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NUMERIC ORDERED WEIGHTED AVERAGING OPERATORS: POSSIBILITIES FOR ENVIRONMENTAL PROJECT EVALUATION

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ABSTRACT

This article outlines aspects of ordered weighted averaging (OWA) aggregation operators in the evaluation of alternative projects with environmental consequences. OWA operators generalize the conventional maximum and minimum aggregation operators commonly use to aggregate fuzzy subsets, here representing the degree of "satisfaction" of factors/impacts by a set of discrete projects. A simple example drawn from Horsak and Damico is given which involves the location of a hazardous waste disposal facility at one of three sites based on ten factors [1]. OWA operators are considered in the context of the aggregation of factors/impacts and the importance weight of those factors/impacts. Consideration is given to maximum entropy OWA (ME-OWA), exponential OWA (E-OWA), and weighted ordered weighted averaging (WOWA) operators, in addition to quantified statements implemented by OWA operators. OWA aggregation operators are considered in the context of the above illustrative example. It is concluded that OWA operators have considerable potential in providing a framework for the aggregation of fuzzy subsets in the evaluation of projects with environmental consequences.

INTRODUCTION

Fuzzy sets have emerged as a new means of representing uncertainty, in particular in expressing the ambiguity of meaning found in natural language—for example, in the definition of a concept or the meaning of a word. Expressions such as "high temperature," "old man," "tall building," "satisfactory project" [2] are ambiguous in nature. A fuzzy set (or more precisely, fuzzy subset [3]) may be represented as a set of ordered pairs, $\mathbf{A} = \{(A(x)|x), x\epsilon X, A(x)\epsilon[0,1]\}$ where x is a generic element of universe X. A(x) is called the *membership value* or *grade of membership* [4, 5]. A fuzzy set is clearly a multivalent generalization of a crisp set whose membership function takes on bivalent values, {0, 1}. Much of human reasoning involves the use of linguistic variables whose values are defined by fuzzy subsets. Thus, a linguistic variable is a variable whose values (linguistic labels) are words rather than numbers with the words defined by fuzzy subsets.

EVALUATION OF PROJECTS IN A FUZZY ENVIRONMENT

Recently, evaluation methods involving fuzzy subsets have been proposed which more adequately acknowledge the uncertainty and imprecision characteristic of project evaluation [6, 7]. In this context, imprecision is of a non-random (deterministic) or ambiguous nature rather than of a random or statistical nature [8]. In project evaluation, a useful expression of such deterministic uncertainty is in terms of linguistic variables which are labels for fuzzy subsets.

The basic structure for the environmental evaluation of projects with multiple environmental consequences is an *outcome matrix,* $\phi = [\phi_{ij}]$ denotes the outcome of project P_i with respect to factor/impact F_j. $P = \{P_1, P_2, ..., P_I\}$ is a set of I mutually exclusive projects and $F = \{F_1, F_2, ..., F_J\}$ is a set consisting of J factors/impacts. Commonly in the evaluation of projects, weights $w = \{w_1, w_2, ..., w_{J,}\}$ are introduced to represent the differential importance of factors/impacts. In terms of fuzzy set theory, $\mathbf{F}_j$ may be construed as a fuzzy subset of the set of projects represented as $\mathbf{F}_j = \{F_j(P_1)|P_1, F_j(P_2)|P_2, ..., F_j(P_I)|P_I\}$, where $F_j(p)$ indicates the degree to which project $p\epsilon P$ satisfies factor/impact F_j. Note that $\phi_{ij} = F_j(P_i)$ and that the outcome of a given project (denoted either as p or P_i) is represented as, $\{F_1(p), F_2(p), ..., F_J(p)\}$, $p\epsilon P$ or as $\{\phi_{i1}\ \phi_{i2}, ..., \phi_{iJ}\}$, $P_i\epsilon P$.

	F_1	F_2	...	F_J
P_1	Φ_{11}	Φ_{12}	...	Φ_{1J}
P_2	Φ_{21}	Φ_{22}	...	Φ_{2J}
⋮	⋮	⋮	...	⋮
P_I	Φ_{I1}	Φ_{I2}	...	Φ_{IJ}

Project evaluation typically involves the identification of a "best" project which satisfies as much as possible each factor/impact. Here "satisfies" implies lower values of negative factors/impacts (e.g., cost, wildlife impact) and higher values of positive factors/impacts (e.g., accident reduction, aesthetics). Rarely will any real project completely satisfy all factors/impacts and will be characterized by variable achievement across factors/impacts. For brevity, the term "factor" will be used, where possible, to include also impacts.

An important issue in the context of discrete fuzzy subsets is the determination of membership grades, which may be derived in a number of ways, mostly involving subjective judgment [9-12]. If objective data is available, then some appropriate transformation may be utilized to normalize the outcomes of a given factor to the (0, 1) interval [13]. Such transformed values are interpreted as membership grades. Weights reflecting the importance of factors may be generated by various means, e.g., [11, 12].

EXAMPLE OF ENVIRONMENTAL PROJECT EVALUATION

Consider an example adapted from Horsak and Damico [1] (also considered by Anandalingam and Westfall [14]) involving the location of a hazardous waste disposal facility with three possible sites assessed against ten factors—*air quality* (dispersive capabilities of site/plant and degree to which waste emissions could concentrate onsite and offsite, F_1), *surface water quality* (potential for surface water degradation due to spills associated with handling storage and waste, F_2), *groundwater quality* (potential for groundwater degradation due to spills associated with handling and storage of waste, including leaching into acquifer, F_3), *impact on ecology* (potential impact on ecological resources of an area due to routine operations or emergency conditions, F_4), *impact on aesthetics* (visual impacts of hazardous waste management operations, including handling, storage, and disposal, F_5), *impact on population* (potential long-term exposure to emissions due to routine operations or emergencies, F_6), *impact on surrounding land use* (compatibility of surrounding land use with the hazardous waste operation, F_7), *possibility of emergency response* (ability of a response team to combat an emergency associated with a spill or other exposure, F_8), *distance from sources of waste* (distance through which the waste should travel to get to the site, F_9), and *political opposition* (political or other organized intervention or opposition to the hazardous waste operation, F_{10}). Factors are fuzzy subsets of the projects (sites), for example, $\mathbf{F}_1 = \{0.9|P_1, 0.7|P_2, 0.3|P_3\}$ for *air quality* (F_1). The matrix, $\phi = [\phi_{ij}]$, is given as follows

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
P_1	0.9	0.8	1.0	0.9	0.8	1.0	0.8	0.8	1.0	0.5
P_2	0.7	0.9	1.0	0.9	0.9	0.5	0.6	0.5	0.6	1.0
P_3	0.3	0.2	1.0	0.2	1.0	1.0	0.2	0.2	0.3	0.3

Note that F_3 (*groundwater quality*) could be excluded as it fails to discriminate between sites, though it is retained here. It is clear from the polygonal profile plot in Figure 1 that site 1 (P_1) is a strong competitor for the overall "best" site.

Further assume factor weights (based on [13]) as follows w = {1, 0.969, 0.919, 0.714, 0.689, 0.658, 0.460, 0.323, 0.286, 0.193}, or in normalized form, w^0 = {0.16, 0.156, 0.148, 0.115, 0.111, 0.106, 0.074, 0.052, 0.046, 0.031} such that $0 \leq w_j^0 \leq 1$ and $\Sigma_{j=1,J} w_j^0 = 1$. Horsak and Damico [1] used *weighted conjunctive aggregation* to select a "best" site and identified a preference order, $P_1 \succ P_2 \succ P_3$.

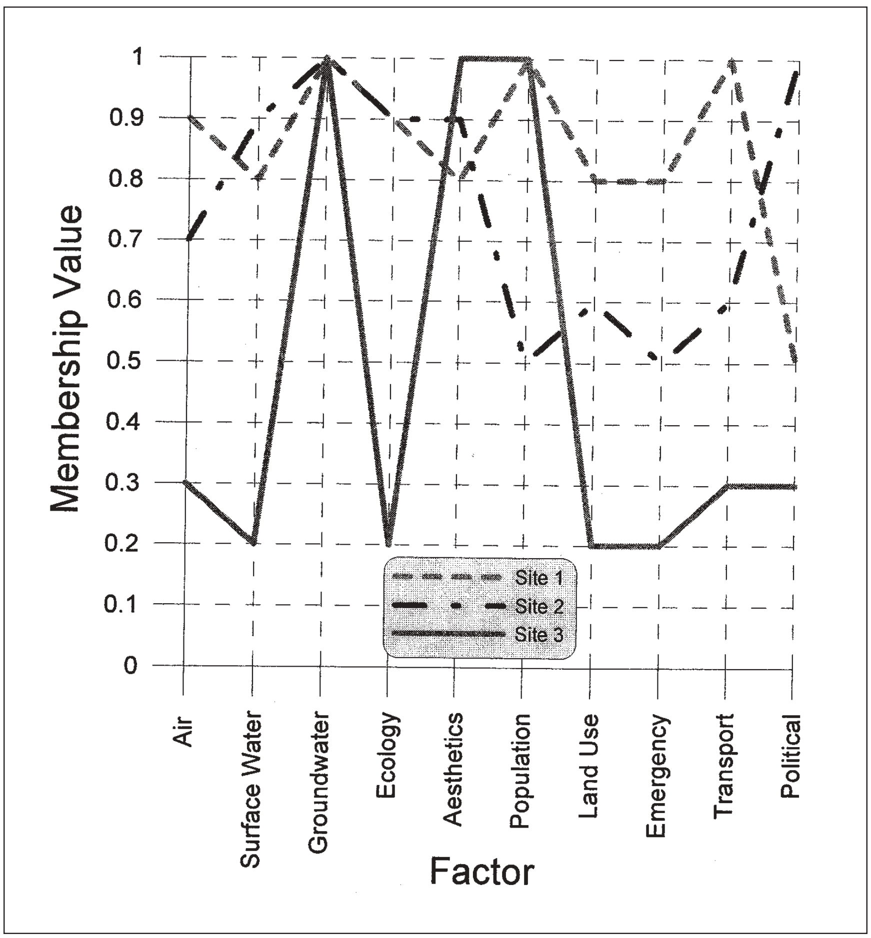

Figure 1. Polygonal profile plot of sites.

ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

The *ordered weighted averaging* (OWA) operator for aggregating fuzzy subsets was introduced by Yager [15]. An OWA operator (of dimension J) is represented as

$$OWA = \Sigma_{j=1,J}\alpha_j b_j$$

where b_j is the jth largest element of the outcomes $\{F_1(p), F_2(p), ..., F_J(p)\}$ for project p. Thus, $b_1 \geq b_2 \geq ... \geq b_J$. OWA operator weights, $\{\alpha_1, \alpha_2, ..., \alpha_J\}$, are associated with the position of b_j and are such that $\alpha_j \in [0,1]$ and $\Sigma_{j=1,J}\alpha_j = 1$. Note that α_j is associated with a particular ordered position j of the arguments (outcomes of project p along factors) and is not a reflection of the importance (salience, significance) of factor F_j in the context of project evaluation. Consider, for example, outcomes (membership grades) {0.3, 0.7, 1} and weights {0.4, 0.5, 0.1}. Then $b_1 = 1$, $b_2 = 0.7$, $b_3 = 0.3$, and the OWA is given as OWA = (0.4)(1) + (0.5)(0.7) + (0.1)(0.3) = 0.78.

It can be shown that the OWA operator includes the commonly used *maximum* and *minimum* operators [15] and the *arithmetic mean* operator for appropriate choice of operator weights represented as $\alpha = \{\alpha_1, \alpha_2 ..., \alpha_J\}$. In particular, the OWA operator is bounded such that, $OWA_{\star} \leq OWA \leq OWA^*$, where the weights $\alpha = \{0, 0, ..., 1\}$ are used in $OWA_{\star}$ and the weights $\alpha = \{1, 0, ..., 0\}$ are used in $OWA_{\star}$. Thus, from the definition of the OWA operator, $OWA_{\star} = \wedge_{j=1,J}F_j(p)$ (minimum operator) and $OWA^* = \vee_{j=1,J}F_j(p)$ (maximum operator) so that extreme OWA operators are the "and" and "or" operators [15]. The arithmetic average corresponds to the OWA operator with weights {1/J, 1/J, ..., 1/J}. The "and" (minimum) provides no compensation in that a high grade of membership with respect to one factor cannot offset (or compensate for) a low grade of membership with respect to another factor. The "or" (maximum) provides full compensation.

With respect to OWA operators, the *orness* of the OWA operator weights, α, is given as $\text{orness}(\alpha) = \Sigma_{j=1,J}\alpha_j(J - j)/(J - 1)$. "Orness" measures the degree to which an aggregation operator is "orlike" or "andlike" and provides some indication of the inclination of the operator to impart more weight to either higher or lower membership grades. Thus, the greater the "orness," the more weight imparted to higher membership grades. It can be shown that $\alpha = \{0, 0, ..., 1\}$ yields $\text{orness}(\alpha) = 0$ (minimum) and that $\alpha = \{1, 0, ..., 0\}$ yields $\text{orness}(\alpha) = 1$ (maximum) [15]. The degree of "andness" of an OWA operator with weights α is defined as $\text{andness}(\alpha) = \neg\text{orness}(\alpha) = 1 - \text{orness}(\alpha)$. Except for the extreme weight sets above, different OWA weight sets can yield the same level of "orness." In particular, all symmetric weight sets yield $\text{orness}(\alpha) = 0.5$. Thus, the "dispersion" given by the *entropy* function, $\text{entropy}(\alpha) = -\Sigma_{j=1,J}\alpha_j \text{Ln}\alpha_j$, further distinguishes between the weight sets

[15]. Entropy is a maximum when all weights are equal to 1/J and a minimum value when one weight is equal to and all others zero.

As has been indicated above, factors are commonly assumed to vary in importance or salience and as a consequence, factor importance weights, $w = \{w_1, w_2, ..., w_J\}$, are introduced. One possibility for including importance is to assume an OWA operator for project p involving transformed membership values, $H(F_j(p),w_j,orness(\alpha))$, the "effective satisfaction" of factor F_j. b_j is now the jth largest element of $H(F_j(p),w_j,orness(\alpha))$ (j = 1, ..., J). One possible function based on Yager [15] is

$$H(F_j(p),w_j,orness(\alpha)) = w_j \vee \neg orness(\alpha)) \wedge (F_j(p))^{(w_j \vee orness(\alpha))}$$

Note that factor importance weights should be such that $\vee_{j=1,J} w_j = 1$. This function reduces to two special cases when orness(α) = 0 and orness(α) = 1. In the former case, $H(F_j(p),w_j,0) = F_j(p)^{w_j}$, used in *weighted conjunctive aggregation,* $D(p) = \wedge_{j=1,J} F_j(p)^{w_j}$ [16], whilst in the latter case, $H(F_j(p),w_j,1) = w_j \wedge F_j(p)$ used in *weighted disjunctive aggregation,* $D(p) = \vee_{j=1,J}(w_j \wedge F_j(p))$ [17]. Aggregation of factors in the example of Horsak and Damico [1] involved weighted conjunctive aggregation, though weights were normalized such that $\Sigma_{j=1,J}\alpha_j = J$. However, an equivalent preference ordering of sites will be obtained by normalization that the maximum weight be unity.

An alternative modification function proposed here as a smoother function of "orness" is

$$G(F_j(p),w_j,orness(\alpha)) = (w_j + \neg orness(\alpha) - w_j*(\neg orness(\alpha)))* (F_j(p))^{(w_j + orness(\alpha) - w_j*orness(\alpha))}$$

which involves the *algebraic sum* (a + b – a * b) instead of the *logical sum* ($a \vee b$) and *algebraic product* (a*b) instead of the *logical product* ($a \wedge b$). Again, when orness(α) = 0, $G(F_j(p),w_j,orness(\alpha)) = F_j(p)^{w_j}$ and when orness(α) = 1, $G(F_j(p),w_j,orness(\alpha)) = w_jF_j(p)$, used in *weighted disjunctive aggregation,* $D(p) = \vee_{j=1,J}(w_jF_j(p))$. This modification function is illustrated in Figure 2 for a variety of importance weights (w) and membership values (m) as a function of "orness." However, neither of these functions recover the arithmetic average when orness(α) = 0.5. Clearly, further investigation is necessary to identify a function with more desirable properties.

SELECTING OWA OPERATOR WEIGHTS

An important issue in the use of OWA operators is the selection of the OWA operator weights, α. One approach recently put forward involves the *maximum entropy* OWA (ME-OWA) operator [see 18]. Given a desired level of "orness,"

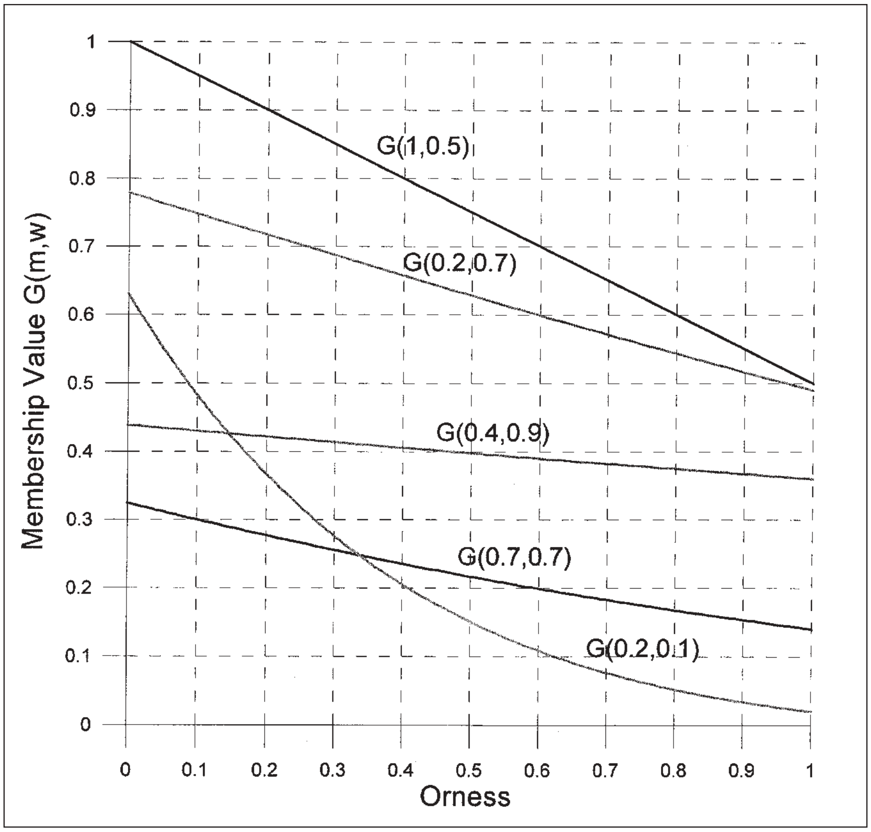

Figure 2. Modification of outcomes.

say orness(α) = ζ, OWA operator weights, α, are identified as the solution to the mathematical programming problem which involves maximizing the entropy (*evenness* of OWA operator weights) subject to constraints (i) that orness(α) = ζ, the desired value, and (ii) that the normalization condition of weights is satisfied. Thus

$$\text{Maximize: entropy}(\alpha) = -\Sigma_{j=1,J}\alpha_j \text{Ln}\alpha_j$$

$$\text{Subject to: } \zeta = \Sigma_{j=1,J}\alpha_j(J-j)/(J-1) \quad (\textit{orness constraint})$$

$$\Sigma_{j=1,J}\alpha_j = 1 \quad (\textit{normality constraint})$$

where $\alpha_j \in [0, 1]$. ME-OWA operator weights have the form

$$\alpha_j = \exp(\lambda(J-j)/(J-1))/\Sigma_{k=1,J}\exp(\lambda(J-k)/(J-1))$$

where the parameter, λ, is derived by substituting α_j in the "orness" constraint equation. When $\lambda = -\infty$, orness(α) = 0 and when $\lambda = +\infty$, orness(α) = 1.

Thus, in project evaluation, given a desired level of "orness" (ζ), identify ME-OWA operator weights and use $G(F_j(p),w_j,\zeta)$ (or, $H(F_j(p),w_j,\zeta)$) to modify $F_j(p)$ based on factor importance weights and ζ. Finally, calculate the OWA operator with arguments $G(F_j(p),w_j,\zeta)$. The project for which the OWA is a maximum is "best."

To illustrate this procedure, consider the above example. Suppose that the desired level of "orness" is 0.3 (i.e., "andness" = 0.7). In terms of the ME-OWA operator (i.e., solving the constrained maximization problem), $\lambda = -2.144$ and ME-OWA weights are given as $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_{10}\} = \{0.03, 0.03, 0.04, 0.06, 0.07, 0.09, 0.11, 0.15, 0.18, 0.23\}$ where $\alpha_j = \exp(-2.144(10-j)/9)/\Sigma_{k=1,J}\exp(-2.144(10-k)/9)$. Applying $G(F_j(p),w_j,0.3)$ defined above yields

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
P_1	0.9	0.796	0.976	0.840	0.762	0.898	0.709	0.709	0.786	0.561
P_2	0.7	0.894	0.976	0.840	0.835	0.530	0.553	0.553	0.609	0.758
P_3	0.3	0.205	0.976	0.252	0.907	0.898	0.342	0.342	0.430	0.449

For example, $G(F_2(p),w_1,0.3) = [(0.969 + (1 - 0.3) - (0.969)(1 - 0.3)]^* (0.8)^{[0.969 + 0.3 - (0.969)(0.3)]} = 0.796$. Then, ME-OWA($P_1$) = 0.723, ME-OWA($P_2$) = 0.662 and ME-OWA($P_3$) = 0.455. Project P_1 is identified as "best."

Rather than solving a constrained nonlinear programming problem as in the derivation of ME-OWA operator weights, a somewhat simpler approach involves *exponential* OWA (E-OWA) operators [19]. Consider $\{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_{J-1}, \alpha_J\} = \{\theta, \theta(1-\theta), \theta(1-\theta)^2, ..., \theta(1-\theta)^{J-2}, (1-\theta)^{J-1}\}$ where $0 \leq \theta \leq 1$. Then $\Sigma_{j=1,J}\, \alpha_j = 1$, and for $\theta = 1$, $\alpha = \{1, 0, 0, ..., 0\}$ and for $\theta = 0$, $\alpha = \{0, 0, 0, ..., 1\}$. It can be shown that orness(α) is a monotonically increasing function of θ [19]. The functional relationship between orness(α) and θ is different for different numbers of arguments (factors). For J = 2, orness(α) = θ, and as J increases, orness(α) is higher than the value of θ. For this reason, the OWA operator is called an *optimistic exponential* OWA operator (OE-OWA) [19] (see Figure 3a).

Now consider $\{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_{J-1}, \alpha_J\} = \{\theta^{J-1}, (1-\theta)\theta^{J-2}, (1-\theta)\theta^{J-3}, (1-\theta)\theta^{J-1}, (1-\theta)\}$ where $0 \leq \theta \leq 1$. Then $\Sigma_{j=1,J}\, \alpha_j = 1$, and for $\theta = 1$, $\alpha = \{1, 0, 0, ..., 0\}$, and for $\theta = 0$, $\alpha = \{0, 0, 0, ..., 1\}$. Again, the functional relationship between orness(α) and

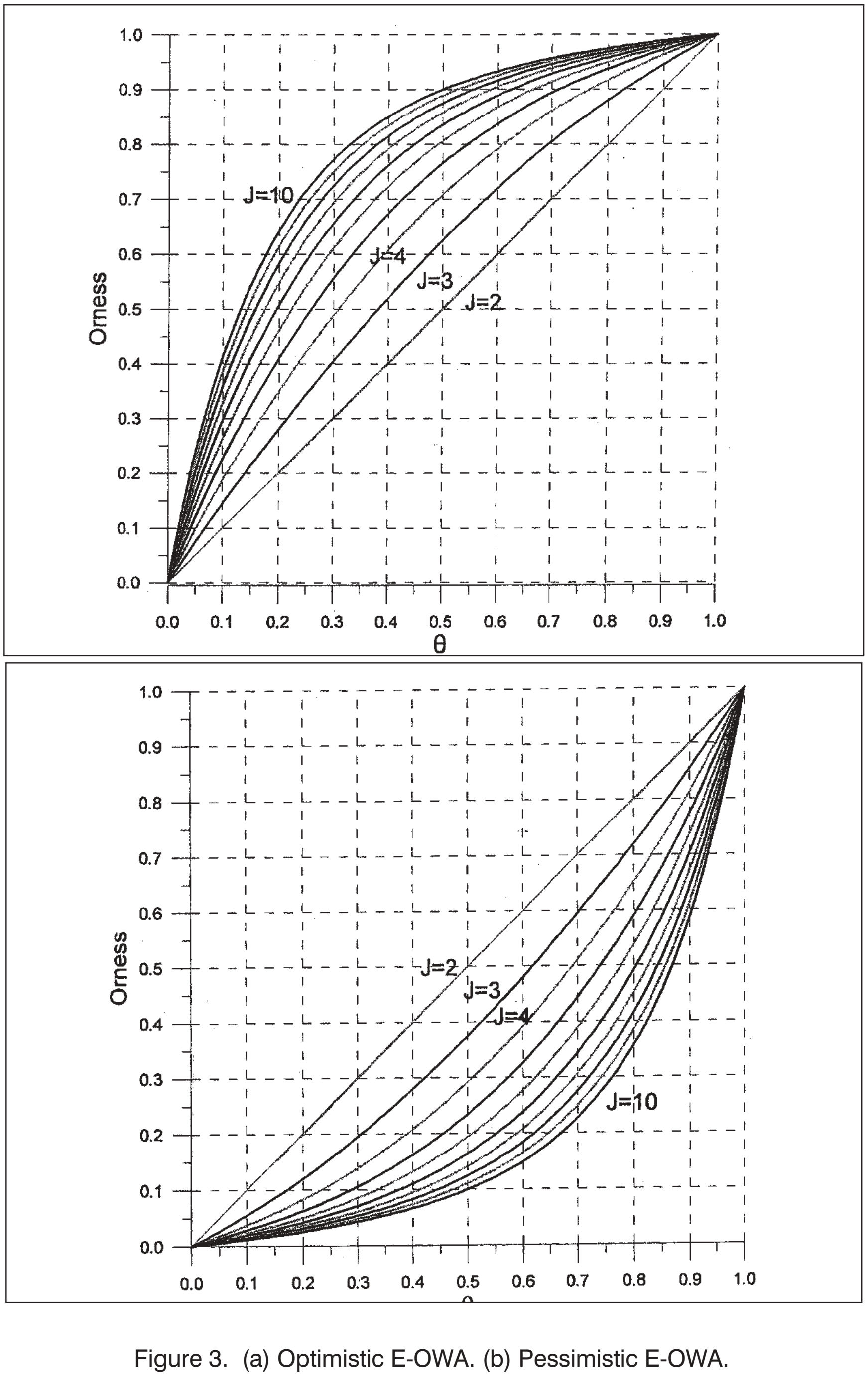

Figure 3. (a) Optimistic E-OWA. (b) Pessimistic E-OWA.

J increases, orness(α) is lower than the value of θ. For this reason, the OWA operator is called a *pessimistic exponential* OWA operator (PE-OWA) [19] (see Figure 3b).

Thus, in project evaluation, given a desired level of "orness," ζ, and number of factors, J, identify the parameter, θ, from charts of the form given in Figure 3a or 3b which are based on the relationship between the parameter, θ, ranging in the interval [0,1] and orness(α) for given number of factors. Use $G(F_j(p),w_j,\zeta)$ (or, $H(F_j(p),w_j,\zeta)$) to modify $F_j(p)$, based on factor weights and ζ. Finally, calculate E-OWA with OWA operator weights given as appropriate. The project for which E-OWA is a maximum is "best."

For a pessimistic OWA operator, assume the modification function as above (i.e., orness(α) = 0.3). Then, since J = 10 and orness(α) = 0.3, Figure 3b shows that θ is approximately 0.7 (more exactly, θ = 0.744) so that OWA operator weights are α = {0.7, 0.024, 0.032, 0.043, 0.058, 0.078, 0.105, 0.142, 0.19, 0.256}. Thus PE-OWA(P_1) = 0.724, PE-OWA(P_2) = 0.703, and PE-OWA(P_3) = 0.427. Again, project P_1 is identified as "best."

LINGUISTIC QUANTIFIED STATEMENTS

In classical logic, *quantifiers* ("for all" and "there exists" ("not none," "at least one")) in statements or propositions may be used to represent the number of items satisfying a given predicate; for example, "*All* factors are satisfied by project p," "*At least one* factor is satisfied by project p." However, classical logic allows for the inclusion of only the above quantifiers. Zadeh introduced *linguistic quantifiers* represented by fuzzy subsets and thus fuzzy subsets provide the basis for linguistically quantified statements or propositions [20]. The general form of a quantified statement is "Q F's are **A**," where Q is a linguistic quantifier (e.g., "few," "most," "at least n"), F is a class of objects and **A**, a fuzzy subset of F, is some property associated with the objects. For example, in the quantified statement "*most* houses are *expensive*," the quantifier Q is "most," F is a set of "houses," and "expensive" is a property/characteristic of houses.

Zadeh distinguished between two types of quantifier, *absolute* and *proportional* (*relative*) [20]. Absolute quantifiers are used to represent amounts that are absolute in nature ("about 5," "more than 10") and are closely related to the concept of the count or number of elements. Proportional quantifiers ("most," "few," "at least half") represent relative amounts. Absolute quantifiers are defined on the set of non-negative reals, $\mathbb{R}^+$, whereas relative quantifiers are defined on the [0,1] interval.

In the context of project evaluation, "Q F's are $\mathbf{A}_p$," where Q is a linguistic quantifier, $\{F_1(p), F_2(p), ..., F_J(p)\}$ is a set of factors against which a project p is assessed and $\mathbf{A}_p$ is a fuzzy subset of F indicating the predicate "satisfied by p." Thus, $\mathbf{A}_p = \{A_p(F_1)|F_1, A_p(F_2)|F_2, ..., A_p(F_J)|F_J\}$. Note that, $A_p(F_j) \equiv F_j(p)$ (j = 1, ..., J), the performance of project p with respect to factor F_j. Examples of linguistic quantified statements in the context of the fuzzy evaluation of projects include "*most* factors are satisfied by project p" or "*at least n* factors are satisfied by project p." This is a *type I* statement [20, 21].

Regular increasing monotone (RIM) quantifiers (e.g., "all," "most," "many," "at least x percent"), are defined by a fuzzy subject, **Q**, defined on the [0,1] interval such that Q(0) = 0, Q(1) = 1, and Q(r) ≥ Q(s) if r > s [22], are used here in the context of project evaluation.

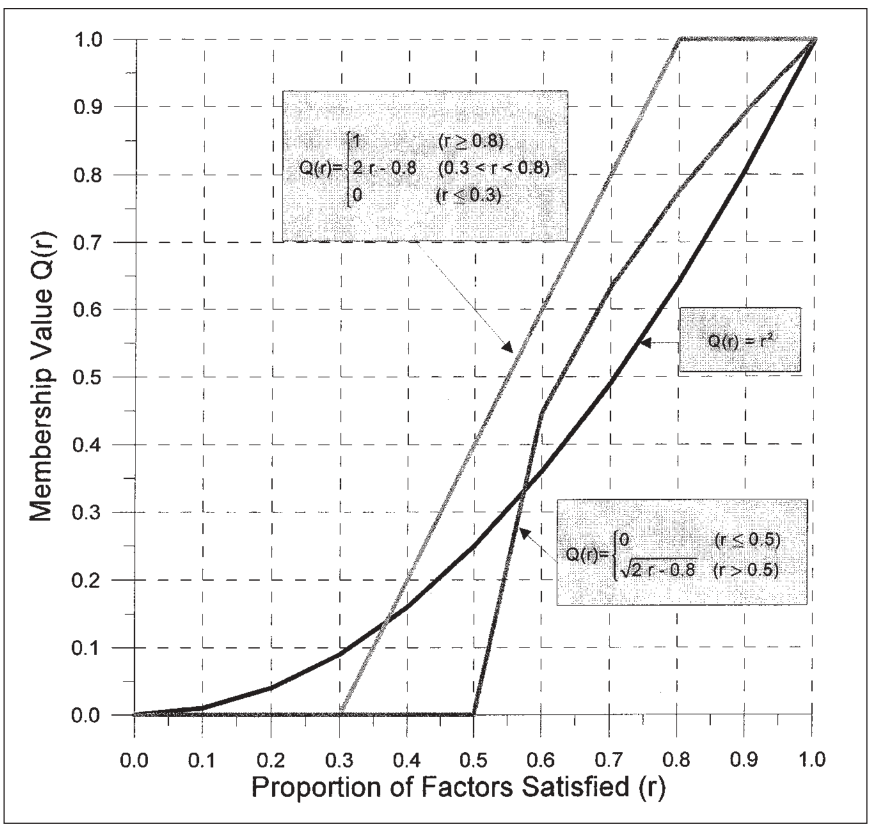

Figure 4. Interpretations of quantifier "most."

In the context of project evaluation, an appropriate quantifier is "most" which has been defined in various ways [23, 24] (see Figure 4). Below, "most" is defined as a RIM quantifier, $Q(r) = r^2$, $r\varepsilon[0,1]$. This quantifier is "andlike" (i.e., "orness" < 0.5) with orness(α) = 1/3 (see Figure 4). The quantifier $Q(r) = r^\beta$ is "andlike" if $\beta > 1$ and "orlike" (i.e., "orness" > 0.5) if $\beta < 1$.

Linguistic quantified statements are readily implemented by OWA operators. Given a linguistic quantifier, OWA operator weights are generated as $\alpha_j = Q(j/J) - Q(j-1)/J)$, (j=1, ..., J) [15] and the "orness" of α is expressed as orness(α) $= (1/(J-1))\Sigma_{j=1,J-1}Q(j/J)$.

As an example of a type I quantified statement in the context of project evaluation, consider "*most* factors are satisfied by project P_1." Then for $p \equiv P_1$, $\mathbf{A}_p = \{A_p(F_1)|F_1, A_p(F_2)|F_2, ..., A_p(F_{10})|F_{10}\} = \{0.9|F_1, 0.8|F_2, ..., 0.5|F_{10}\}$. Let Q be the relative quantifier, "most." Since J = 10, $\alpha_j = Q(j/10) - Q(j-1)/10)$ (j = 1, ..., 10) and $\alpha = \{0.01, 0.03, 0.05, 0.07, 0.09, 0.11, 0.13, 0.15, 0.17, 0.19\}$, where, for example, $\alpha_1 = Q(1/10) - Q(0) = 0.01$. Here, the orness($\alpha$) = 0.317 and will approach 1/3 as J increases. The *truth* of the quantified statement, "*most* factors are satisfied by project P_1", is given as $OWA_Q(P_1) = 0.777$, where OWA_Q denotes an OWA operator when quantifier, Q, is used to define the operator weights. Also $OWA_Q(P_2) = 0.645$ and $OWA_Q(P_3) = 0.299$. Thus, project P_1 is "best."

The classical quantifier, *all,* is Q(j/J) = 0 for j < J and Q(J/J) = Q(1) = 1, in which case $\alpha = \{0, 0, ..., 1\}$ and the OWA operator identifies $\wedge_{j=1,J}F_j(p)$. The classical quantifier, *at least one,* is Q(j/J) = 1 for j ≥ 1 in which case $\alpha = \{1, 0, ..., 0\}$ and the OWA operator identifies $\vee_{j=1,J}F_j(p)$.

An extension of the above quantified statement is "Q B F's are $\mathbf{A}_p$" where $\mathbf{B} = \{B(F_1)|F_1, B(F_2)|F_2, ..., B(F_J)|F_J\}$ is a fuzzy subset of F, such that $B(F_j)$ indicates the importance of factor F_j, i.e., $B(F_j) \equiv w_j$. Examples of this type of quantified statement are "*most important* factors are satisfied by project p" and "*at least n important* factors are satisfied by project p." These are *type II* statements [20-22]. Type II statements may be implemented by using the modification functions given above or by *weighted ordered weighted averaging operators.*

WEIGHTED ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

Rather than modifying the membership grades for a given project p with respect to factor importance weights (as in the case of ME-OWA, OE-OWA, and PE-OWA operators), it is possible to use importance weights to modify the OWA operator weights. This approach is natural when some linguistic quantifier is available to guide the aggregation of factors.

A weighted OWA (WOWA) operator which generalizes both the OWA operator and the weighted average has been defined [23-26] as

$$\text{WOWA} = \Sigma_{j=1,J}\beta_j\, b_j$$

Again, b_j is the jth largest element of $\{F_1(p), F_2(p), ..., F_J(p)\}$. Weights, w_j, reflecting the importance of factors are normalized such that $w_j \varepsilon [0,1]$ and $\Sigma_{j=1,J} w_j = 1$. WOWA operator weights are given as $\beta_j = W(\Sigma_{k=1,j}u_k) - W(\Sigma_{k=1,j-1}u_k)$ where u_j is the importance weight associated with b_j. Thus, for example, if $b_1 = F_3(p)$, then $u_1 = w_3$. $W(\bullet)$ is a monotonic nondecreasing function that interpolates the points $(j/J, \Sigma_{k=1,j}\alpha_k)$ together with point (0,0), if the weights used in the OWA operator (such that $\alpha_j \varepsilon [0,1]$ and $\Sigma_{j=1,J}\alpha_j = 1$) are given. If factor weights are all equal (i.e., $w_j = 1/J$, j=1, ..., J), then the WOWA operator reduces to the OWA operator. Alternatively, given $W(\bullet)$, it is possible to define β_j from $W(\bullet)$ without the initial step of defining OWA operator weights, α [26]. This latter approach is used below.

The WOWA operator is useful in the context of type II quantified statements of the form "Q B F's are $\mathbf{A}_p$" where $\mathbf{B} = \{B(F_1)|F_1, B(F_2)|F_2, ..., B(F_J)|F_J\} \equiv \{w_1|F_1, w_2|F_2, ..., w_J|F_J\}$ is a fuzzy subset of factors, such that $B(F_j) \equiv w_j$ indicates the degree of importance of factor F_j. The function $W(\bullet)$ is given by the quantifier $Q(\bullet)$. The "orness" of $\beta = \{\beta_1, \beta_2, ..., \beta_J\}$ is then given as $\text{orness}(\beta) = (1/(J-1))\Sigma_{j=1,J-1}Q(\Sigma_{k=1,j}u_k)$. The "orness" of β will be different for different β resulting from the ordering of the outcomes for each project p.

Consider, for example, the type II quantified statement "*most important* factors are satisfied by project P_1," For $p \equiv P_1$, $\mathbf{A}_p = \{A_p(F_1)|F_1, A_p(F_2)|F_2, ..., A_p(F_J)|F_J\} = \{0.9, 0.8, ..., 0.5\}$. Let $\mathbf{B} = \{1|F_1, 0.969|F_2, ..., 0.193|F_{10}\}$ be the fuzzy subset of F representing the importance of factors, e.g., $B(F_1) \equiv w_1 = 1$. In the calculation of the β_j's, weights are normalized such that $\Sigma_{J=1,J} w_j = 1$ (i.e., w^0). Again, "most' is defined as $Q(r) = r^2$ which may be used for the function $W(\bullet)$. Thus, for project P_1 with membership grades, {0.9, 0.8, 1, 0.9, 0.8, 1, 0.8, 0.8, 1, 0.5}, ordered arguments are $\{b_1, b_2, ..., b_{10}\}$ = {1, 1, 1, 0.9, 0.9, 0.8, 0.8, 0.8, 0.8, 0.5} and therefore $\{u_1, u_2, ..., u_{10}\}$ = {0.046, 0.148, 0.106, 0.161, 0.115, 0.074, 0.111, 0.052, 0.156, 0.031}. Also WOWA operator weights, β, are shown in Figure 5 for each project (site).

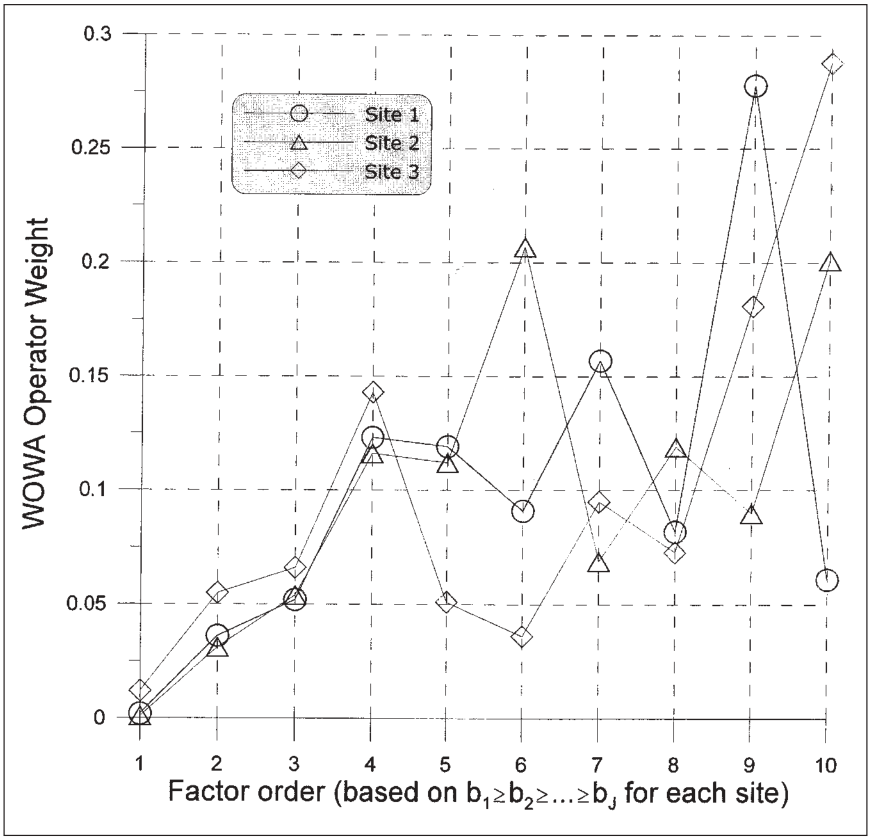

Figure 5. WOWA operator weights based on quantifier "most."

The truth of the quantified statement "*most important* factors are satisfied by P_1," under quantifier, Q, is given as $WOWA_Q(P_1) = 0.845$. Similarly, $WOWA_Q(P_2) = 0.737$ and $WOWA_Q(P_3) = 0.382$. Again, P_1, is "best."

CONCLUSION

This article has outlined some features of OWA operators in the evaluation of alternative projects with significant environmental consequences. An example adapted from Horsak and Damico [1] involving the location of a hazardous waste disposal facility with three possible sites assessed against ten factors was considered in terms of the operators. Each method identifies project (site) 1 as "best," a conclusion also found by Horsak and Damico. However, this results from the strong performance of site 1 with respect to many factors and such agreement between methods need not be expected in general. Various OWA aggregation operators methods have been considered including *maximum entropy* OWA operators and (*optimistic* and *pessimistic*) *exponential* OWA operators [27]. Using these operators, a given level of "orness" ("andness") may be readily acknowledged and used to guide selection of a "best" project. However, search for a more acceptable modification function for incorporating factor weights and level of "orness" (or, conversely, "andness") into project outcomes (membership values) is desirable. Some progress in this direction has been made by Yager [28] using fuzzy systems modeling (*direct fuzzy reasoning*) involving rules of the form: "If orness = α_i then modification function = $G_i(w, m)$." Possible functions emerged from plausible fuzzy systems.

Quantifier guided propositions or statements have also been considered and implemented through weighted OWA operators. The "orness" of WOWA operator weights are then determined by the nature of the quantifier. A disadvantage of this approach is that WOWA operator weights will be different for different factors. Further, the appropriate numeric characterization of quantifiers of various descriptions (e.g., "most," "many") suitable in environmental evaluation requires some exploration. However, it is expected that appropriate quantifiers display a high level of "andness" and thus give due recognition to the wide range of environmental consequences of projects. Possibilities for eliciting from decision makers the parameters characterizing particular type quantifiers might also be usefully explored [29].

Clearly, the issue of aggregation of fuzzy subsets is critical to the results obtained and much further research is necessary on the appropriate choice of aggregation operator.

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