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ANDNESS-DIRECTED WEIGHTED AVERAGING OPERATORS: POSSIBILITIES FOR ENVIRONMENTAL PROJECT EVALUATION

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ABSTRACT

Some of the conventional methods of aggregating the performance of infrastructure projects with respect to multiple factors/impacts are considered. It is suggested that alternative forms of aggregation might be more useful; in particular, the ordered weighted averaging (OWA) operator introduced by Ronald Yager. Factor importance weights and fuzzy satisfaction of factors by projects may be aggregated prior to aggregation via an OWA operator. In this case OWA operator weights may be based on the "attitudinal character" of the decision-maker expressed in terms of the degree of "orness" and "andness" of the aggregation. One approach is maximum entropy aggregation where weights are derived to be as "even" (or as minimally dispersed) as possible subject to satisfying a given "orness" or "andness" constraint. Recently aggregation processes based on "andness" have been proposed by Henrik Larsen which have several desirable properties and may also be considered as alternative forms of aggregation. A simple example based on a hypothetical but realistic example by Horsak and Damico is given which involves the location of a hazardous waste disposal facility (PCB-contaminated transformer fluids) at one of three sites based on ten factors.

INTRODUCTION

On many occasions, decisions relating to major projects (e.g., the siting of hazardous waste storage facilities [1, 2] or solid waste disposal facilities [3]) must be made based on available data and information that are vague, imprecise, and uncertain by nature. The decision-making process in, for example, the field of waste management is one of these typical occasions which frequently calls for a method of treating uncertain and ill-defined data and information. The nature of vagueness, imprecision, and uncertainty is fuzzy rather than random, especially when subjective assessments are involved in the decision making process. Fuzziness derives from the lack of precise boundaries in some of the subsets of data and information considered in a given situation. Fuzzy set theory offers a possibility of handling these sorts of data and information which involve subjective characteristics of the human decision making process.

Following [4, 5], the basic structure for decisions relating to projects with multiple (ecological, social, economic, aesthetic, etc.) consequences is a decision matrix which shows the satisfaction of project p_i with respect to factor/impact F_j. $P = \{p_1, p_2, ..., p_I\}$ is a set of I mutually exclusive projects and $F = \{F_1, F_2, ..., F_J\}$ is a set consisting of J factors/impacts. Commonly in the decision process, weights $\underline{w} = [w_1, w_2, ..., w_J]$ are introduced to represent the differential importance (salience, significance) of factors/impacts.

In terms of fuzzy set theory, each factor, F_J, may be construed as a fuzzy subset of the set of projects represented as $\mathbf{F_j} = \{F_j(p_1)|p_1, F_j(p_2)|p_2, ..., F_j(p_I)|p_I\}$, where $F_j(p)$ indicates the degree to which project $p \varepsilon P$ satisfies factor/impact F_j. Note that the satisfaction of a given project (denoted either as p or p_i) is represented as, $\underline{F}(p) = [F_1(p), F_2(p), ..., F_j(p)]$, $p \varepsilon P$ (see Table 1 for the "outcome matrix" of projects with respect to factors).

Project evaluation typically involves the identification of a "best" project which satisfies as much as possible each factor/impact. "Satisfies" implies lower values of negative factors/impacts (e.g., cost, ecological impact) and higher values of positive factors/impacts (e.g., accident reduction, aesthetic impact, savings in travel time). Rarely will any real project completely satisfy all factors/impacts and will be characterized by variable achievement across factors/impacts. For brevity, the term "factor" will be used below, where possible, to include also impacts.

Table 1.

	F_1	F_2	...	F_J
p_1	$F_1(p_1)$	$F_2(p_1)$	...	$F_J(p_1)$
p_2	$F_1(p_2)$	$F_2(p_2)$	...	$F_J(p_2)$
⋮	⋮	⋮		⋮
p_I	$F_1(p_I)$	$F_2(p_I)$	...	$F_J(p_I)$

EXAMPLE OF ENVIRONMENTAL PROJECT EVALUATION

Consider an example adapted from Horsak and Damico [1] (also considered by Anandalingam and Westfall [2] and see Smith [4, 5]) involving the location of a hazardous waste disposal facility with three possible sites assessed against ten factors: *air quality* (dispersive capabilities of site/plant and degree to which waste emissions could concentrate onsite and offsite, F_1); *surface water quality* (potential for surface water degradation due to spills associated with handling storage and waste, F_2), *groundwater quality* (potential for groundwater degradation due to spills associated with handling and storage of waste, including leaching into acquifer, F_3); *impact on ecology* (potential impact on ecological resources of an area due to routine operations or emergency conditions, F_4); *impact on aesthetics* (visual impacts of hazardous waste management operations, including handling, storage, and disposal, F_5); *impact on population* (potential long-term exposure to emissions due to routine operations or emergencies, F_6); *impact on surrounding land use* (compatibility of surrounding land use with the hazardous waste operation, F_7); *possibility of emergency response* (ability of a response team to combat an emergency associated with a spill or other exposure, F_8); *distance from sources of waste* (distance through which the waste should travel to get to the site, F_9); and *political opposition* (political or other organized intervention or opposition to the hazardous waste operation, F_{10}). Factors are fuzzy subsets of the projects (sites); for example, $\mathbf{F_1} = \{0.9|p_1, 0.7|p_2, 0.3|p_3\}$ for *air quality* (F_1). The outcome matrix is given in Table 2.

Note that F_3 (*groundwater quality*) could be excluded as it fails to discriminate between sites, though it is retained here. It is clear that site 1 (p_1) is a strong competitor for the overall "best" site [4].

Further assume factor weights (based on [1]) as follows $\underline{w}$ = [1, 0.969, 0.919, 0.714, 0.689, 0.658, 0.460, 0.323, 0.286. 0.193], Horsak and Damico used *weighted conjunctive aggregation* to select a "best" site and identified a preference order, $p_1 \succ p_2 \succ p_3$ [1]. The weighted fuzzy decision $\mathbf{D} = \{D(p_1)|p_1, D(p_2)|p_2, ..., D(p_I)|p_I\}$ is $\mathbf{D} = \cap_{j=1,J}\mathbf{F}_j^{wj}$ with membership grade $D(p) = \min_{j=1,J}F_j(p)^{wj}$ for $p \varepsilon P$ (see [4]).

Table 2.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
p_1	0.9	0.8	1.0	0.9	0.8	1.0	0.8	0.8	1.0	0.5
p_2	0.7	0.9	1.0	0.9	0.9	0.5	0.6	0.5	0.6	1.0
p_3	0.3	0.2	1.0	0.2	1.0	1.0	0.2	0.2	0.3	0.3

ORDERED WEIGHTED AVERAGING (OWA) OPERATORS

Though weighted conjunctive aggregation is a useful aggregation method, flexible aggregation operators exist which more explicitly recognize the "attitudinal character" of the decision-maker expressed in terms of the degree of "orness" and "andness" of the aggregation. The ordered weighted averaging (OWA) operator for aggregating fuzzy subsets was introduced by Yager [6]. The OWA has been elaborated in the context of project evaluation in numeric terms [4] and in linguistic terms [5]. An OWA operator (of dimension J) is represented as $OWA(\underline{\alpha}, \underline{F}(p)) = \Sigma_{j=1,J}\alpha_j b_j$ where b_j is the j^{th} largest element of the values $\underline{F}(p) = [F_1(p), F_2(p), ..., F_J(p)]$.

OWA operator weights, $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$, are associated with the position of b_j and are such $\alpha_j \varepsilon [0,1]$ and $\Sigma_{j=1,J}\alpha_j = 1$. That is, α_j is associated with a particular ordered position j of the arguments (project satisfaction with respect to a factor) and is not a reflection of the importance (salience, significance) of the factor in the context of project evaluation.

The OWA operator includes the commonly used *maximum* and *minimum* operators [4, 6] and the *arithmetic mean* operator for appropriate choice of operator weights, $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$. In particular, the OWA operator is bounded such that $OWA([0, 0, ..., 1], \underline{F}(p)) \leq OWA(\underline{\alpha}, \underline{F}(p)) \leq OWA([1, 0, ..., 0], \underline{F}(p))$. Thus from the definition of the OWA operator, $OWA([0, 0, ..., 1], \underline{F}(p)) = \min_{j=1,J} F_j(p)$, and $OWA([1, 0, ..., 0], \underline{F}(p) = \max_{j=1,J} F_j(p)$ so that extreme OWA operators are the "and" and "or" operators [4, 6]. The arithmetic average corresponds to $OWA([1/J, 1/J, ..., 1/J], \underline{F}(p))$ operator. Again, the "and" (*minimum*) provides no compensation in that a high grade of membership with respect to one factor cannot offset (or compensate for) a low grade of membership with respect to another factor. The "or" (*maximum*) provides full compensation. "And" reflects a conservative/pessimistic attitudinal character on the part of the decision maker, "or" reflects a risk-taking/optimistic character.

With respect to OWA operators, the "orness" of the OWA operator weights is given as $orness(\underline{\alpha}) = \Sigma_{j=1,J}\alpha_j(J - j)/(J - 1)$. "Orness" measures the degree to which an aggregation operator is "orlike" or "andlike" and provides some indication of the inclination of the operator to impart more weight to either higher or lower satisfaction levels. Thus, the greater the "orness," the more weight imparted to higher satisfaction levels and the lower the "orness," the more weight imparted to lower satisfaction levels. "Orness" is itself an OWA aggregation with $b_j = (j - 1)/(J - 1)$. The degree of "andness" of an OWA operator with weights $\underline{\alpha}$ is defined as $andness(\underline{\alpha}) = \neg orness(\underline{\alpha}) = 1 - orness(\underline{\alpha})$.

INCLUDING FACTOR IMPORTANCE IN OWA OPERATOR AGGREGATION

One possibility for including factor importance is to assume an OWA operator for project p involving transformed membership values, $H(w_j,F_j(p))$, (the "effective satisfaction" of factor F_j), a function of both satisfaction level ($F_j(p)$) and factor importance weight (w_j). $H(w_j,F_j(p))$ is also a function of the "orness" of the aggregation (i.e., the OWA dispositional weights, $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$) though this is not explicitly shown in the expression [4, 6]. b_j is now the j^{th} largest element of $H(w_j,F_j(p))$ ($j=1,\dots,J$). One undesirable feature of these prior aggregation functions of factor weights and fuzzy performance is that proposed methods do not yield the conventional weighted average when "orness" and "andness" are equal (orness = andness = ½).

Recently, an aggregation process has been proposed by Larsen based on an OWA operator that recovers the weighted average when andness = ½ [7]. Larsen suggests that project satisfaction of factor j, $F_j(p)$, and factor importance weights, w_j, be combined as

$$h(w_j,F_j(p)) = \text{andness} + w_j(F_j(p) - \text{andness})$$

For andness = 1, $h(0,F_j(p)) = 1$ (implying that, when $w_j = 0$, $h(0,F_j(p))$ will have no influence in the minimum operation associated with andness = 1), $h(1,F_j(p)) = F_j(p)$ (implying the full impact of $F_j(p)$, when a factor has the maximum importance, $w_j = 1$). For andness = 0, $h(0,F_j(p)) = 0$ (implying that, when $w_j = 0$, $h(0,F_j(p))$ will have no influence in the maximum operation associated with andness = 0), $h(1,F_j(p)) = F_j(p)$ (implying full impact of $F_j(p)$ when a factor has the maximum importance, $w_j = 1$). When $w_j = 0$, $h(0, F_j(p))$ = andness. Also, $\partial h(w_j,F_j(p))/\partial w = -\rho + F_j(p)$, so that, if $F_j(p) > \rho$, change in $h(w_j,F_j(p))$ is positive with respect to w_j and if $F_j(p) < \rho$, change in $h(w_j,F_j(p))$ is negative with respect to w_j (here, ρ = andness).

The (normalized) importance weighted OWA is

$$\text{OWA}^N(\underline{\alpha},\underline{h}) = \frac{\text{OWA}(\underline{\alpha},\underline{h}) - \text{OWA}(\underline{\alpha},[0,0,\dots,0])}{\text{OWA}(\underline{\alpha},[1,1,\dots,1]) - \text{OWA}(\underline{\alpha},[0,0,\dots,0])}$$

Here, $\underline{h} = [h_1, h_2, ..., h_J]$, $\underline{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_J]$ are the OWA positional weights, $\underline{w} = [w_1, w_2, ..., w_J]$ are the factor importance weights and $h_j = h(w_jF_j(p))$. Also, $\text{OWA}(\underline{\alpha},\underline{h}) = \Sigma_{j=1,J}\, \alpha_j b_j$, where b_j is the j^{th} largest h_j for a particular project. The importance weighted OWA operator generalizes the OWA operator such that, when $w_j = 1$ ($j=1,2,...,J$), the latter is retained. Further, it can be shown that (see [7]), when andness = ½, then

$$\text{OWA}^N\left(\left[\frac{1}{J},\frac{1}{J},\dots,\frac{1}{J}\right],\underline{h}\right) = \frac{\sum_{j=1,J} w_j F_j(p)}{\sum_{j=1,J} w_j}$$

A requirement of the above formulation is that factor weights be normalized such that the maximum weight assigned to a factor is 1, that is, $\max_{j=1,J} w_j = 1$. When andness = 0, OWA([1,0,...,0],$\underline{h}$) = $\max_{j=1,J} w_j F_j(p)$, an *importance weighted disjunction* and when andness = 1, OWA([0,0,...,1],$\underline{h}$) = $\min_{j=1,J}(\underline{1} - w_j(1 - F_j(p)))$, an *importance weighted conjunction.*

The relationship identical to that presented by Larsen [7, 8], $h(w_j, F_j(p))$ = andness + $w(F_j(p)$ – andness), was also derived by Yager [9] via the use of (2-rule, 1-input) *Takagi-Sugeno-Kang* (TSK) fuzzy systems modeling (see [10]). Yager [9, 11, 12] showed that, in the context of OWA operators, when positional weights in the OWA are $\underline{\alpha}$ = [1/J, 1/J, ..., 1/J], this expression yields an equivalent ordering to the weighted average though not the weighted average itself. Yager presents a variation of the above as a (3-rule, 2-input) TSK system yielding the same result [13].

MAXIMUM ENTROPY ORDERED WEIGHTED AVERAGING OPERATORS

Except for the extreme weight sets above, different OWA weight sets can yield the same level of "orness" or "andness." In particular, all symmetric weight sets yield orness($\underline{\alpha}$) = andness($\underline{\alpha}$) = ½. Thus, the dispersion given by the *entropy* function, entropy($\underline{\alpha}$) = $-\Sigma_{j=1,J}\alpha_j \ln\alpha_j$, may be used to further distinguish between the weight sets [4, 6, 14, 15]. Entropy is a maximum when all weights are equal to 1/J and a minimum value when one weight is equal to unity and all others zero (that is, when $w_j = 1$ and $w_k = 0$, $k \neq j$).

The maximum entropy OWA (MEOWA) operator is derived as follows. Given a desired level of "andness," say andness(α) = ρ, OWA operator weights are identified as the solution to the mathematical programming problem which involves maximizing the entropy ("evenness" of OWA operator weights) subject to constraints: (i) that andness($\underline{\alpha}$) = ρ, the desired value; and (ii) that the normalization condition of weights is satisfied. Thus,

Maximize: entropy($\underline{\alpha}$) = $-\Sigma_{j=1,J}\alpha_j \ln\alpha_j$
Subject to:
$\rho = 1 - \Sigma_{j=1,J}\alpha_j(J - j)/(J - 1)$ (andness constraint)
$\Sigma_{j=1,J}\alpha_j = 1$ (normality constraint)
$\alpha_j \geq 0 \qquad (j = 1, ..., J)$

Note that the above nonlinear programming problem is expressed using an "andness" constraint, rather than the more familiar "orness" constraint [14, 15]. MEOWA operator weights thus have the form

$$\alpha_j = \frac{e^{-\mu\left(\frac{J-j}{J-1}\right)}}{\sum_{k=1,J} e^{-\mu\left(\frac{J-k}{J-1}\right)}}$$

where the parameter, μ, is derived by substituting α_j in the "andness" constraint equation. When $\mu \to \infty$, andness($\underline{\alpha}$) = 1 and when $\mu \to -\infty$, andness($\underline{\alpha}$) = 0.

Given $h(w_j, F_j(p))$, importance weighted maximum entropy ordered weighted average (MEOWA) operator weights may be generated for a given level of "andness." Maximum entropy weights for J = 10 factors and 0.5 ≤ andness ≤ 0.9 are shown in Figure 1. Maximum entropy weights for 0.1 ≤ andness ≤ 0.5 are symmetrical to those shown in Figure 1. MEOWA weights for andness = 1 are $\underline{\alpha}$ = [0, 0, ..., 1] and for andness = 0, $\underline{\alpha}$ = [1, 0, ..., 0].

It should be noted that alternative methods to maximum entropy OWA weights have been proposed. Fuller and Majlender propose minimal variability weighting vector under a given level of "orness" [16]. Wang and Parkan proposed a *minimax* disparity approach for obtaining OWA operator weights [17]. Minimax disparity weights for 0.5 ≤ andness ≤ 0.9 are shown in Figure 2. Minimax disparity weights for J = 10 factors and for 0.1 ≤ andness ≤ 0.5 are symmetrical to

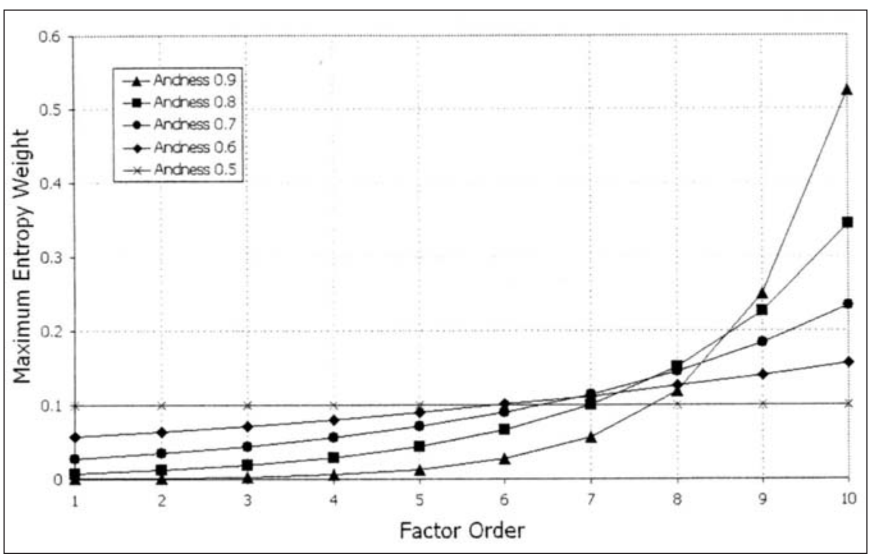

Figure 1. Maximum entropy weights for J = 10 factors and 0.5 ≤ andness ≤ 0.9.

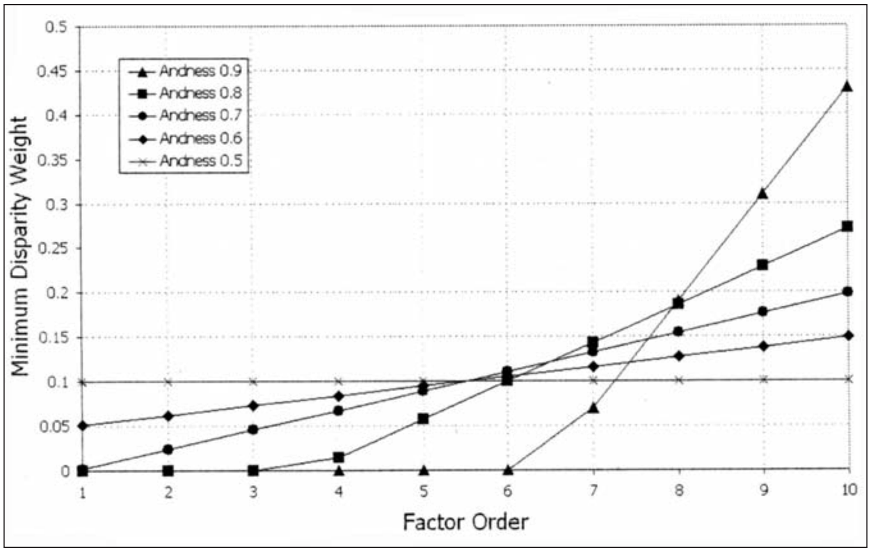

Figure 2. Minimax disparity weights for J = 10 factors and $0.5 \leq$ andness ≤ 0.9.

those shown in Figure 2. Minimax disparity weights for andness = 1 are $\underline{\alpha} = [0, 0, ..., 1]$ and for andness = 0, $\underline{\alpha} = [1, 0, ..., 0]$.

MEAN OPERATORS

Larsen also presented an extended (in the sense that the parameter is restricted to $\xi \geq 1$ so that arguments can assume the value zero) generalized mean operator which assumes prior weighting of performance scores, $h(w_j,F_j(p)) = h_j =$ andness $+ w_j(F_j(p) -$ andness), as above [8]. This is expressed as

$$\lambda(\underline{h}) = \begin{cases} \max_{j=1,J} h_j \text{ if andness} = 0 \\ \left(\frac{1}{J}\sum_{j=1,J} h_j^{\xi}\right)^{\frac{1}{\xi}} \text{ if andness } \varepsilon\ (0, \tfrac{1}{2}] \\ 1-\left(\frac{1}{J}\sum_{j=1,J} (1-h_j)^{\frac{1}{\xi}}\right)^{\xi} \text{ if andness } \varepsilon\ [\tfrac{1}{2}, 1) \\ \min_{j=1,J} h_j \text{ if andness} = 1 \end{cases}$$

The parameter, ξ, is given as = (1 – ρ)/ρ (ρ = andness). Here "andness" is different to that used in OWA aggregation (see [18, 19]) for elaboration in this respect). "Andness" is assumed as monotonic function of the parameter, ξ. Again, to get the weighted average when andness = ½, $\lambda_\rho(\underline{h})$ is normalized so that (ρ = andness)

$$\lambda_\rho^N(\underline{h}) = \frac{\lambda_\rho(\underline{h}) - \lambda_\rho([0,0,\ldots,0])}{\lambda_\rho([1,1,\ldots,1]) - \lambda_\rho([0,0,\ldots,0])}$$

Then, when ρ = 0.5,

$$\lambda_{0.5}^N(\underline{h}) = \frac{\sum_{j=1,J} w_j F_j(p)}{\sum_{j=1,J} w_j}$$

Also for andness = 0 and andness = 1, we have respectively, an *importance weighted disjunction* λ_0^N (F(p)) = $\max_{j=1,J} w_j F_j(p)$, and an *importance weighted conjunction,* λ_1^N (F(p)) = $\min_{j=1,J} (1 - w_j (F_j(p) - 1))$.

The (unnormalized) EGM operator performs somewhat similar to the unnormalized MEOWA operator (see [8]).

Larsen presented an alternative "andness"-directed importance weighted averaging (AIWA) operator defined as [18]:

$$\text{AIWA}(\underline{w}, \underline{F}(p)) = \begin{cases} \max_{j=1,J} w_j F_j(p) & \text{if andness} = 0 \\ \left(\dfrac{\sum_{j=1,J} (w_j F_j(p))^{\xi}}{\sum_{j=1,J} w_j^{\xi}} \right)^{\frac{1}{\xi}} & \text{if andness } \varepsilon\ (0, \tfrac{1}{2}] \\ 1 - \left(\dfrac{\sum_{j=1,J} (w_j (1 - F_j(p)))^{\frac{1}{\xi}}}{\sum_{j=1,J} w_j^{\frac{1}{\xi}}} \right)^{\xi} & \text{if andness } \varepsilon\ [\tfrac{1}{2}, 1) \\ \min_{j=1,J} (1 - w_j (1 - F_j(p))) & \text{if andness} = 1 \end{cases}$$

Again, it can be shown that when andness = ½, then

$$\text{AIWA}(\underline{w}, \underline{F}(p)) = \frac{\sum_{j=1,J} w_j F_j(p)}{\sum_{j=1,J} w_j}$$

Also for andness = 0 and andness = 1 we have, respectively, an *importance weighted disjunction* and *importance weighted conjunction* as for the EGM operator.

Aggregations involving the normalized importance weighted MEOWA (NORM_MEOWA), the normalized extended generalized mean operator (NORM_EGM) and the "andness"-directed importance weighted average (AIWA) as a function of andness are shown in Figures 3 through 5.

Clearly, site p_1 appears to be the better project followed by p_2 and p_3 for all levels of "andness."

Relationships between the unnormalized MEOWA, the normalized MEOWA (NORM_MEOWA), the normalized extended generalized mean (NORM_EGM), and the "andness"-directed weighted average (AIWA) as a function of "andness" are shown in Figures 6 through 8 and show close similarities.

It is clear from Figures 6 through 8 that p_1 dominates both p_2 and p_3, and that p_2 dominates p_3, irrespective of the level of "andness." However, this need not necessarily be the case, in that a project may be superior at from one perspective related to one level of "andness" and inferior at from another perspective. That is, that the "trajectories" of projects cross at particular levels of "andness" corresponding to the decision-makers attitude. For example, consider the decision matrix shown in Table 3, in which projects (sites) p_2 and p_3 are more "competitive" with p_1 than previously. Then, for example, the normalized EGM

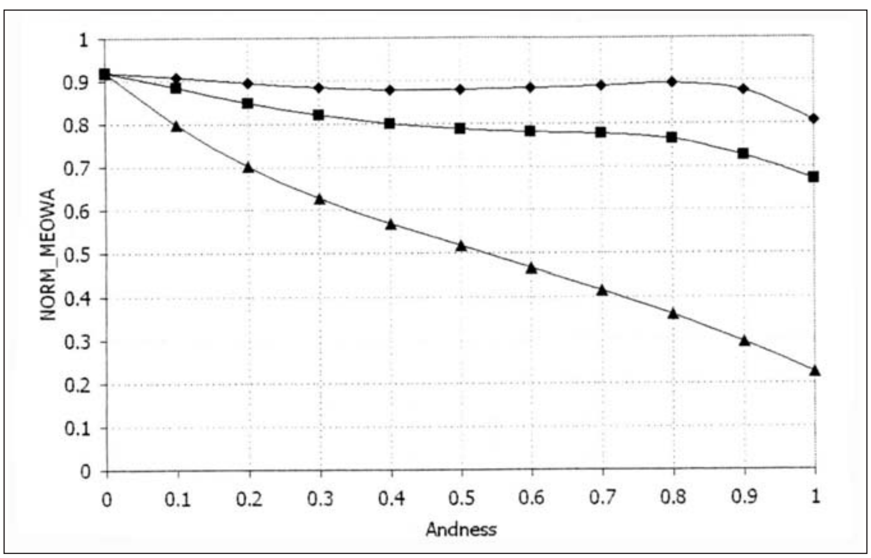

Figure 3. Normalized MEOWA (NORM_MEOWA) for projects p_1-p_3 and $0 \leq$ andness ≤ 1.

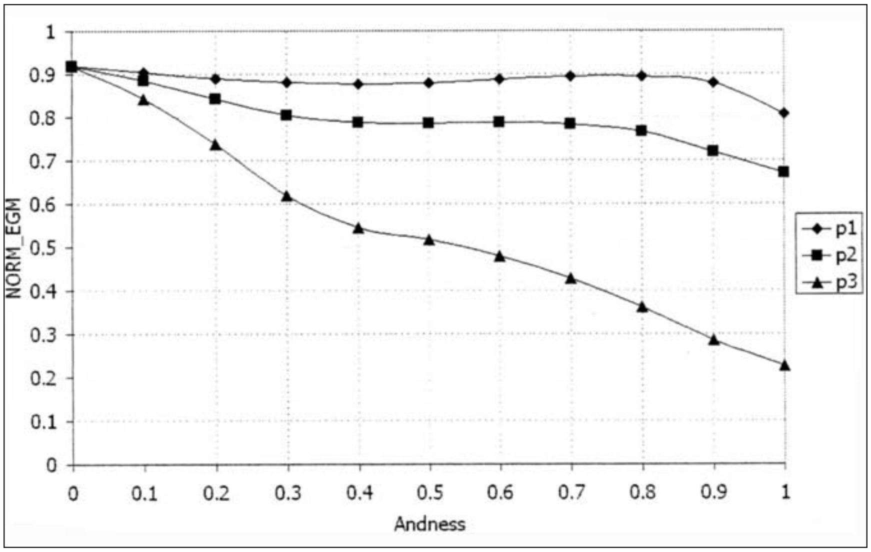

Figure 4. Normalized EGM (NORM_EGM) for projects p_1-p_3 and $0 \leq$ andness ≤ 1.

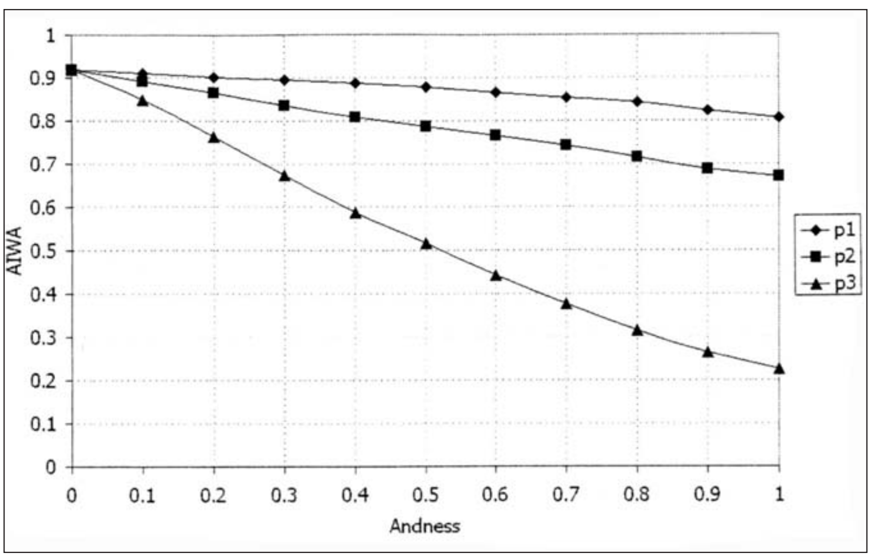

Figure 5. AIWA for projects p_1-p_3 and $0 \leq$ andness ≤ 1.

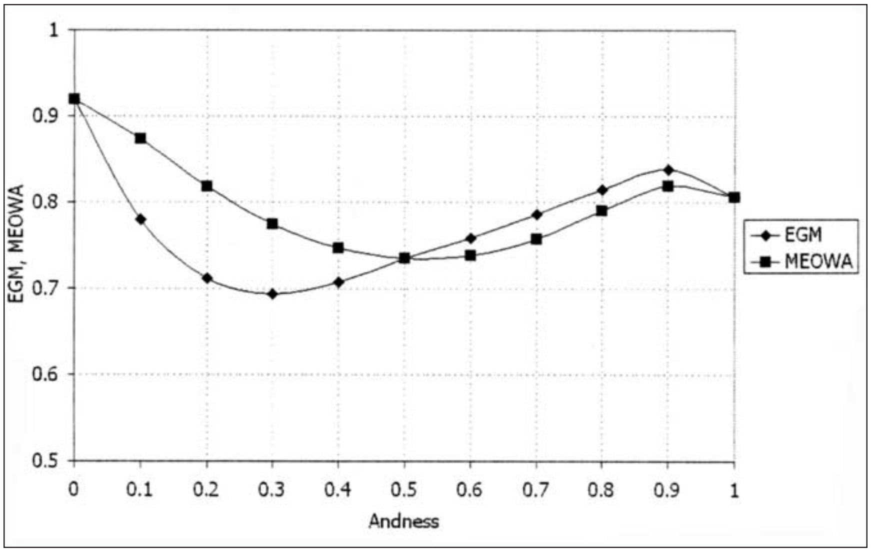

Figure 6. (Unnormalized) MEOWA and (unnormalized) EGM for project p_1 and $0 \leq$ andness ≤ 1.

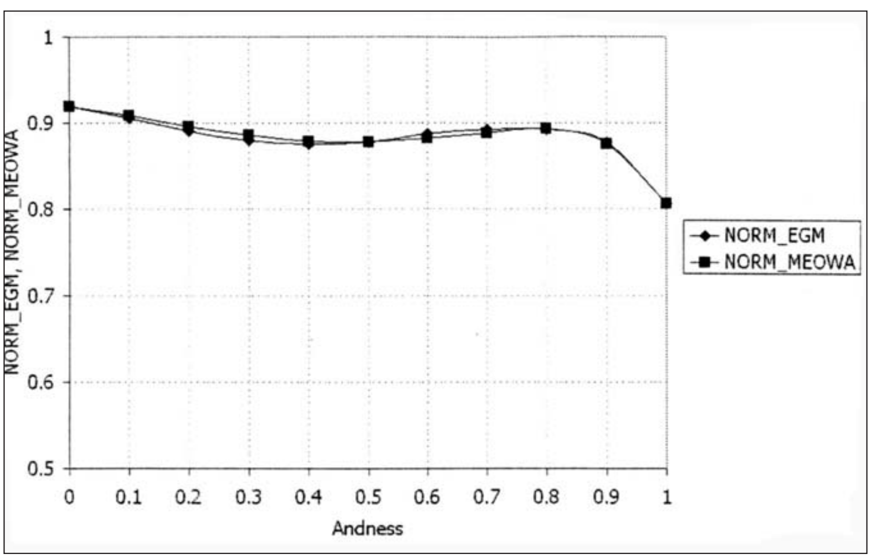

Figure 7. Normalized MEOWA (NORM_MEOWA) and normalized EGM (NORM_EGM) for project p_1 and $0 \leq$ andness ≤ 1.

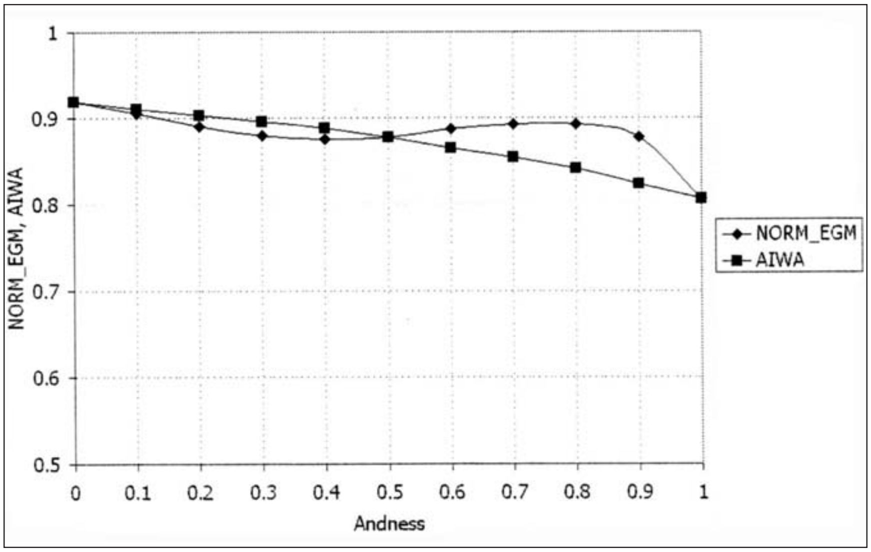

Figure 8. AIWA and normalized) EGM (NORM_EGM) for project p_1 and $0 \leq$ andness ≤ 1.

Table 3.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
p_1	1.0	0.3	0.9	0.2	0.2	0.8	0.8	0.8	0.7	0.5
p_2	0.7	0.9	0.9	0.4	0.9	0.6	0.5	0.5	0.6	1.0
p_3	0.9	0.8	0.2	1.0	0.3	0.2	0.5	0.5	0.5	0.7

(NORM_EGM) as a function "andness" is shown in Figure 9. Here, p_1 or p_2 or p_3 could be identified as "best" depending on the level of "andness" reflecting the decision-maker's degree of optimism/pessimism.

CONCLUSION

More flexible aggregation (between minimum and maximum) of weights and satisfaction of factors has been investigated. Factor weights and project satisfaction of factors have been aggregated prior to aggregation using an OWA operator. In this case, OWA operator weights are based on the "attitudinal

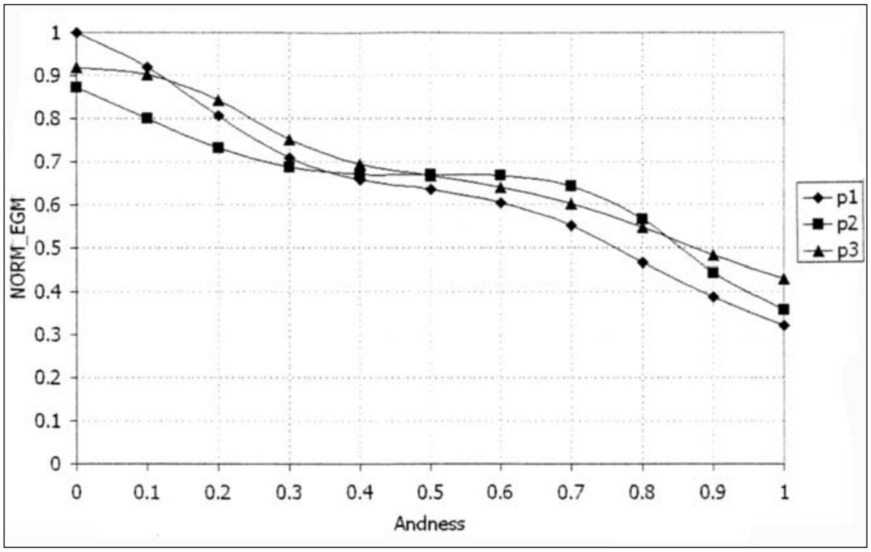

Figure 9. Normalized EGM for projects (sites) p_1-p_3 and $0 \leq$ andness ≤ 1 based on decision matrix involving more "competitive" performance between sites.

character" of the decision maker expressed in terms of the degree of "orness" and "andness," of the aggregation. Further, the decision-maker is able to see the implication of factor weighted project performance graphically as a function of the "andness" of the aggregation. Mostly, with complex projects, a high degree of "andness" will be desired but perhaps not the extreme degree (minimum operator). The maximum entropy aggregation involves weights derived to be as "even" (or as minimally dispersed) as possible while satisfying a given "andness" constraint. The aggregation process proposed by Larsen based on MEOWA operators recovers the weighted average when andness = ½ [7]. In addition, an extended generalized mean operator (EGM) (involving prior aggregation of factor weights and performance scores) proposed by Larsen is closely related to the MEOWA operator [8]. The "andness"-directed importance weighted averaging (AIWA) operator, incorporating factor importance weights directly, proposed by Larsen [18] has also been considered and shown to yield similar results to the normalized EGM operator and MEOWA operator. Though there is considerable scope for further investigation of aggregation processes relating to factor importance and satisfaction in discriminating between projects with environmental consequences, it is claimed that such "andness"-directed operators provide a useful and flexible range of aggregation possibilities available to the decision-maker.

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