

**OPTIMAL RECYCLING OF
ALUMINUM BEVERAGE CANS:
AN EMPIRICAL APPROACH**

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ABSTRACT

Recently, environmentalists have increased pressure for better solid waste disposal for ecological reasons. Consequent to such pressure, government agencies have evoked stricter measures of solid waste disposal. Apart from the ecological basis for better solid waste disposal, economic motives have been nurtured too.

This paper surveys recycling of aluminum beverage cans as a method of solid waste disposal in the light of the growing importance of beverage cans in solid wastes. It seeks economic justification for recycling which if found will be evoked to complement ecological reasons for better disposal of solid wastes. Only then will the environment be better protected.

The analysis employs the Simplex Method. It shows that of the three main products from recycling the beverage cans, copper is the most important followed by aluminum and zinc respectively. Recycling is economically justifiable and the largest profit is attained when recycling is for copper and aluminum only.

Introduction

The recent years have witnessed mounting pressure from the environmentalists for better disposal of solid waste materials as a means of preserving the ecology. To justify their pressure, they have astutely pointed out the environmental, social, and economic implications of solid waste recycling. The pressure from them has begun to yield fruitful results, for they have successfully stimulated government agencies into adopting stricter standards of solid waste

disposal [1]. Furthermore, consequent to their pressure, many industrialists have begun to recycle solid wastes for purely economic reasons. It can be pointed out that though the industrialists have not embarked upon recycling primarily for environmental reasons, their recycling activities for economic reasons doubtlessly have led to the attainment of certain environmental objectives too.

Numerous analyses of solid waste recycling have been carried out by scholars especially in the present decade. As if it were by design, most of such studies were simplistic in nature, and often couched in general assumptions. Furthermore, such studies have emphasized the environmental, physical, and social implications of solid waste recycling to the detriment of the economic considerations.¹ It is important to expose the economic consequences of solid waste recycling. For once exposed, it will be used to kindle industrial interests in solid waste recycling activities in our free enterprise economy.

The purpose of this paper is to make an in-depth economic analysis of recycling one major component of solid waste materials—aluminum beverage cans. This component of solid waste material is singled out for study because, industrial sources have discovered that the aluminum beverage can, a relative newcomer to the beverage container market, is one of the most rapidly growing containers in the entire beverage industry, and, therefore, is assuming greater importance as a solid waste. Aluminum cans constitute over 40 per cent of all the cans in the beverage industry in 1976 [2]. Aluminum beverage cans found in wastes contain among others, three highly valued nonferrous metals, namely aluminum, copper, and zinc. Recycling is a good way to recover such valuable metals. Furthermore, such recovery through recycling constitute an effective means of reducing not only import expenditures on primary aluminum, copper, and zinc, but also an energy saver, as it takes only 2 to 4 per cent of electricity used in producing a ton of virgin aluminum to recycle a ton of aluminum cans [3].

The economic analysis of recycling aluminum beverage cans is expected to serve two vital objectives. In the first instance, it will determine whether recycling such beverage cans is profitable, and if so, what the optimal recycling level should be for a plant. Secondly, assuming that recycling such cans is found profitable,

¹ This paper intends to concentrate on the economic implications of solid waste recycling only. It has chosen recycling aluminum beverage cans as a specific case for study. The rationale for such a choice is to complement scholarly works on non-economic motives for recycling.

this finding can be used as an incentive to open up an economic frontal attack on the problem of solid waste disposal. It is only by this economic frontal attack complementing the noneconomic frontal attacks could the problem of solid waste disposal be solved.

Section one introduces the Simplex Method as the main model to be employed in this study. It explains the parameters, the variables, and the constraints of the study. In Section two, the constraints are transformed into equations. Using them in conjunction with the objective function, basic feasible solutions are obtained. The best of such solution is the most-sought-after optimal basic feasible solution. This solution guarantees maximization of the objective function. As the quest for optimal solution necessitates trade-offs in input mix known as the marginal rate of substitution, the value of such a substitution at each stage is shown in this section. Section three provides a conclusion for the study.

Section One: The Simplex Method

Recycling aluminum beverage cans yield multiproducts whose major elements are aluminum, copper, and zinc. Because the Simplex Method has been a very effective instrument of choice in a multi-product industry, its use in aluminum beverage can recycling will reveal what combination of aluminum, copper, and zinc yields optimal profit. This method employs a system of linear equations whose solutions form basic feasible solutions. The best solution is optimal and maximizes the objective function. It is the one most often sought for.

The Simplex Method is unique in attempting to attain the optimal basic feasible solution in a multiproduct industry. It does so by ferreting out from the totality of basic feasible solution a specific solution that is in a sense optimal, thereby saving both time and cost. It is a forward looking method with built-in-ratchet which assures that later solutions are superior to their predecessors. As Daniel C. Vandermeulen has succinctly put it, the simplex method achieves its computational efficiency by screening and passing over all inconsistent, redundant, and infeasible subsets and thereby eliminates much needless computations [4].

As stated previously, recycling aluminum beverage cans yields three very valuable nonferrous metals—aluminum, copper, and zinc.² These variable outputs are X_1 , X_2 , and X_3 respectively.

² Industrial chemical analysis shows other possible metals which can be delivered from recycling aluminum beverage cans. Because industries value them low, this study will ignore them.

Because a recycling industry markets these three main products, the revenue function is:

$$R = 22x_1 + 42x_2 + 14x_3 \quad (1.1)$$

where the coefficients are the respective prices per pound of the aluminum, copper, and zinc derived from recycling used aluminum beverage cans.³ Also recycling such beverage cans creates costs. After sharing the cost among aluminum, copper, and zinc products, the weights for allocation being those of the prices found in function 1.1 above, the cost function derived is:

$$C = 4X_1 + 8X_2 + 3X_3 \quad (1.2)$$

Implicit in the revenue cost function is the profit function. It is in this function that an aluminum beverage recycling industry tries to maximize. Symbolically, the profit function is:

$$\begin{aligned} &= 22x_1 + 42x_2 + 14x_3 - 4x_1 - 8x_2 - 3x_3 \\ &= 18x_1 + 34x_2 + 11x_3 \end{aligned} \quad (1.3)$$

The main problem which confronts the industry producing aluminum, copper, and zinc from aluminum beverage cans is what combination of the three products maximize profit. Assuming that producing three of the products should lead to maximized profit.

Certain constraints on profit maximization in the aluminum beverage can recycling industry have been imposed by capital, labor, and supplies inputs. On the assumption that land building are given, industrial source indicates that the average capital cost of an aluminum recycling plant is \$275,000 [6]. By allocating the cost of capital to aluminum, copper, and zinc, the capital input constraint is:

$$85x_1 + .001x_2 + .005x_3 \leq \$275,000 \quad (1.4)$$

where the coefficients represent the percentages of aluminum, copper, and zinc contents in a typically recycled aluminum scrap.

An authentic industrial source indicated that labor of all kinds received \$13,000,000 in fifty-eight plants in 1976. The average labor cost per recycled plant is \$224,137.93 [8]. Given this labor

³ In *Iron Age* [5, p. 64] prices were given for dealer scraps. Though these prices vary with time, the above prices are adopted as proxies for the year 1977.

⁴ The basis for deriving the capital input-constrain coefficients above is [7, p. 26] in which typically recycled aluminum scrap shows 85 per cent to be aluminum, .1 per cent to be copper, and .06 per cent to be zinc. Because other chemicals found are of low value, all the capital cost is assigned to aluminum, copper, and zinc.

cost constraint, and assuming that the price of each of the products—aluminum, copper, and zinc, is equal to the value of the marginal product of labor used in producing the output, the price of the products form the basis for the computation of the coefficients of labor-input constraint. Because only aluminum, copper, and zinc possess high value, all the labor resource cost is charged to them. The labor coefficient of each product is its price presented in 1.1 as a percentage of all prices. The labor input constraints thus derived is:

$$28x_1 + 5x_2 + 18x_3 \leq \$224,137.93 \tag{1.5}$$

An industrial source quoted that the average cost of collecting aluminum beverage cans in an industry of fifty-eight plants is \$196,776.61 in 1976.⁵ This cost is incurred not for aluminum, copper, and zinc alone but for all the alloy chemically found in a beverage can. The chemical analysis of all the alloys reveals that aluminum, copper, and zinc, constitute 94 per cent, 1 per cent, and 1 per cent respectively. Using these weights for the sharing of the supply, the supply constraint is:

$$94x_1 + 1x_2 + 1x_3 \leq \$196,776.61 \tag{1.6}$$

Finally, the variables to be produced x_1, x_2, x_3 are constrained to being equal to or greater than zero. The reason is that a plant will never produce a negative output. It can produce nothing or something at any point in time. The product constraint is:

$$x_1, x_2, x_3 \geq 0 \tag{1.7}$$

A synthesis of the objective function and the constraints results in a model whose solutions yield the combinations of products in a multiproduct industry from which maximization of the objective function can be determined.

$$\text{Max} = 18x_1 + 34x_2 + 11x_3$$

Subject to

$$85x_1 + .001x_2 + .0005x_3 \leq \$275,000 \tag{1.8}$$

$$28x_1 + 54x_2 + 18x_3 \leq 224,137.91$$

$$94x_1 + 1x_2 + 1x_3 \leq 196,776.61$$

$$x_1, x_2, x_3 \geq 0$$

⁵ [8, p. 59], Mr. Reynolds was quoted as saying that his company of fifty-eight recycling plants collected one and three-quarter million beverage cans. Given that twenty-three cans weigh a pound and that a pound of cans costs 15 cents to collect, the average collecting cost per plant is \$196,776.61.

Section Two: Determination of Basic Feasible and Optimal Feasible Solutions

A precondition to the solution of 1.8 is the linear transformation of the constraints into equations. Such a transformation is performed by the addition of a slack variable to each of the constraints.⁶ Also the objective function is modified by the addition of a slack variable for every constraint in the model. The result is:

$$85x_1 + 0.001x_2 + 0.0005x_3 + x_4 = \$275,000 \quad (1.9)$$

$$28x_1 + 54x_2 + 18x_3 + x_5 = \$224,137.91$$

$$94x_1 + 1x_2 + 1x_3 + x_6 = \$196,776.61$$

$$18x_1 + 34x_2 + 11x_3 = P \text{ (max)}$$

The above model of three equations in six variables has $C(3,3)! = 20$ different ways in which to combine the three main products of aluminum beverage cans so as to maximize profit. However, certain of such combinations are likely to be infeasible because they may involve negative output of certain metals in a case where the x_1 variables are constrained to being equal or greater than zero. Luckily, the use of the Simplex Method reduces the number of solutions by weeding out the infeasible solutions.

It is advisable always to begin by finding the initial basic feasible solution and in a model like 1.9 above, where the resource inputs on the right side of the equal signs are non-negative, the slack variable provide the initial basic feasible solution. This easy solution obtains because the constraints in the original model were of the form \leq the resource constraints. The initial basic feasible solution read from 1.9 above in which no aluminum, copper or zinc is produced is:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_5 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_6 = \begin{bmatrix} 275,000 \\ 224,137.91 \\ 196,776.61 \end{bmatrix} \quad (1.10)$$

A solution without output though feasible is the least desirable in the recycling industry because it allows resource inputs to stand costlessly idle. In the face of positive price coefficients in the

⁶ Slack variables are measures of the unused capacities of the input variables. Because maximization of the objective function is the goal sought, additions of positive slack variables are made. Had minimization been the goal sought slacks would have been negative values.

objective function, management can earn profit by producing any of the three main outputs. An endeavour to improve upon the profit recommends the displacement of one of the non-basic variables in 1.10 by one of the previously excluded basic variables. A choice has to be made of which variable to include and which one to drop. The variable in the objective function with the largest positive coefficient is the basic variable to be entered first. Its column is the pivot column. In 1.9 the pivot column is x_2 . In order to determine which non-basic variable to drop a certain transformation is necessary to avoid linear dependence between the entering variable and the remaining variables. The transformation involves dividing each resource input on the right of 1.9 by the corresponding coefficient of the pivot column and choosing the row with the least quotient as the pivot row. The element at the intersection of the pivot row and column is called the pivot element. The pivot element has the unique role of keeping production within bounds of the constraints. The pivot element is found in x_2 and is fifty-four. The x_2 column will displace the slack variable on the pivot row; this is x_5 .

The process of displacement involves, first multiplying each element of the pivot row including the resource input by the reciprocal of the pivot element to obtain a new first equation. Second, add to each element of each row the product of the inverse of its positive element on the pivot column and the corresponding element of the new first equation. This process is carried out with respect to the objective function too. The result of this conversion yields the new basic feasible solution. The conversion results of 1.9 are:

$$\begin{aligned}
 84.9994x_1 + 0x_2 + .0001x_3 + x_4 - .00001x_5 &= & (1.11) \\
 274995.8492 & \\
 .5185x_1 + 1x_2 + .3333x_3 + .0185x_5 &= \\
 4150.7024 & \\
 93.4815x_1 + 0x_2 + .6667x_3 - .0185x_5 + 1x_6 &= \\
 192625.9076 & \\
 .3710x_1 + 0x_2 - .3322x_3 - .6290x_5 &= \\
 R \ 141123.8816 &
 \end{aligned}$$

Columns x_2 , x_4 , and x_6 form an identity matrix when columns x_1 , x_3 , x_5 are excluded. Because the variables x_1 , x_3 , and x_5 have been excluded, the column vectors from which the solution values are easily read are:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_6 = \begin{bmatrix} 274995.8492 \\ 4150.7024 \\ 192625.9076 \end{bmatrix} \quad (1.12)$$

The only basic variable being produced as can be read off from 1.12 is x_2 . The quantity produced is 4150.7024. If this number of pounds of copper is sold at a profit of \$.34 per pound, total profit is \$1,411.24.⁷

The production of copper alone does not yield optimal profit to the recycling industry because a certain coefficient of the objective function is still greater than zero. Though labor input is fully employed, underutilization still exists in capital and supply inputs. The amount of excess capital input is \$275,000 - (.001)(4150.7024) = \$274,995.8492. Similarly the amount of excess supply input is \$196,776.61 - 1(4150.7024) = \$192,625.9076.

Astute management requires the introduction of either aluminum or zinc production to absorb the unused resources. The next metal to produce, based on the positivity of the objective function, is aluminum. Aluminum production uses labor input just as does copper. Being that no excess labor exists, aluminum production would be made possible only by a cut-back in labor input in copper production. However, as labor is cut-back in copper production, other labor cooperant factors in production are cut-back too. The rate of labor cut-back in copper with respect to increased aluminum production is $-\frac{28}{54}$. The net capital requirement due to increased aluminum production is $85 - .001(\frac{28}{54}) = 84.9994$.

Similarly, the net supplies requirement is $94 - 1(\frac{28}{54}) = 93.4815$.

The cut-back in copper results in loss of revenue to the extent of $-(34)(\frac{28}{54}) = 17.63$. However, each additional aluminum produced is sold for 18¢. The net unit increased in profit from such switching of resources is $18¢ - 17.63¢ = .37¢$. To the extent that this is positive, a cutback in copper production and an increased output of aluminum lead to higher profit.

It is profitable to produce more aluminum at the expense of copper. Pivoting operation which had been previously illustrated

⁷ It should be pointed out that this amount of profit could be read from the last equation of 1.11. The figure there is in cents and when put into dollars will yield a figure of \$1411.24.

serves a useful purpose of determining which column vector in 1.11 should be dropped as the aluminum vector x_1 goes in. The result of pivoting and the required transformation is:

$$\begin{aligned}
 0x_1 + 0x_2 - .6033x_3 + 1x_4 + .0084x_5 - .9009x_6 &= & (1.13) \\
 99847.9725 & \\
 0x_1 + 1x_2 + .3296x_3 + .0185x_5 - .0054x_6 &= & \\
 3082.2928 & \\
 1x_1 + 0x_2 + .0071x_3 - .0001x_5 + .0106x_6 &= & \\
 2060.5778 & \\
 0x + 0x - .3348x - .6290x - .0039x &= & \\
 R - 141,888.3559 &
 \end{aligned}$$

Because the column vectors x_3 , x_5 , and x_6 have been dropped, the vectors that yield the basic feasible solution are x_1 , x_2 , and x_4 . The solution values for variables x_1 , x_2 , and x_4 can easily be read from the column vectors given below since they form an identity matrix.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_4 = \begin{bmatrix} 99847.9725 \\ 3082.2928 \\ 2060.5778 \end{bmatrix} \tag{1.14}$$

The recycling industry should produce in this plant 2060.58 pounds of aluminum x_1 and 3082.29 pounds of copper x_2 . Being that per unit profit for aluminum and copper are 18¢ and 34¢ respectively, total profit is \$1,418.88. A look at the profit equation in 1.13 shows that no coefficient in the objective function is positive. Therefore, no further addition can be made to profit by cutting down the production of one kind of product and increasing the production of another. It can, therefore, be safely concluded that the basic feasible solution which offers a profit of \$1,418.88 is simultaneously the much sought-after optimal basic feasible solution. This profit level is also read off the last line of 1.13.

Conclusion

This paper has investigated the procedure for invoking economic incentives as a means to kindle industrial interest in solid waste recycling activities. This new approach is necessary because the already existing one socio-environmental frontal attack on the problem of solid waste disposal, has not been very fruitful.

Employing this often neglected economic frontal attack will undoubtedly complement the already existing socio-environmental frontal attack in adequately solving the problem solid waste disposal.

The case selected for illustrating the economics of solid waste recycling is that of aluminum beverage cans recycling. It is a multi-product recycling industry. Given the three main valuable products it produces, the study indicates that in the quest for the largest profit, the industry should not produce the three main products. As is shown by the optimal basic feasible solution, the largest profit exists when 3082 lbs. copper and 2061 lbs. aluminum are produced. Also implicit in the fact that copper price and quantity exceed the price and quantity of aluminum is the feeling that branding this industry the aluminum beverage can recycling industry is a misnomer.

The general conclusion is that economically speaking, solid waste recycling is beneficial. This discovery has to be stressed. When this motive for recycling is used conjointly with the non-economic motives, the problem solid waste disposal, is likely to be solved.

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