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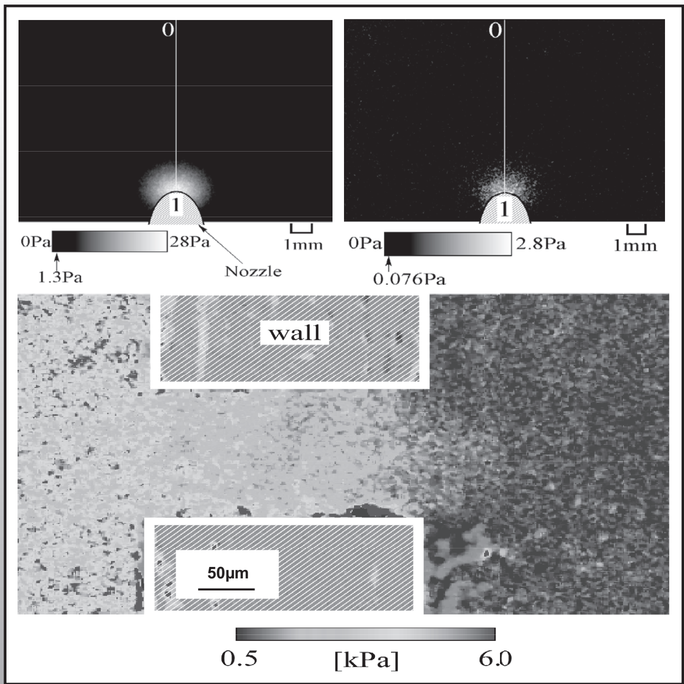

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Front Cover Window: **Pressure distribution measured in high Knudsen number flows over a flat surface using Pressure Sensitive Paints (page 223) and in a micro-channel using Pressure Sensitive Molecular Film (page 225).**

The Law of Multi-Scale Turbulence

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ABSTRACT
This is a study of the classical physical and mathematical principles that govern turbulent fluid dynamics. Contrary to the general conception that views turbulent flows as incomprehensible; it can, in fact, be cast as a deterministic phenomenon that obeys a universal law. This universal law was introduced by A. N. Kolmogorov in the work that has now been immortalized as K41, widely accepted as among the most significant contributions to fluid dynamics research. A presentation of the universal law of multi-scale turbulence is given here with the objective of assuaging apprehensions in regard to turbulent fluid dynamics and to garner interest in this very fascinating concept.

1. INTRODUCTION

Multi-scale turbulent flow stands as among the classical, unresolved mysteries of science, whose secrets remain closely guarded to the present time, bedeviling over a century of searching investigation [1]. It is a topic of notorious difficulty and abounds in academic value that has accrued over longstanding time, as it staunchly obviates aspiring attempts to understand its nature. The inherent difficulties to comprehend the nature of turbulent flow have led the general academic community to withdraw from the topic with trepidation. But in reality, turbulent fluid dynamics can be cast as a deterministic phenomenon that is, in fact, governed by a universal law. The presentation of this universal law of turbulence by Kolmogorov, 1941a [2] is now recognized to be among the most significant contributions in the field. In 1991, a special edition of the Proceedings: Mathematical and Physical Sciences (also known as Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences) was issued to commemorate the 50th anniversary of this paper that has become the fulcrum for turbulent fluid dynamics research, Kolmogorov, 1941a [2], *The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers*, better known as K41. In the preface of the special edition, J. C. R. Hunt, O. M. Phillips, and D. Williams describe K41 as *"probably the most famous paper ever written on turbulence"* [3]. It would then seem fitting to the author to take time to present in this study an appreciation of the universal law of K41 along with the fundamental physics and mathematics upon which it is founded, as well as, reactions of the fluid dynamics community to this singularly historic work.

The organization of this study is outlined as follows. Section 2 provides an overview of the fundamental physics and mathematics of turbulent fluid dynamics. Key concepts include the length scale of turbulence, definition of the inertial range, and statistical properties of turbulence. Section 3 presents the works of Kolmogorov, namely K41, and the universal law of turbulence. Section 4 discusses a major challenge to K41 theory in the form of intermittency of turbulence. Section 5 gives an overview of contemporary reactions to K41. Among the objectives of this study is to carefully divest the general academic community of the misconception that turbulence is incomprehensible and to recast it as a fascinating topic open to engagement.

2. FUNDAMENTAL PHYSICS AND MATHEMATICS

Before the universal law of K41 can be presented, the fundamental physics and mathematics of turbulent fluid dynamics need first be established. To begin, let us consider the concept of the "Mischungsweg", the length scale of turbulence, as presented in the now classical series Taylor, 1935 I, II, III, and IV [4-7].

2.1 The "Mischungsweg"

Perhaps, the most important defining parameter of turbulent flow is its length scale. Turbulent flows generate eddies and vortices that possess definitive physical dimensions known as their length scales. Taylor and Prandtl independently termed this length scale the classical name, "Mischungsweg" [4]. Qualitatively speaking, the "Mischungsweg" is likened to, but not exactly, the concept of the mean free path. In the case of the "Mischungsweg", it depicts a parcel of fluid that traverses a certain path length while preserving a certain intrinsic identity [4]. Beyond this path length, the parcel of fluid mixes with its neighboring medium, taking up the properties of its surroundings and shedding its intrinsic identity. The preceding vague description hints towards the idea of correlated motion among fluid parcels within a given defined space. Figure 1 shows an example of correlated rotational motion of flow in the wake of a cylinder known as the von Karman vortex street. The length scale of turbulence, its "Mischungsweg", is thus the spatial dimension over which a parcel of fluid travels with correlated, synchronized motion. In mathematical terms, the length scale ℓ can be defined as [4]:

$$\ell = \int_0^{\infty} R_y \, dy = \int_0^{Y} R_y \, dy \tag{1}$$

where y is the spatial distance between two points, R_y is a correlation function for the fluid motion between the two points such that $R_y = 1$ at $y = 0$ and $R_y \rightarrow 0$ as y becomes large. According to Taylor, 1935 I, ℓ could then be taken to represent the *"average size of the eddies"* in the flow [4]. The magnitudes of the length scales of these eddies can span over wide-ranging orders of magnitude, hence, the moniker multi-scale turbulence. The universal law of Kolmogorov addresses a particular range of length scales within the spectrum known as the inertial range.

Figure 1. Von Karman vortex street in the wake of a cylinder [8].

2.2 The Inertial Range

Turbulent eddies can be generated in flows by increasing the Reynolds number (for example, increase of flow velocity). As the Reynolds number increases, turbulent eddies of variegated ranges of length scales form sequentially [9]. Eddies with large length scales will form first, driven by mechanisms such as the introduction of external disturbances to the flow or magnification of non-linear internal fluid interactions within it [9-11]. These eddies, termed first-order disturbances, have length scales ℓ_1 that are on the same order of magnitude as the length scale L of the external geometric features that shape the flow. Furthermore, the first-order eddies draw their energies predominantly from the prevailing macroscopic flow, termed the mean flow. Therefore, first-order eddies are highly sensitive to the macroscopic characteristics of the mean flow and inherit many of these characteristics onto themselves. For instance, if the mean flow contains anisotropic velocity fields within it (differing statistical properties for the velocities in differing coordinate directions), then the first-order eddies will likewise exhibit an anisotropic velocity field [9]. These eddies are also known as the energy-containing eddies.

Due to their unstable sustenance, the first-order eddies are progenitors in that they readily break down to beget succeeding generations of eddies, beginning with second-order eddies of smaller length scales ℓ_2. The second-order eddies themselves are unstable and hence break down to form third-order eddies of smaller length scales and so forth. The posterior eddies derive their kinetic energy from that of their antecedents in a process of cascading energies from the large scales to the small scales. With each succeeding generation, the length scales of eddies decrease $(\ell > \ell_1 > \ell_2 > \ell_3)$ [9]. Concomitantly, the time scales of the eddies, their durations of existence, also decrease with decreasing length scales $(t > t_1 > t_2 > t_3)$. The progeneration of eddies will finally reach scales that are stable and do not break down to smaller eddies. These eddies will attain qualities of microscopic length scales, minuscule time scales, and stability. At these diminutive length scales, dissipative frictional forces act prominently to convert the energies of these eddies into heat and eventually quench them back into the flow. The time scales of these terminal eddies will be vastly smaller than the time scales of the rates of energy cascade and frictional dissipation associated with their formation and depletion, respectively [11]. So, the flux of energy through these eddies would appear to be steady and constant throughout their duration. For this reason, these eddies exist in a state of quasi-equilibrium where the statistical properties associated with their motion remain apparently constant in time [11]. This range of length scales is famously named the equilibrium range, and its existence is a major tenet of Kolmogorov's theories on turbulence [12-16].

The equilibrium range contains within itself a sub-range of length scales. The act of frictionally dissipating the kinetic energy of the prevailing flow into heat is accomplished mostly by the eddies of the equilibrium range. However, the work of energy dissipation may not be distributed uniformly throughout the eddies of various length scales in the equilibrium range. It may be possible that the energy dissipation takes place within concentrated sub-ranges of eddies in the equilibrium range, while the remaining sub-ranges of eddies do not participate appreciably [11]. In the sub-range where frictional dissipation is largely absent, the flux of energy is driven primarily by inertial forces responsible for the eddy formation and breakup, and so this sub-range is given the classical name, the inertial range [11]. The universal law of turbulence of Kolmogorov applies to the inertial range [17-21].

Beyond the inertial range, the viscous forces become increasingly prominent, and the range of scales in this region is known as the dissipation range. There is a point in the dissipation range where the viscous dissipation is equal to the countervailing energy contribution from non-linear fluid momentum effects. The scale at this point is famously known as the Kolmogorov micro-scale η:

$$\eta = \left(\frac{v^3}{\varepsilon} \right)^{\frac{1}{4}} \tag{2}$$

where v is the viscosity and ε is the kinetic energy dissipation rate. Figure 2 summarizes the ranges of scales in the flow.

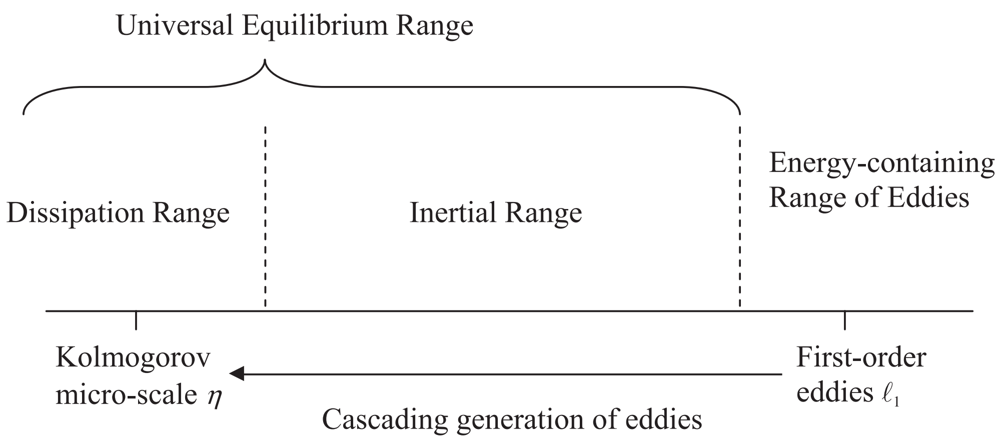

Figure 2. Summary of the ranges of scales in the flow (scales decrease logarithmically from right to left).

A most important facet of the cascading generation of eddies is that with each successive generation, information regarding the external geometry of the mean flow is progressively lost in the descent to smaller scales and may finally be completely lost after a sufficient number of generations [9, 11]. So, while the large scale eddies are highly sensitive to the external conditions; inertial range eddies are highly insensitive to these conditions. For example, anisotropy in the velocity field of the mean flow are not inherited by the inertial range eddies. The buffering of the inertial range from influence by the external conditions bestows upon it two pivotal attributes: isotropy and homogeneity [9, 11]. By isotropy, the statistical properties of the fluid motion of the inertial range eddies are not dependent upon the coordinate direction of consideration. By homogeneity, these statistical properties are also not dependent upon their spatial location within the flow. Further explanations of the statistical properties of turbulence will be given in Section 2.3. So, in summary, the inertial range possesses the consummate traits of quasi-equilibrium, isotropy, homogeneity, and negligible dissipation. These traits render the inertial range to be conducive to the manifestation of a universal law of turbulence. L. F. Richardson poetically captures the eddy formation and breakup cascade in his classic rhyme [22]:

"Big whirls have little whirls
which feed on their velocity;
Little whirls have smaller whirls
and so on to viscosity."

2.3 Statistical Properties of Turbulence

A perplexing aspect of turbulent flow is that its fluid motions appear to be random. In experimental measurements of turbulence at a fixed point, the flow velocity fluctuates vicissitudinously as a function of time. Furthermore, when the same flow experiment is repeated under the same experimental conditions, a different pattern of fluctuations will be realized. The non-repeating vagaries of turbulence arise due to non-linear interactions in the flow that magnify any slight perturbations or instabilities, causing the velocity profile to diverge down different patterns of evolution, even as the repeated experimental conditions are held constant [11, 23]. The remedy to this conundrum would then appear to be to consider the fluid velocity profile at a given point to comprise a distribution of possible values and to study the statistical properties of such a distribution.

Taking a statistical perspective, a useful tool for treating the stochastic turbulent velocity is to compute its ensemble average. The ensemble average, often denoted by $\langle u \rangle$, is the average of the flow velocities across a number of repeated experiments under the same conditions, termed realizations. A beneficial effect of ensemble averaging is that the random fluctuations in the velocity are conveniently eliminated, and the ensemble-averaged velocity profile at a point as a function of time is converted to a smooth function [11, 23]. Moreover, if sufficient realizations are included in the ensemble average, the resulting averaged value will converge to a constant value across further repeated realizations. Therefore, it is customary in the study of turbulent flows to circumvent the anomalous velocity fluctuations by working with the ensemble average $\langle u \rangle$.

Next, a concept of great interest in studies of turbulent flows is its kinetic energy (given here as per unit mass):

$$K.E. = \frac{1}{2}\left\langle u^2 \right\rangle \cdot \tag{3}$$

More specifically, a hotly debated question in turbulent fluid dynamics is the nature by which the kaleidoscopic spectrum of eddies of diverse length scales in a turbulent flow contributes to the overall kinetic energy. What is the distribution of the kinetic energy that originates from the macroscopic range of the spectrum of eddies, and what is the distribution from the microscopic range? To answer this question, another useful tool to use is the Fourier transform.

The Fourier transform converts a given function of interest into an integral of oscillating functions that are defined by periodic parameters such as a wave number. In studies of turbulent flows, the Fourier transform of interest is shown below, and it is related to the kinetic energy of the flow [11, 23]:

$$F_{ij}\left(\vec{k}\right)=\frac{1}{\left(2\pi\right)^3}\int G_{ij}\left(\vec{x}\right)e^{-i\vec{k}\cdot\vec{x}}d\vec{x} \tag{4a}$$

$$G_{ij}\left(\vec{x}\right)=\int F_{ij}\left(\vec{k}\right)e^{i\vec{k}\cdot\vec{x}}d\vec{k} \tag{4b}$$

where $\vec{k}$ is a wave number vector with components of [k_x, k_y, k_z] (the length scales of eddies ℓ are approximately related to the wave number $\vec{k}$ by the relationship $\ell \sim 1/\left\|\vec{k}\right\|$), $\vec{x}$ is the separation vector between two points, $F_{ij}\left(\vec{k}\right)$ is known as the spectrum tensor whose properties will be given in eqn (6), $G_{ij}\left(\vec{x}\right)$ is a tensor of velocity correlation functions whose diagonal elements are of special interest and are defined as [11, 23]:

$$G_{ii}\left(\vec{x}\right)=\left\langle u\left(\vec{x}_0\right)u\left(\vec{x}_0+\vec{x}\right)\right\rangle_{ii} \tag{5}$$

where $u\left(\vec{x}_0\right)$ and $u\left(\vec{x}_0+\vec{x}\right)$ are velocities at two points separated by the vector $\vec{x}$ and $\vec{x}_0=\left(x_0,y_0,z_0\right)$ is the location of the first point. In sum, $F_{ij}\left(\vec{k}\right)$ is the Fourier transform of $G_{ij}\left(\vec{x}\right)$, and the diagonal elements of $F_{ij}\left(\vec{k}\right)$ effectively define the kinetic energy of the flow by combining eqns (4b and 5) and taking the limit of $\vec{x}\rightarrow 0$ to give [11, 23]:

$$\frac{1}{2}\int F_{ii}\left(\vec{k}\right)d\vec{k}=\frac{1}{2}\left\langle u^2\right\rangle. \tag{6}$$

A notable variant of $F_{ii}\left(\vec{k}\right)$ is often used in the form of [11, 23]:

$$E\left(\kappa\right)=2\pi\kappa^2 F_{ii}\left(\vec{k}\right) \tag{7}$$

where $\kappa=\left\|\vec{k}\right\|$. The quantity $E\left(\kappa\right)$ is the three-dimensional energy spectrum of the flow field and is a very important entity in turbulent fluid dynamics. It provides an indication and approximation of the distribution of contributions to the kinetic energy from the array of eddies in the flow. So, $E\left(\kappa\right)$ approximately describes the spectrum of incremental contributions to the kinetic energy from eddies of length scales $\ell \sim \kappa^{-1}$. The universal law of Kolmogorov pertains to $E\left(\kappa\right)$.

The backdrop for the presentation of Kolmogorov's universal law of turbulence is now set. To summarize, in the inertial range, where the conditions of quasi-equilibrium, isotropy, homogeneity, and negligible dissipation are satisfied, the three-dimensional energy spectrum $E\left(\kappa\right)$, which gives indication to the distribution of contributions to the kinetic energy from the array of eddies in the flow, obeys a universal law. This universal law, given in the classical work K41, will be considered next.

3. KOLMOGOROV'S UNIVERSAL LAW OF TURBULENCE

The 1941 classic of Kolmogorov, *The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers*, has been bestowed recognition as among the greatest works in the study of turbulent fluid dynamics [2]. In it, Kolmogorov presents a universal law of turbulence that has become the most celebrated theory in the field of turbulent fluid dynamics. In 1991, the Proceedings:

Mathematical and Physical Sciences commemorated the 50th anniversary of this historically influential work, now known as K41, with a special issue dedicated to discourses on the works of Kolmogorov [2, 3, 24-26]. The most distinguished personalities in the field participated in delivering their perspectives and reactions to the theories of Kolmogorov [3, 27-31]. Though regarded as a classic, some questions remained regarding the work of K41. The esteemed participants offered their admiration, while not hesitating to probe towards further refining the theories of Kolmogorov. Aside from the theoretical insights, this special issue provides rare illustration of the unique dynamics underlying this particular academic community. An overview of K41 theory and its universal law of turbulence will be recounted here.

To begin with discussions on K41, it is necessary to first introduce the concept of the second-order longitudinal structure function of the flow field. Consider two points in the domain of a turbulent flow that are separated by the distance r. The second-order longitudinal structure function $D_{\ell\ell}(r)$ can be defined in regards to the relative velocity between the two points as [9]:

$$D_{\ell\ell}(r) = \left\langle \left[u_\ell(\vec{x}_0 + \vec{x}) - u_\ell(\vec{x}_0) \right]^2 \right\rangle \tag{8}$$

where ℓ is the longitudinal direction of the vector $\vec{x}$ connecting the two points and $r = \|\vec{x}\|$. The second-order longitudinal structure function $D_{\ell\ell}(r)$ is closely related to the kinetic energy of the flow as given in eqn (3) and can be converted to the energy spectrum $E(\kappa)$ of eqn (7) by Fourier transform. Qualitatively speaking, the second-order longitudinal structure function provides an approximate indication of the kinetic energy contained in eddies of length scales of r or below. The universal law of turbulence of Kolmogorov addresses the second-order longitudinal structure function. K41 states that, in the inertial range, the second-order longitudinal structure functions obey a universal law given as [2]:

$$D_{\ell\ell}(r) \sim C \langle \varepsilon \rangle^{2/3} r^{2/3} \tag{9}$$

where C is a universal constant with value ~ 2 and ε is the rate of dissipation of kinetic energy. Equation (9) is the universal 2/3 law of Kolmogorov given in K41. Fourier transform of eqn (9) into its spectral counterpart in terms of the energy spectrum $E(\kappa)$ gives [9, 11]:

$$E(\kappa) = C_k \langle \varepsilon \rangle^{2/3} \kappa^{-5/3} \tag{10}$$

where C_k is a universal constant with value ~ 1.5. Here it is. Equation (10) is the very famous universal 5/3 law of Kolmogorov. It states that, in the inertial range, under the conditions of quasi-equilibrium, isotropy, homogeneity, and negligible dissipation, the energy spectrum $E(\kappa)$, an approximate measure of the distribution of contributions to the kinetic energy by eddies of variegated length scales, obeys the universal 5/3 law characterized by a universal constant C_k and a universal exponent -5/3 [2]. By universality, this law applies to all turbulent flows under these underlying conditions irregardless of differences in the external aspects of the flows such as flow geometries and Reynolds numbers.

The major implication of the universality of the 5/3 law of Kolmogorov is that the energy spectrums $E(\kappa)$ in the inertial range of turbulent flows will collapse onto one universal relationship, irregardless of differences in external flow geometries or differences in Reynolds numbers. Experimental measurements and numerical simulations have shown evidence for the validity of the 5/3 law. Vivid examples of the universal law are given in Figures 3 and 4 with log-log plots of the energy spectrum $E(\kappa)$ as a function of the wave number κ for varying separation distances and Reynolds numbers R_λ, respectively. By using a log-log plot, the 5/3 scaling law manifests as a region with a constant slope of -5/3. In both Figures 3 and 4, a definitive inertial range is discernible with a universal -5/3 slope, even as the separation distances r and Reynolds numbers R_λ are varied respectively. The 5/3 law also applies to flows of other configurations such as boundary layer flows. In boundary layer flows, the 5/3 scaling can begin to break down at locations near to the bottom wall. Close to the wall, the viscous forces would be dominant, flow velocity is substantially reduced, and the velocity gradient is high. The combination of these factors can cause deviations from the 5/3 law very near the wall of boundary layer flows [32]. However, in the middle regions of the boundary layer, flow conditions are such that the 5/3 has been observed to be valid [32].

Figure 5 shows a comparison between experimental results of the one-dimensional longitudinal energy spectra $E_{11}(\kappa_1)$ for different flow configurations: middle region of the boundary layer, grid turbulence, wake of a cylinder, homogeneous shear flow, pipe flow, round jet, tidal channel, return channel, and channel centerline. The 5/3 scaling is clearly shown to be upheld in all cases, as the energy spectra collapses into a congruent relationship for all the varying flow configurations. So, contrary to the customary view that depicts turbulence to be random caprices, embedded within the morass of heteroclitic behavior lurks a universal law that is undegirded by the qualities of quasi-equilibrium, isotropy, and homogeneity and that can be experimentally measured and numerically simulated with consistency.

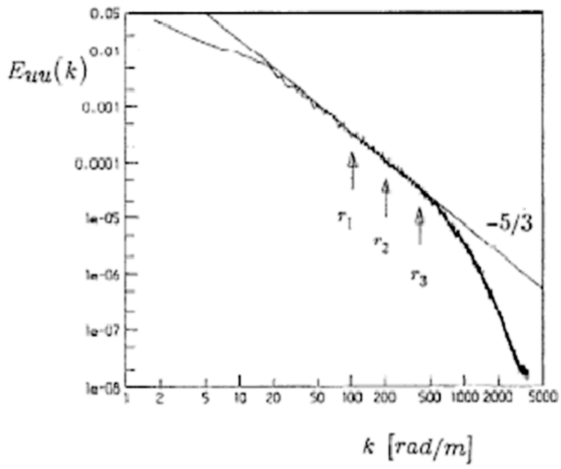

Figure 3. Experimental results of turbulent flow in the wake of a cylinder with $Re_D = 10^5$. Log-log plot of the energy spectrum as a function of wave number with varying length scales r for the separation distance between two points in eqn (9). Non-dimensionalized length scales with respect to the cylinder diameter are: $r_1 = 0.72$, $r_2 = 0.36$, and $r_3 = 0.18$. The straight line with -5/3 slope indicates the inertial range. Reprinted from Thoroddsen and Van Atta, 1992 [13], with permission from American Institute of Physics.

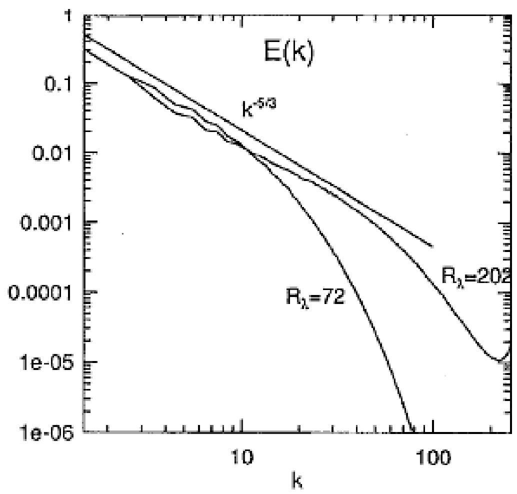

Figure 4. Direct numerical simulation results of turbulent flows in a cubic domain. Log-log plot of the energy spectrum as a function of wave number for Reynolds numbers of $R_\lambda = 72$ and 202. The straight line region with -5/3 slope indicates the inertial range. Reprinted from Chen, et al., 1993 [19], with permission from American Institute of Physics.

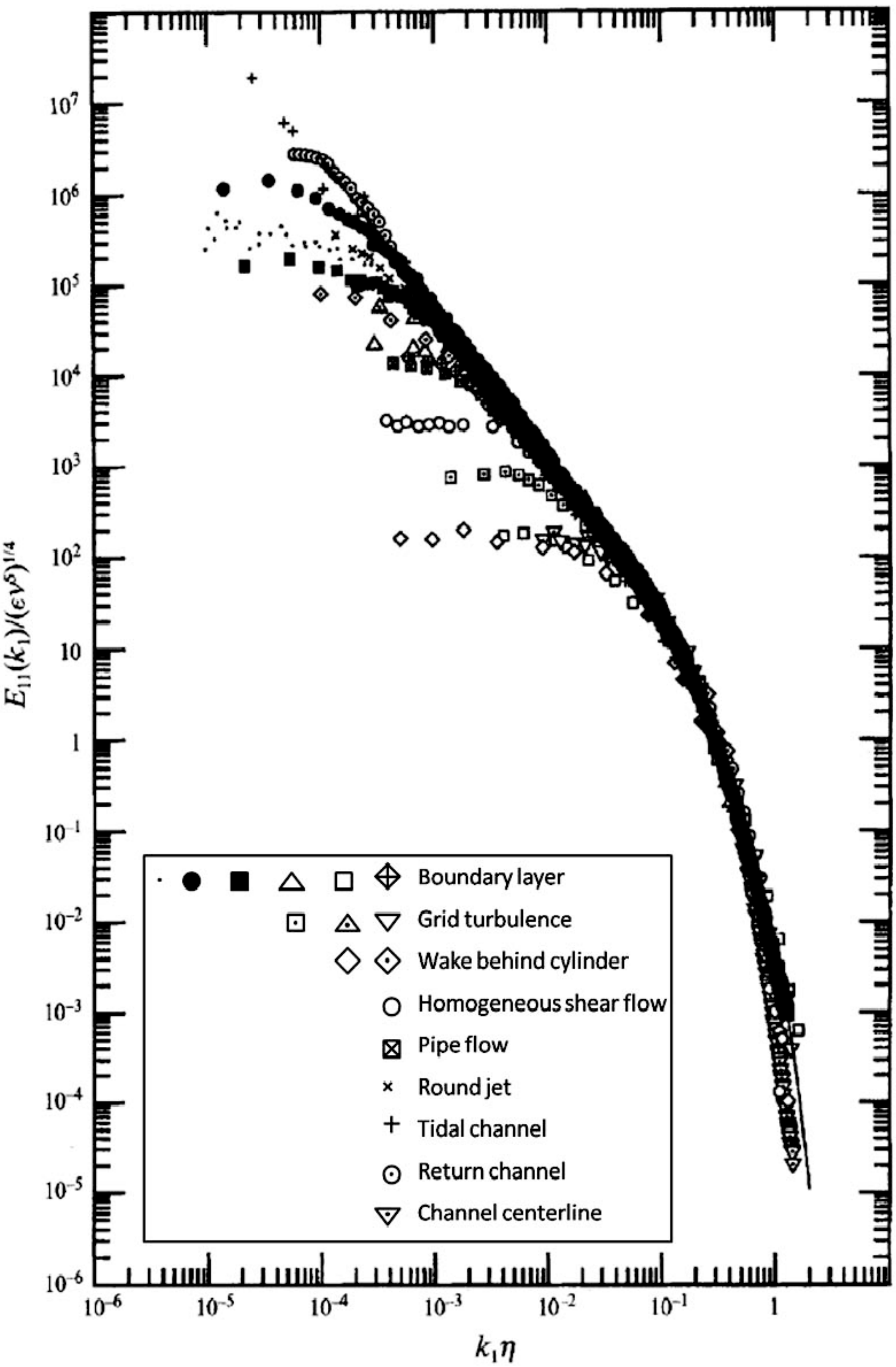

Figure 5. Comparison of experimental results of the one-dimensional longitudinal energy spectra $E_{11}(\kappa_1)$ for boundary layer flows in the middle region of the boundary layer, grid turbulence, wake of a cylinder, homogeneous shear flow, pipe flow, round jet, tidal channel, return channel, and channel centerline (denoted by different symbols) [32]. Reprinted from Saddoughi and Veeravali, 1994 [32], with permission from Cambridge University Press.

4. MAJOR CHALLENGE TO K41 THEORY

4.1 Intermittency

The theories presented forth in K41, though accepted as classical, originated as outgrowths from astute phenomenological and heuristic arguments. The perspicacity of the formulations have been later proven through validation of the theories by experimental results, for example in the classical works of Grant, et al., 1962 [33] and Gibson, 1963 [34]. Nevertheless, questions have arisen speculating the soundness of K41 theory and the validity of the 5/3 law. A major question surrounding K41 theory is the issue of intermittency, which is conspicuously not considered in K41. Intermittency is the tendency of turbulent structures to cluster in concentrated pockets of highly correlated motion characterized by steep velocity gradients. So, in a turbulent flow, the turbulent eddies and vortices will cumulate in intermittent regions rather than be uniformly disbursed throughout. An example of intermittent turbulence can be seen in Figure 1 with intermittent concentrations of vorticity in the wake of the cylinder. The extent of the intermittency will depend on the external flow conditions such as Reynolds number and flow geometry. Furthermore, the influence of intermittency extends to affect the cascading of energy that generates eddies in the inertial range. In this way, the unique features of the external flow conditions are translated to the inertial range. Therefore, the statistical properties of the inertial range such as the energy spectrum $E(\kappa)$ cannot be universal as the 5/3 law stipulates.

In the now legendary 1942 meeting of the Academy of Sciences in Kazan, USSR, L. D. Landau first hinted at the idea of intermittency by remarking during discussions regarding Kolmogorov's work that the eddies in a turbulent field are distributed in limited regions of the flow [25, 35]. Landau subsequently formally presented an argument for intermittency in the 1944 first Russian edition of Landau and Lifshitz, 1987 [36]. The objection of Landau to the universality of the 5/3 law has become part of the lore of turbulent fluid dynamics. The intended meaning of Landau's statements on intermittency remains the subject of rigorous study to the present day. It is the classical counter-argument to K41 theory and the universal 5/3 law. With intermittency, the assumptions underlying K41 theory of isotropy and homogeneity in the inertial range would no longer hold true. In the 1991 special edition of the Proceedings: Mathematical and Physical Sciences, She, et al., 1991 stresses that the issue of intermittency *"presents a serious challenge to the K41 theory"* [30]. The challenge of intermittency will be considered here.

4.2 Fractal Nature of Turbulent Intermittency

Theories abound aiming to capture the nature of intermittency. A classical representation is to depict turbulence as fractals. The definition of a fractal is aptly given by Mandelbrot, 1991 [37] as:

> *"Fractal sets and multi-fractal measures are geometric objects such that each part is very much like a reduced image of the whole."*

This description conjures up images of the children's puzzle that contains a pattern embedded within the same pattern that, in turn, is embedded within the same pattern. Such an intricate fractal nature augments the fascination and bewilderment that turbulence provokes. Figure 6 shows an example of the fractal nature of turbulence, as vortices are formed in atmospheric turbulence off the coast of Chile. Notice the striking resemblance between the flow patterns in Figures 1 and 6, in spite of their vastly different orders of magnitude in the length scales.

Novikov and Stewart, 1964 [39] presents a simple and clear paradigm of turbulence as fractals by considering the intermittent distribution of the kinetic energy dissipation rates in the flow [9]. K41 theory and the universal 5/3 law are predicated upon the assumption of a homogeneous and uniform distribution of the kinetic energy dissipation rate throughout the flow. Thus, intermittency in ε directly opposes K41 theory and the universality of the 5/3 law. Novikov and Stewart, 1964 depicts intermittency in ε by beginning first with a cube of dimension L_0 whose magnitude is on the order of the length scale of the external flow geometry L [9, 39]. This cube is then sub-divided into n numbers of first-order inner cubes whose dimensions are $L_1 = L_0/n^{1/3}$. To account for intermittency, the kinetic energy dissipation rate ε is assumed to occur in only a subset m number of the n cubes with $m \ll n$. In keeping with the fractal nature of intermittency, each of the m first-order cubes are again sub-divided into n cubes of dimension $L_2 = L_1/n^{1/3}$ and of which m subset numbers of second-order cubes contain concentrations of ε [9, 39]. The sequence of sub-divisions continues until the length scales are reached where viscous forces are dominant. At that point, ε is distributed homogeneously throughout all cubes.

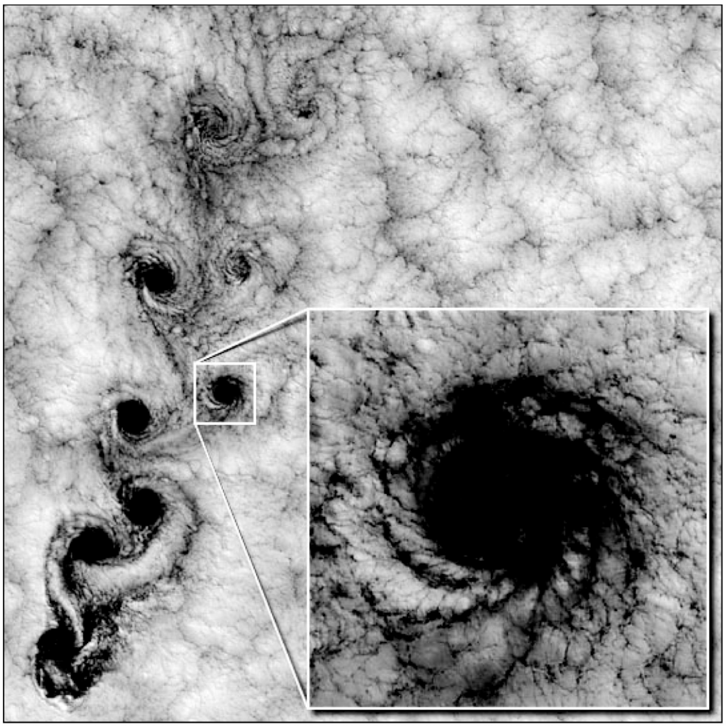

Figure 6. Satellite image of vortex formation in clouds off the Chilean coast near the Juan Fernandez Islands (also known as Robinson Crusoe Islands) [38].

Of course, the model presented by Novikov and Stewart, 1964 [39] is a simplistic facsimile of the fractal nature of turbulent intermittency, and further refinements can be made to achieve better convergence with the realistic phenomenon. Yaglom, 1966 [40] presents a more sophisticated model where, again, an initial cube is sub-divided into n numbers of inner cubes. In this case, each cube is assumed to contain a kinetic energy dissipation rate that is a random variable whose probability distribution function is the same for all the inner cubes [9, 40]. So, with each flow experiment or realization, each inner cube will exhibit a varying with frequencies of occurrence in accordance with a probability distribution function. Next, each inner cube is sub-divided further to n numbers of second-order cubes with each containing that is a random variable defined by a probability distribution function [9, 40]. The sequence proceeds until the final stage at length scales of viscous dissipation.

Using a similar fractal perspective, the classical β-model of turbulence intermittency has been derived. In the β-model, as eddies break up to form smaller scale eddies, the volume occupied by each successive generation of eddies is depicted to be progressively less than previous generations. In this way, each successive generation of eddies will become increasingly spatially intermittent. Assume that the length scales of the succession of eddies scale as [9]:

$$\ell_0 = 2\ell_1 = 2^2\ell_2 = \cdots = 2^n\ell_n \tag{11}$$

where ℓ_n is the length scale of the eddies of generation n. The reduction in volume occupied by successive generations of eddies can then be described by the parameter β such that [9]:

$$\beta = \frac{volume\, occupied\, by\, generation\, n+1\, eddy}{volume\, occupied\, by\, generation\, n\, eddies} = \frac{N\ell_{n+1}^3}{\ell_n} = \frac{2^D}{2^3} \tag{12}$$

where D is a fractal dimension and N is the number of $n+1^{\text{th}}$ generation eddies formed by the breakup of one n^{th} generation eddy. Consequently, the second-order structure function in this scenario would scale as [9]:

$$D_{\ell\ell}(r) \sim \langle \varepsilon \rangle^{\frac{2}{3}} r^{(5-D)/3}. \tag{13}$$

More generally, the p^{th}-order structure function would scale as [9]:

$$\left\langle \left[u_\ell(\vec{x}_0 + \vec{x}) - u_\ell(\vec{x}_0) \right]^p \right\rangle \sim \langle \varepsilon \rangle^{p/3} r^{\xi} \tag{14a}$$

$$\xi = \frac{p}{3} + (3 - D)\left(1 - \frac{p}{3}\right). \tag{14b}$$

Even further refinements to the model are possible. Kolmogorov, 1962 [26] and Obukhov, 1962 [41] consider a spherical volume and a related quantity ε_r, taken to be the kinetic dissipation rate averaged over a sphere of radius r. Using a similar argument as Yaglom, 1966 [40] except with spherical geometries, Kolmogorov, 1962 [26] and Obukhov, 1962 [41] suggest a log-normal probability distribution for ε_r. By that, the quantity $1\text{n}\varepsilon_r$ follows a normal or Gaussian distribution with a variance of [9, 41]:

$$\sigma^2 = A(\vec{x}, t) + 9\alpha \ln\left(\frac{L}{\eta}\right) \tag{15}$$

where $A(\vec{x}, t)$ is a variable parameter that depends on the external flow geometry, $\vec{x} = [x, y, z]$ is the spatial variable, t is the time variable, L is the length scale of the external flow, η is the length scale of viscous dissipation, and α is a universal constant. With the log-normal distribution, the Kolmogorov 2/3 law given in eqn (9) is refined to become [9, 41]:

$$D_{\ell\ell}(r) = C(\vec{x}, t)\langle \varepsilon \rangle^{\frac{2}{3}} r^{\frac{2}{3}} \left(\frac{L}{r}\right)^{-\alpha}. \tag{16}$$

The quantity $C(\vec{x}, t)$ is no longer a universal constant but is a variable parameter. Equation (16) has become well-known as Kolmogorov's Refined Similarity Hypothesis, and Kolmogorov, 1962 [26] is now better known as K62, a follow-up to K41, where Kolmogorov acknowledges the objection of Landau and includes a refinement to K41 theory to account for intermittency.

Debate continues to the present time regarding the nature of intermittency and its effects on turbulent fluid dynamics. A confounding factor of intermittency is that an extreme level of it to the extent where turbulent eddies are highly concentrated in very small fractions of the flow is necessary to exert even an appreciable impact on the turbulent properties such as the 2/3 law [9]. This is one reason for the dichotomy of opinions regarding the validity of K41 theory. Experiments that do not produce strong intermittency would tend to support K41 theory, whereas, those that do generate sufficient intermittency would contend for the need to correct for it [12-21, 33, 34]. Contemporary debate regarding K41 theory and intermittency will now be considered.

5. CONTEMPORARY THOUGHT ON K41

A forerunner in contemporary thought on K41 theory is V. R. Kuznetsov. In Kuznetsov and Praskovsky, 1992 [42], the impact of intermittency on the universality of the 2/3 law is investigated with experimental measurements of wind tunnel turbulence. The experimental results aver definitively that both the Kolmogorov's constant C of the 2/3 law in eqn (9) and the exponent α in the K62 Refined Similarity Hypothesis in eqn (16) are, in fact, highly sensitive to variations in the extent of intermittency [42]. The constant C and exponent α are not universal constants as hypothesized in K41 and K62, respectively. Extensive discourses on intermittency effects on K41 and K62 theory can be found in Kuznetsov and Praskovsky, 1992 [42] and Kuznetsov and Sabel'nikov, 1990 [43].

The 1991 Special Edition of the Proceedings: Mathematical and Physical Sciences commemorating the 50th anniversary of K41 is rich with manifold perspectives and reactions from among the most distinguished personalities in the field in regard to this very significant accomplishment [3]. Notable contributions include Hunt and Vassilicos, 1991 [28] that offers an overview of K41 theory. Van Atta, 1991 [27] examines the legitimacy of the assumption of isotropy of turbulent statistical properties in the inertial range that underlies K41 theory and offers support for it. Sreenivasan, 1991 [31], on the other hand, raises doubts and concerns regarding the assumption of isotropy in K41 and presents a justifiable rebuttal against it. The complicating mystery of intermittency is discussed in She, et al., 1991 [30]. Fractal and multi-fractal models applicable for representing intermittency are presented in Mandelbrot, 1991 [37]. The legendary objection of Landau to the universality of K41 theory is recounted by Spalding, 1991 [35] and Frisch, 1991 [44]. The 1991 Special Edition of the Proceedings: Mathematical and Physical Sciences is a valuable resource indeed for contemporary thought on K41 and also offers a glimpse into the unique interactions within the turbulent fluid dynamics academic community.

An excellent text for fundamental background on turbulent fluid dynamics and K41 theory is Monin and Yaglom, 1971 [23] and 1975 [9]. Both A. S. Monin and A. M. Yaglom are former students of Kolmogorov. A biographic look into the life and works of Kolmogorov is portrayed in Yaglom, 1994 [45].

Other contemporary works of significance include Saddoughi and Veeravalli, 1994 [32] that experimentally studies the assumption of isotropy in the inertial range. She and Leveque, 1994 [16], She and Jackson, 1993 [21], and She, et al., 1993 [15] consider the universality of turbulent statistical properties and K41 theory. Wang, et al., 1996 [46], Chen, et al., 1993 [19], and Thoroddsen and Van Atta, 1992 [13] investigate the K62 Refined Similarity Hypothesis. Finally, Sreenivasan and Kailasnath, 1993 [47] expounds upon the issue of intermittency. Of course, the aforementioned works are a mere modicum sample of works from the library of turbulent fluid dynamics research, a topic of illustrious history and lore.

6. CONCLUSION

This has been an enlightening walk through the museum of turbulent fluid dynamics, with an educational study of its classical concepts and works. There is the “Mischungsweg” that gives indications of the length scale of the flow. Within the spectrum of turbulent length scales, an equilibrium range of scales exists. Within the equilibrium range lies the inertial sub-range. The inertial range possesses the consummate traits of quasi-equilibrium, isotropy, homogeneity, and negligible dissipation from which the universal 2/3 and 5/3 laws of Kolmogorov would manifest, as espoused in the classical work K41. By universality, this law applies to all turbulent flows under the underlying conditions of the inertial range irregardless of differences in the external aspects of the flows such as flow geometries and Reynolds numbers. However, the universality of the 5/3 law has come under question, beginning with the thought-provoking objection of Landau. The problematic issue is the intermittent clustering of turbulent eddies in concentrated regions in the flow. Intermittency can be modeled in the form of fractals and multi-fractals with chains of patterns embedded within patterns. Correction to K41 theory to account for intermittency is given in the form of the Kolmogorov Refined Simliarity Hypothesis of K62.

This study is only a brief discussion of turbulent fluid dynamics. It is the aspiration of the author that the present work would contribute to lifting the intimidating aura of turbulent fluid dynamics from the perceptions of the general academic community. As demonstrated by the voluminous classical works of the past and present, turbulence is, in fact, a problem that can be systematically broken down by a sustained progression of thought. It is truly a fascinating challenge decorated with history and wonderment. To this day, turbulent fluid dynamics remains among the classical unsolved mysteries of science whose presence is ubiquitous in the natural world. In fact, it has been observed that turbulent motion among celestial bodies causes the stars in the galaxies to be clustered in the form of vortices with each galaxy forming an intermittent turbulent eddy [48-50]. The size of a galaxy would then define a type of turbulent length scale of the universe, if you will. Such is the fractal nature of turbulence. Scale up, scale down, zoom in, zoom out; the same sacrosanct law is upheld consistently in all circumstances, from vortex formation behind a cylinder to the stars in the galaxies. So, if the sizes of the galaxies represent the turbulent length scales of the universe; and turbulent length scales are painstakingly small; then how infinitely great must be the universe?

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