unsatisfactory** as fuzzy subset **US** = 1-S = {1.0|0.0, 0.9|0.1, 0.8|0.2, 0.7|0.3, 0.6|0.4, 0.5|0.5, 0.4|0.6, 0.3|0.7, 0.2|0.8, 0.1|0.9, 0.0|1.0}. **Fairly unsatisfactory** is defined as fuzzy subset **FUS** = $\mathbf{US}^{1/2}$ = {1.0|0.0, 0.95|0.1, 0.89|0.2, 0.84|0.3, 0.77|0.4, 0.71|0.5, 0.63|0.6, 0.55|0.7, 0.45|0.8, 0.32|0.9, 0.0|1.0}. **Perfect** is defined as fuzzy subset **P** = {0.0|0.0, 0.0|0.1, 0.0|0.2, 0.0|0.3, 0.0|0.4, 0.0|0.5, 0.0|0.6, 0.0|0.7, 0.0|0.8, 0.0|0.9, 1.0|1.0}. These fuzzy subsets are illustrated in Figure 1 (note that the base set is {0, 1, . . . , 10} and not the interval [0,10]).

Let the factors be measured on the base set P = {P_1, P_2, P_3, P_4} of projects as follows

HTTS = {0.5|P_1, 0.6|P_2, 0.7|P_3, 1.0|P_4}
LSI = {0.7|P_1, 0.6|P_2, 0.3|P_3, 0.2|P_4}
LNI = {0.7|P_1, 0.7|P_2, 0.4|P_3, 0.3|P_4}
LFFI = {1.0|P_1, 0.5|P_2, 0.5|P_3, 0.3|P_4}
LWQI = {0.7|P_1, 0.4|P_2, 0.3|P_3, 0.1|P_4}
LCC = {0.0|P_1, 0.5|P_2, 0.6|P_3, 0.8|P_4}

where **HTTS = High Travel-Time Savings, LSI = Low Social Impact, LNI = Low Noise Impact, LFFI = Low Flora/Fauna Impact, LWQI = Low Water Quality Impact** and **LCC = Low Capital Cost**. The relative performance of the projects against different factors is illustrated by the polygonal profile plot in Figure 2. Thus P_1 is the most environmentally sensitive project and P_4 is the least environmentally sensitive project emphasizing engineering/economic factors. The calculation of the fuzzy subsets, $\mathbf{A}_i$, of projects associated with fragment d_i (i=1, . . . , 6) are shown in Table 1. $\mathbf{A}_i = \mathbf{A}_{i1} \cap \mathbf{A}_{i2} \cap \mathbf{A}_{i3} \cap \mathbf{A}_{i4} \cap \mathbf{A}_{i5} \cap \mathbf{A}_{i6}$ is given in the right column of Table 1.
Thus

$\mathbf{A}_1$ = {0.25|P_1, 0.36|P_2, 0.30|P_3, 0.20|P_4}
$\mathbf{A}_2$ = {0.50|P_1, 0.40|P_2, 0.30|P_3, 0.10|P_4}
$\mathbf{A}_3$ = {0.00|P_1, 0.36|P_2, 0.09|P_3, 0.04|P_4}
$\mathbf{A}_4$ = {0.00|P_1, 0.50|P_2, 0.30|P_3, 0.20|P_4}
$\mathbf{A}_5$ = {0.00|P_1, 0.36|P_2, 0.30|P_3, 0.20|P_4}
$\mathbf{A}_6$ = {0.03|P_1, 0.40|P_2, 0.30|P_3, 0.00|P_4}

The fragments are therefore:

d_1: If G = $\mathbf{A}_1$, then S = **FS**
d_2: If G = $\mathbf{A}_2$, then S = **MS**
d_3: If G = $\mathbf{A}_3$, then S = **P**
d_4: If G = $\mathbf{A}_4$, then S = **VS**
d_5: If G = $\mathbf{A}_5$, then S = **S**
d_6: If G = $\mathbf{A}_6$, then S = **US**

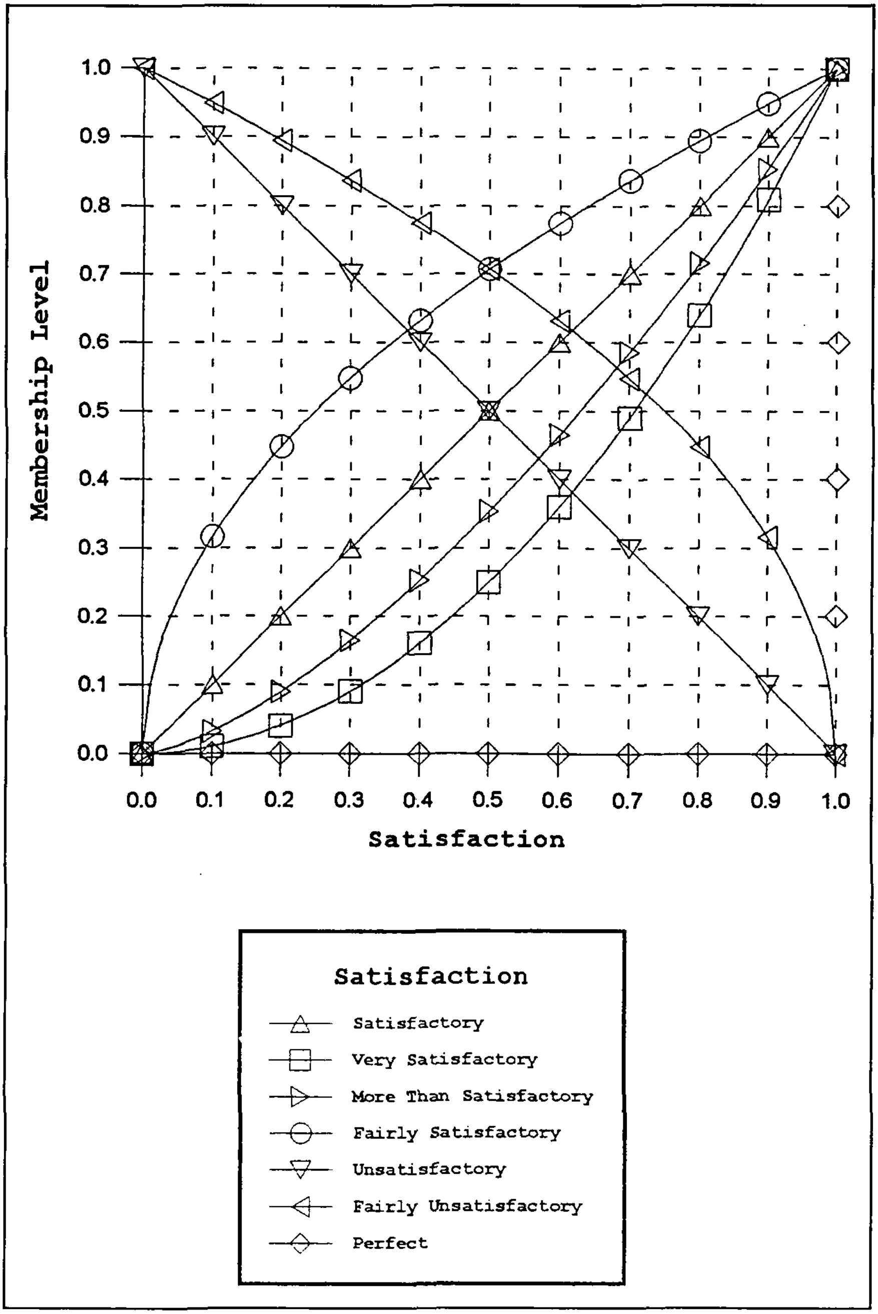

Figure 1. Levels of satisfaction.

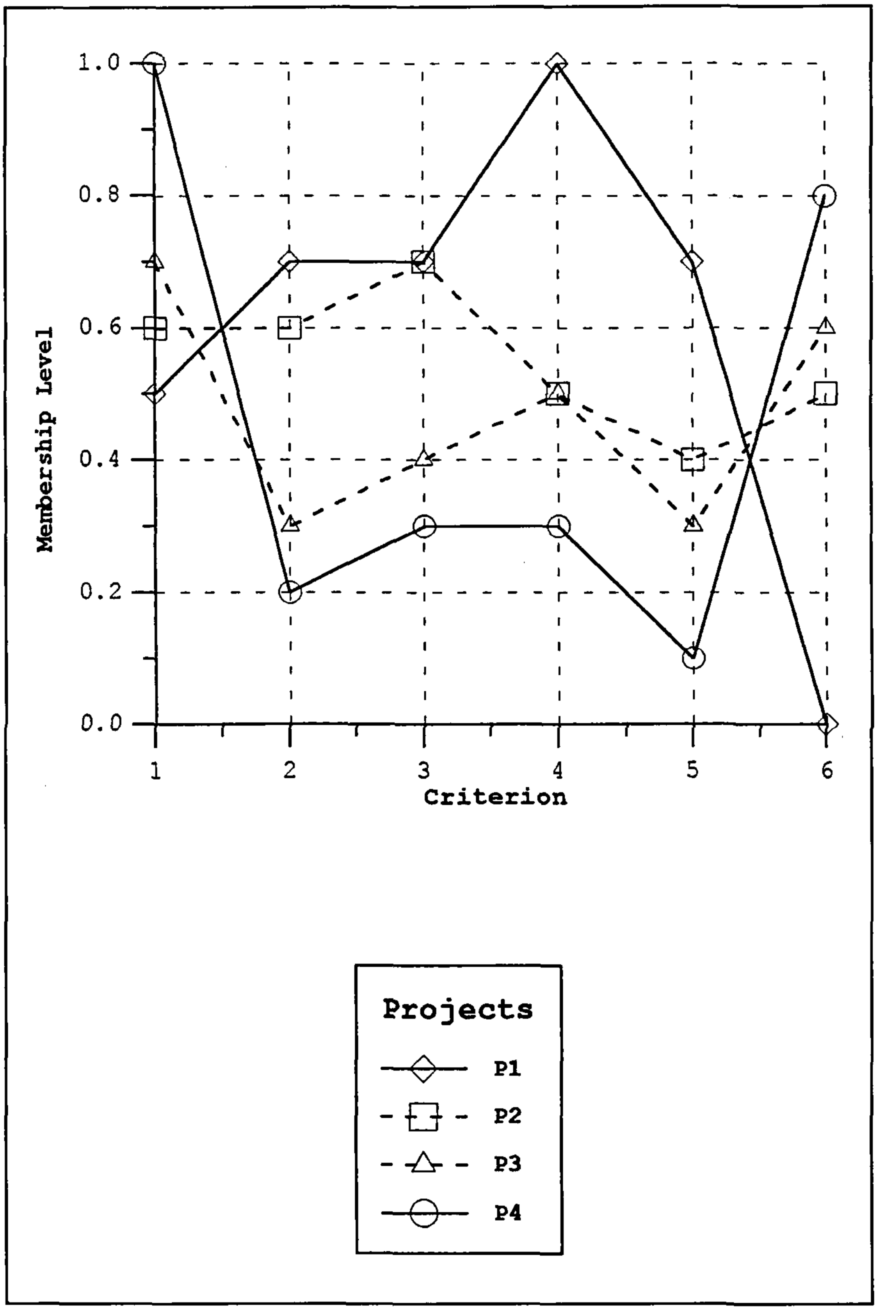

Figure 2. Profile plot of projects.

d_1	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_1
P_1	0.25	0.70	0.70	1.00	1.00	1.00	0.25
P_2	0.36	0.60	0.70	1.00	1.00	0.50	0.36
P_3	0.49	0.30	0.40	1.00	1.00	0.40	0.30
P_4	1.00	0.20	0.30	1.00	1.00	0.20	0.20
	G_1^2	G_2	G_3			G_6^c	

d_2	A_{21}	A_{22}	A_{23}	A_{24}	A_{25}	A_{26}	A_2
P_1	0.50	0.70	0.70	1.00	0.70	1.00	0.50
P_2	0.60	0.60	0.70	0.50	0.40	1.00	0.40
P_3	0.70	0.30	0.40	0.50	0.30	1.00	0.30
P_4	1.00	0.20	0.30	0.30	0.10	1.00	0.10
	G_1	G_2	G_3	G_4	G_5		

Table 1. [d_i for $i = 1, \ldots, 6$].

d_3	A_{31}	A_{32}	A_{33}	A_{34}	A_{35}	A_{36}	A_3
P_1	0.25	0.49	0.70	1.00	0.70	0.00	0.00
P_2	0.36	0.36	0.70	0.50	0.40	0.50	0.36
P_3	0.49	0.09	0.40	0.50	0.30	0.60	0.09
P_4	1.00	0.04	0.30	0.30	0.10	0.80	0.04
	G_1^2	G_2^2	G_3	G_4	G_5	G_6	

d_4	A_{41}	A_{42}	A_{43}	A_{44}	A_{45}	A_{46}	A_4
P_1	0.50	0.70	0.70	1.00	1.00	0.00	0.00
P_2	0.60	0.60	0.70	1.00	1.00	0.50	0.50
P_3	0.70	0.30	0.40	1.00	1.00	0.60	0.30
P_4	1.00	0.20	0.30	1.00	1.00	0.80	0.20
	G_1	G_2	G_3			G_6	

Table 1. (Cont'd.)

d_5	A_{51}	A_{52}	A_{53}	A_{54}	A_{55}	A_{56}	A_5
P_1	0.25	0.70	0.70	1.00	1.00	0.00	0.00
P_2	0.36	0.60	0.70	0.50	1.00	0.50	0.36
P_3	0.49	0.30	0.40	0.50	1.00	0.60	0.30
P_4	1.00	0.20	0.30	0.30	1.00	0.80	0.20
	G_1^2	G_2	G_3	G_4		G_6	

d_6	A_{61}	A_{62}	A_{63}	A_{64}	A_{65}	A_{66}	A_6
P_1	0.50	0.30	1.00	1.00	1.00	1.00	0.30
P_2	0.40	0.40	1.00	1.00	1.00	1.00	0.40
P_3	0.30	0.70	1.00	1.00	1.00	1.00	0.30
P_4	0.00	0.80	1.00	1.00	1.00	1.00	0.00
	G_1^c	G_2^c					

Table 1. (Cont'd.)

D_1	0.0	0.1	0.2	0.3	0.4	0.5
P_1	0.75	1.00	1.00	1.00	1.00	1.00
P_2	0.64	0.96	1.00	1.00	1.00	1.00
P_3	0.70	1.00	1.00	1.00	1.00	1.00
P_4	0.80	1.00	1.00	1.00	1.00	1.00

D_1(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	1.00	1.00	1.00	1.00	1.00
P_2	1.00	1.00	1.00	1.00	1.00
P_3	1.00	1.00	1.00	1.00	1.00
P_4	1.00	1.00	1.00	1.00	1.00

Table 2. $[D_i(p,y) = 1 \wedge (1 - A_i(p) + B_i(y))$ for fragments $i = 1, \ldots, 6]$.

D_2	0.0	0.1	0.2	0.3	0.4	0.5
P_1	0.50	0.53	0.59	0.66	0.75	0.85
P_2	0.60	0.63	0.69	0.76	0.85	0.95
P_3	0.70	0.73	0.79	0.86	0.95	1.00
P_4	0.90	0.93	0.99	1.00	1.00	1.00

D_2 (Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	0.96	1.0	1.0	1.0	1.0
P_2	1.0	1.0	1.0	1.0	1.0
P_3	1.0	1.0	1.0	1.0	1.0
P_4	1.0	1.0	1.0	1.0	1.0

Table 2. (Cont'd.)

D_3	0.0	0.1	0.2	0.3	0.4	0.5
P_1	1.00	1.00	1.00	1.00	1.00	1.00
P_2	0.64	0.64	0.64	0.64	0.64	0.96
P_3	0.91	0.91	0.91	0.91	0.91	0.91
P_4	0.96	0.96	0.96	0.96	0.96	0.96

D_1(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	1.00	1.00	1.00	1.00	1.00
P_2	0.64	0.64	0.64	0.64	1.00
P_3	0.91	0.91	0.91	0.91	1.00
P_4	0.96	0.96	0.96	0.96	1.00

Table 2. (Cont'd.)

D_4	0.0	0.1	0.2	0.3	0.4	0.5
P_1	1.00	1.00	1.00	1.00	1.00	1.00
P_2	0.50	0.60	0.70	0.80	0.90	1.00
P_3	0.70	0.80	0.90	1.00	1.00	1.00
P_4	0.80	0.90	1.00	1.00	1.00	1.00

D_4(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	1.00	1.00	1.00	1.00	1.00
P_2	1.00	1.00	1.00	1.00	1.00
P_3	1.00	1.00	1.00	1.00	1.00
P_4	1.00	1.00	1.00	1.00	1.00

Table 2. (Cont'd.)

D_5	0.0	0.1	0.2	0.3	0.4	0.5
P_1	1.00	1.00	1.00	1.00	1.00	1.00
P_2	0.64	0.65	0.68	0.73	0.80	0.89
P_3	0.70	0.71	0.74	0.79	0.86	0.95
P_4	0.80	0.81	0.84	0.89	0.96	1.00

D_5(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	1.00	1.00	1.00	1.00	1.00
P_2	1.00	1.00	1.00	1.00	1.00
P_3	1.00	1.00	1.00	1.00	1.00
P_4	1.00	1.00	1.00	1.00	1.00

Table 2. (Cont'd.)

D_6	0.0	0.1	0.2	0.3	0.4	0.5
P_1	1.00	1.00	1.00	1.00	1.00	1.00
P_2	1.00	1.00	1.00	1.00	1.00	1.00
P_3	1.00	1.00	1.00	1.00	1.00	1.00
P_4	1.00	1.00	1.00	1.00	1.00	1.00

D_6(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	1.00	1.00	1.00	1.00	0.70
P_2	1.00	1.00	1.00	0.92	0.60
P_3	1.00	1.00	1.00	1.00	0.70
P_4	1.00	1.00	1.00	1.00	1.00

Table 2. (Cont'd.)

D	0.0	0.1	0.2	0.3	0.4	0.5
P_1	0.50	0.53	0.59	0.66	0.75	0.85
P_2	0.50	0.60	0.64	0.64	0.64	0.64
P_3	0.70	0.71	0.74	0.79	0.86	0.91
P_4	0.80	0.81	0.84	0.89	0.96	0.96

D(Cont'd)	0.6	0.7	0.8	0.9	1.0
P_1	0.96	1.00	1.00	1.00	0.70
P_2	0.64	0.64	0.64	0.64	0.60
P_3	0.91	0.91	0.91	0.91	0.70
P_4	0.96	0.96	0.96	0.96	1.00

Table 3. $[D = D_1 \cap D_2 \cap D_3 \cap D_4 \cap D_5 \cap D_6]$.

Then the fuzzy subsets of $P \times Y$, $\mathbf{D}_i$, $i=1, \ldots, 6$ are shown in Table 2. The final decision is $\mathbf{D} = \mathbf{D}_1 \cap \mathbf{D}_2 \cap \mathbf{D}_3 \cap \mathbf{D}_4 \cap \mathbf{D}_5 \cap \mathbf{D}_6$ where $D(p,y) = \wedge_i D_i(p,y)$. These calculations are shown in Table 3. Note that $\mathbf{C}_k = \{C_k(P_1)|P_1, C_k(P_2)|P_2, C_k(P_3)|P_3, C_k(P_4)|P_4$. Thus, if $k=1$, then $C_1 = \{C_1(P_1)|P_1, C_1(P_2)|P_2, C_1(P_3)|P_3, C_1(P_4)|P_4 = \{1|P_1, 0|P_2, 0|P_3, 0|P_4, 0|P_5\}$. Calculating the point values for $\mathbf{H}_k$, $k=1, \ldots, 4$, where $\mathbf{H}_k = \mathbf{C}_k \text{ o } D$, yields $\mathbf{F}(\mathbf{H}_1) = 0.59$, $\mathbf{F}(\mathbf{H}_2) = 0.51$, $\mathbf{F}(\mathbf{H}_3) = 0.53$, and $\mathbf{F}(\mathbf{H}_4) = 0.55$. Thus P_1 is the "best" project.

The above approach has assumed that the weights of factors in the fragments are equal. When differential weights are introduced, the "aggregate worth" of projects change. For example, consider the weights $W^{eng/econ} = \{w_1, w_2, w_3, w_4, w_5, w_6\} = \{1.0, 0.1, 0.1, 0.1, 0.1, 1.0\}$ emphasizing non-environmental or engineering/ economic factors (travel-time savings, capital cost) at the expense of environmental criteria and $W^{env} = \{w_1, w_2, w_3, w_4, w_5, w_6\} = \{0.1, 1.0, 1.0, 1.0, 1.0, 0.1\}$ emphasizing environmental factors at the expense of engineering/economic factors. In the former case, $F(\mathbf{H}_1) = 0.57$, $F(\mathbf{H}_2) = 0.55$, $F(\mathbf{H}_3) = 0.68$, and $F(\mathbf{H}_4) = 0.91$; that is P_4, the least environmentally sensitive project is "best." In the latter case, $F(\mathbf{H}_1) = 0.69$, $F(\mathbf{H}_2) = 0.54$, $F(\mathbf{H}_3) = 0.48$, and $F(\mathbf{H}_4) = 0.43$; that is, P_1, the most environmentally sensitive project is "best." Though, the above weights sets are extreme, they do illustrate the potential for differentially weighting antecedents. In practice, more realistic weight sets could be evolved.

CONCLUSION

An application of a fuzzy logic method for the evaluation of projects characterized in terms of multiple environmental factors has been given. The method facilitates the incorporation of fragments of imprecise information (implications) involving some or all of the factors as antecedents and a level of satisfaction as a consequent.

REFERENCES

1. L. W. Canter, *Environmental Impact Assessment*, McGraw-Hill, New York, 1977.
2. J. K. Ford, N. Schmitt, S. L. Schechtman, B. Hults, and M. L. Dohrty, Process Tracing Methods: Contributions, Problems and Neglected Research Questions, *Organisational Behaviour and Human Decision Processes, 43*, pp. 75-117, 1989.
3. D. Timmermans, The Impact of Task Complexity on Information Use in Multiattribute Decision Making, *Journal of Behavioural Decision Making, 6*, pp. 95-111, 1993.
4. N. Dee, J. Baker, N. Drobny, K. Duke, I. Whitman, and D. Fahringer, An EES for Water Resources Planning, *Water Resources Research, 9*, pp. 523-535, 1973.
5. H. Odum, *Optimum Pathway Matrix Analysis Approach to the Environmental Decision Making Process. Testcase: Relative Impact of Proposed Highway Alternatives*, Institute of Ecology, University of Georgia, 1971.

6. E. P. Odum, G. A. Bramlett, H. Ike, J. R. Champlin, J. C. Zieman, and H. H. Shugart, Totality Indexes for Evaluating Environmental Impacts of Highway Alternatives, *Transportation Research Record, 561*, pp. 57-67, 1976.
7. M. Prasartee, A Conceptual Development of Quantitative Environmental Impact Assessment Methodology for Decision-Makers, *Journal of Environmental Management, 14*, pp. 301-307, 1982.
8. P. N. Smith, Environmental Evaluation: Fuzzy Impact Aggregation, *Journal of Environmental Systems, 24*, pp. 165-178, 1995-96.
9. L. A. Zadeh, Fuzzy Sets, *Information and Control, 8*, pp. 338-353, 1965.
10. A. Kandel, *Fuzzy Mathematical Techniques with Applications*, Addison-Wesley, Reading, Massachusetts, 1986.
11. A. Kaufmann, *Introduction to Fuzzy Subsets, Vol. 1*, Academic Press, New York, 1975.
12. R. R. Yager, On the Implication Operator in Fuzzy Logic, *Information Sciences, 31*, pp. 141-164, 1980.
13. R. R. Yager, An Approach to Inference in Approximate Reasoning, *International Journal of Man-Machine Studies, 13*, pp. 323-338, 1980.
14. E. H. Mamdani, Advances in the Linguistic Synthesis of Fuzzy Logic Controllers, *International Journal of Man-Machine Studies, 8*, pp. 669-678, 1976.
15. R. R. Yager, A Procedure for Ordering Fuzzy Subsets of the Unit Interval, *Information Sciences, 24*, pp. 143-161, 1981.
16. R. R. Yager, Multicriteria Decisions with Soft Information: An Application of Fuzzy Set and Possibility Theory (I and II), *Fuzzy Mathematics, 3*, pp. 7-16, 21-28, 1982.
17. L. A. Zadeh, Fuzzy Logic and Approximate Reasoning, *Synthese, 30*, pp. 407-428, 1975
18. L. A. Zadeh, A Theory of Approximate Reasoning, in *Machine Intelligence, 9*, J. Hayes, D. Michie, and L. I. Mikulich (eds.), Halstead Press, New York, 1979.
19. R. E. Bellman and L. A. Zadeh, Local and Fuzzy Logics, in *Modern Uses of Multiple-Valued Logic*, J. M. Dunn and G. Epstein (eds.), Reidel, Dordrecht, Holland, pp. 105-165, 1977.
20. G. J. Klir and T. A. Folger, *Fuzzy Sets, Uncertainty, and Information*, Prentice-Hall, Englewood Cliffs, 1988.
21. L. A. Zadeh, Outline of a New Approach to the Analysis of Complex Systems and Decision Processes, *IEEE Transactions on Systems, Man and Cybernetics*, SMC-3, pp. 28-44, 1973.
22. M. Mizumoto and H.-J. Zimmerman, Comparison of Fuzzy Reasoning Methods, *Fuzzy Sets and Systems, 8*, pp. 253-283, 1982.
23. E. Sanchez, Importance in Knowledge Systems, *Information Systems, 14*, pp. 455-464, 1989.
24. R. R. Yager, Approximate Reasoning and Possibilistic Models in Classification, *International Journal of Computer and Information Sciences, 10*, pp. 141-175, 1981.
25. C. C. Lee, Fuzzy Logic Control Systems: Fuzzy Logic Controller, Part I, Part II, *IEEE Transactions on Systems, Man and Cybernetics, 20*, pp. 404-435, 1990.

26. J. F. Baldwin and B. W. Pilsworth, A Model of Fuzzy Reasoning through Multi-valued Logic and Set Theory, *International Journal of Man-Machine Studies, 11*, pp. 351-380, 1979.

Direct reprint requests to:

P. N. Smith
Department of Geographical Sciences and Planning
The University of Queensland
St. Lucia, Queensland
Australia 4072