Effect of Bimodularity on Dynamic Response of Cross-ply Conical Panels

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Abstract
The effect of bimodularity ratio on free vibration characteristics of cross-ply conical panels of various geometry and lamination scheme is studied. Forced vibration response is also studied for a typical case. The formulation is based on first order shear deformation theory and Bert's constitutive model. An iterative eigenvalue approach is employed to obtain the positive and negative half cycle free vibration frequencies. Galerkin's approach in time domain is used to obtain the frequency response. It is interesting to note that there is a significant difference between positive and negative half cycle frequencies depending on panel parameters. Also, there is a significant difference in positive and negative half cycle forced response amplitudes due to bimodularity.

Key words: Bimodular, cross-ply, conical panels

1. INTRODUCTION

Bimodularity, the different behavior of material in tension and compression, affects the static and dynamic response of structures. Few studies on static analysis of bimodulus laminated cross-ply composite shells and dynamic analysis of cross-ply panels have been presented [1-4]. To the best of the authors' knowledge, the work on the analysis of bimodular laminated cross-ply conical shell panels is not dealt in the literature. The effect of bimodularity ratio on free and forced vibration characteristics of cross-ply conical panel is important for design of such structures under dynamic loading condition.

Here, the dynamic analysis of cross-ply laminated conical panels of bimodulus material is carried out using finite element method and Bert's constitutive model. The effect of semi-cone angle, number of layers and bimodularity ratio (E_{2t}/E_{2c}) on free vibration frequencies and frequency response is investigated.

2. FORMULATION

The geometry and dimensions of a conical panel is shown in Fig. 1 with total thickness h, small end radius r_1, large end radius r_2, meridional length L, circumferential length b at small end, sector angle ψ, and semi-cone angle α. Displacements u, v, w at a point (s, θ, z) are expressed as functions of middle surface displacements u_0, v_0, w_0 and independent rotation β_s, β_θ of the meridional and hoop sections, respectively, as:

$$
\begin{aligned}
u(s, \theta, z, t) &= u_0(s, \theta, t) + z\,\beta_s(s, \theta, t) \\
v(s, \theta, z, t) &= v_0(s, \theta, t) + z\,\beta_\theta(s, \theta, t) \\
w(s, \theta, z, t) &= w_0(s, \theta, t)
\end{aligned}
\tag{1}
$$

where t is time.

A C^0 eight-noded serendipity quadrilateral shear flexible shell element with 5 nodal degrees of freedom ($u_0, v_0, w_0, \beta_s, \beta_\theta$) based on field consistency approach is used for the analysis. Using Lagrange's equations of motion, the element level governing equations are generated. Following the usual finite element assembly procedure, the governing equations of motion for the panel can be written as:

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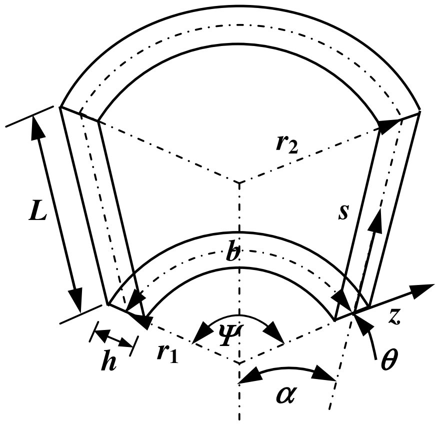


Figure 1. Geometry and coordinate system of a conical shell panel

$$[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}=\{F\} \tag{2}$$

where [M], [C] and [K] are global mass, damping and stiffness matrices and {F} is global load vector. Assuming the solution $\{\delta\}=\{\bar{\delta}\}e^{i\omega t}$ for free vibration analysis, Eq. (2) can be rewritten as:

$$[K]\{\bar{\delta}\}=\omega^2[M]\{\bar{\delta}\} \tag{3}$$

The free vibration frequencies are extracted using iterative eigenvalue approach for simply supported and clamped cross-ply conical shell panels.

The steady state forced response of panels subjected to uniformly distributed harmonic excitation ($q = q_0$Cos $\omega_F\, t$) is obtained using Galerkin method in time domain [5]. The steady state solution is assumed as:

$$\{\delta\}=\{\delta_0\}+\sum_{i=1}^{M}\{\delta_{ci}\}Cos\, i\omega_F t+\{\delta_{si}\}Sin\, i\omega_F t \tag{4}$$

where ω_F is forcing frequency.

Substituting Eq. (4), Eq. (2), can be rewritten as:

$$-\omega_F^2[M]\sum_{i=1}^{M}i^2\left(\{\delta_{ci}\}Cos\, i\omega_F t+\{\delta_{si}\}Sin\, i\omega_F t\right)-\omega_F[C]\sum_{i=1}^{M}i\left(\{\delta_{ci}\}Sin\, i\omega_F t-\{\delta_{si}\}Cos\, i\omega_F t\right)+[K]\{\delta_0\}$$
$$+[K]\sum_{i=1}^{M}\left(\{\delta_{ci}\}Cos\, i\omega_F t+\{\delta_{si}\}Sin\, i\omega_F t\right)-\{F_0\}Cos\,\omega_F\, t=\{0\} \tag{5}$$

Let the L.H.S of Eq. (5) be $\{R\}$. In Galerkin method, for each weighting function (say Cos $i\omega_F$t), the integration w.r.t time is performed piecewise as follows:

$$\int_0^{t_1}\{R\}Cos\, i\omega_F t\, dt+\int_{t_1}^{t_2}\{R\}Cos\, i\omega_F t\, dt+\int_{t_2}^{2\pi/\omega_F}\{R\}Cos\, i\omega_F t\, dt=0$$

Here t_1 and t_2 are time instants within a cycle when displacement changes from positive/negative to negative/positive. It may be noted that $\{R\}$ is different for positive and negative displacements. Since t_1 and t_2 are not known *a priori*, solution is obtained using Newton-Raphson iterative method.

3. RESULTS AND DISCUSSION

The results are presented for CCCC (all edges clamped: $u_0 = v_0 = w_0 = \beta_s = \beta_\theta = 0$) cross-ply conical panels for fundamental mode of vibration.

The material properties used are:

In tension: $E_{1t}/E_{2t} = 25$, $E_{2t} = E_{3t}$, $E_{3t} = E_{2t}$, $G_{12t}/E_{2t} = G_{13t}/E_{2t} = 0.5$, $G_{23t}/E_{2t} = 0.2$

$\nu_{12t} = \nu_{23t} = \nu_{13t} = 0.25$.

In compression: $E_{1c}/E_{2c} = 25$, $E_{2c} = E_{3c} = 1$ GPa, $G_{12c}/E_{2c} = G_{13c}/E_{2c} = 0.5$, $G_{23c}/E_{2c} = 0.2$,

$\nu_{12c} = \nu_{23c} = \nu_{13c} = 0.25$.

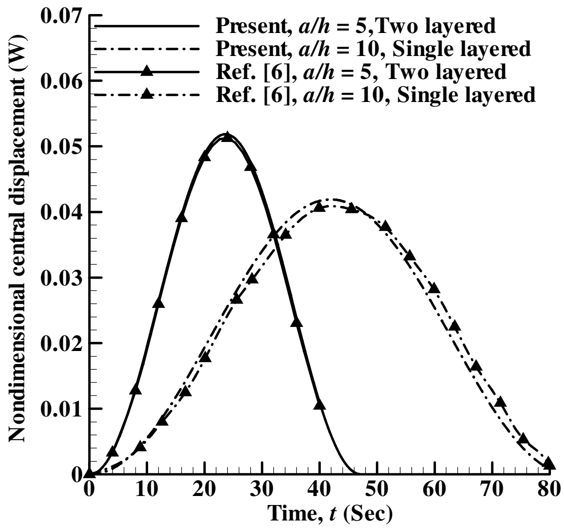


Figure 2. Comparison of nondimensional central displacement response for single (0°) and two (0°/90°) layered cross-ply square plate of bimodular material

Based on the convergence study, 10 × 10 mesh discretization of full panel and $M = 2$ in Eq. (4) are found to yield accurate results. The free vibration frequencies corresponding to positive and negative half cycles are plotted in the non-dimensional form as: $(\Omega_1, \Omega_2) = (\omega_1, \omega_2) b^2 (\rho / E_{2c} h^2)^{1/2}$. The other nondimensional quantities are defined as: central displacement $W = [w_0 h^3 E_{2c} / q_0 L^4]$, nondimensional stresses: $(S_{11}, S_{22}, S_{12}) = (\sigma_{11}, \sigma_{22}, \tau_{12}) / (q_0 S^2)$; where $S = b/h$. The average frequency (ω) over the entire cycle is given by: $\omega = 2(1/\omega_1 + 1/\omega_2)^{-1}$. The transient response obtained from present formulation using Newmark's direct time integration is compared with that of Ref. [6] in Fig. 2 and is found to be in good agreement.

The positive and negative half cycle frequency parameters are plotted for two-and eight-layered cross-ply panels with $\Psi = 15°$, $r_1/h = 50$, 100 for different L/b and E_{2t}/E_{2c} ratio in Figs. 3-4.

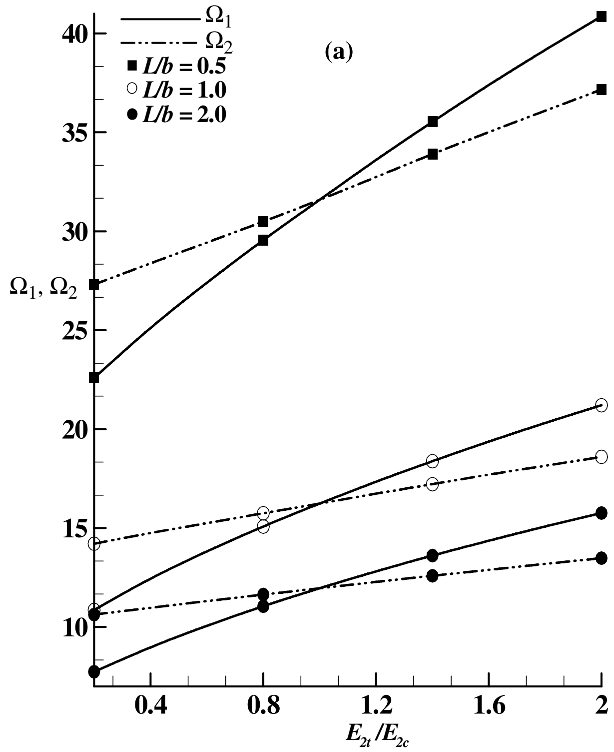
As the bimodularity ratio increases, the material properties increase making the structure more stiff and hence frequency parameters increase. The positive half cycle frequency is smaller than negative half cycle frequency for $E_{2t}/E_{2c} < 1$ and is greater for $E_{2t}/E_{2c} > 1$. At $E_{2t}/E_{2c} = 1$, Ω_1 and Ω_2 are same.

The difference between positive and negative half cycle frequencies is highest for $E_{2t}/E_{2c} = 0.2$. It is observed from these figures that as number of layers increases, the frequency parameters increase.

The r_1/h ratio has a little effect on frequency parameter but L/b ratio has significant effect on it. It is observed that as L/b increases, the frequency parameters decrease.

The positive and negative half cycle frequency parameters are plotted for two- and eight-layered cross-ply panels with $\Psi = 45°$, $r_1/h = 50{,}100$ for different L/b and E_{2t}/E_{2c} ratios in Figs. 5-6. The nature of free vibration frequency variation with E_{2t}/E_{2c} is similar to that for $\Psi = 15°$.

The effect of bimodularity ratio on the frequency response is studied for CCCC conical panel ($L/b = 0.5$, $r_1/h = 50$, $b/h = 10$, $\Psi = 15°$) subjected to uniformly distributed harmonic force. The proportional damping is taken as: $[C] = 0.02\omega[M]$. The central displacement amplitude versus forcing frequency curve is shown in Fig. 7.



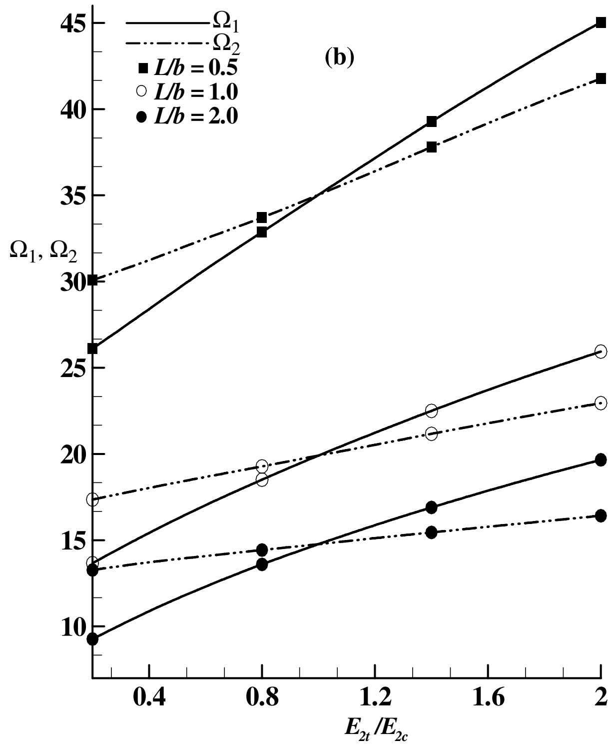


Figure 3. Variation of positive and negative half cycle frequencies for various L/b ratios with E_{2t}/E_{2c} of cross-ply panels (r_1/h = 50, ψ = 15°, b/h = 10) (a) two-layered: (0°/90°) (b) eight-layered: $(0°/90°)_4$

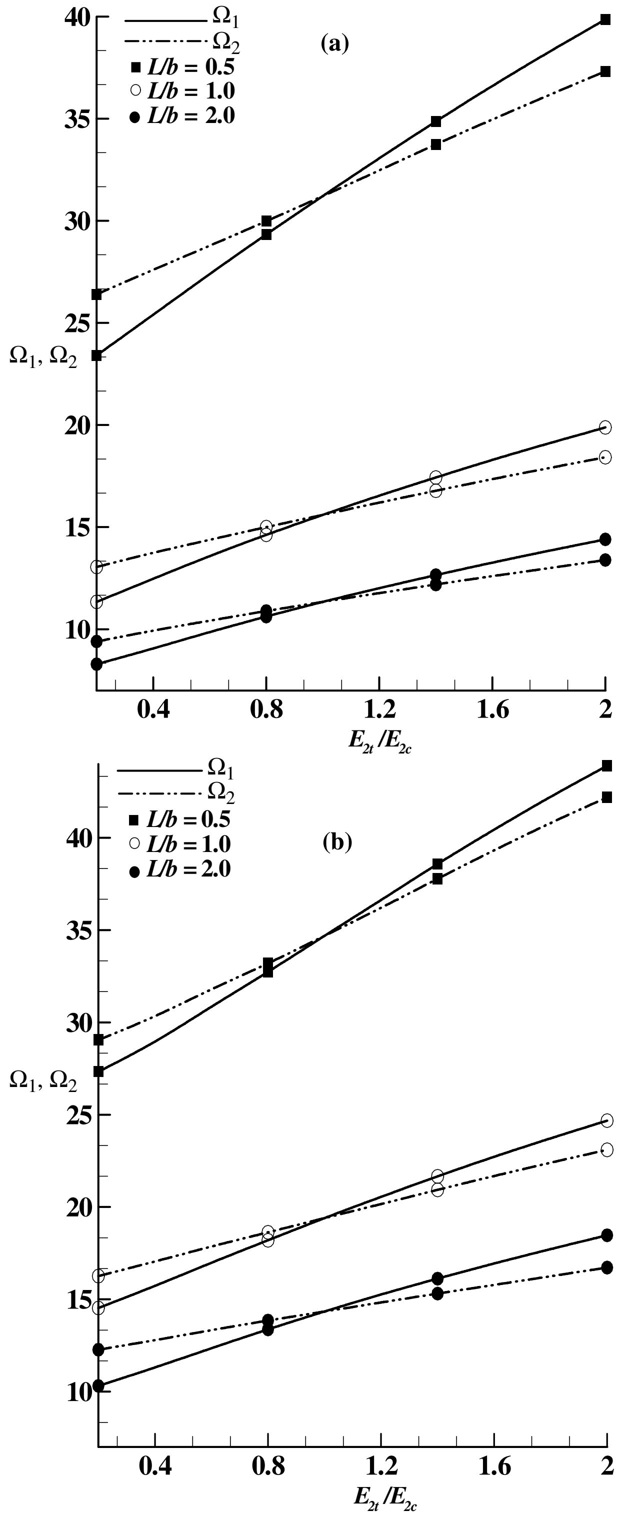
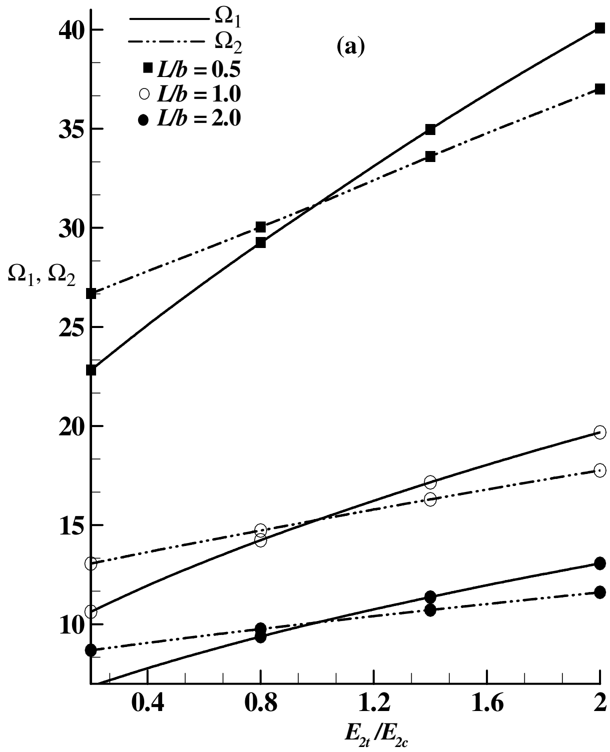


Figure 4. Variation of positive and negative half cycle frequencies for various L/b ratios with E_{2t}/E_{2c} of cross-ply panels ($r_1/h = 100$, $\Psi = 15°$, $b/h = 10$) (a) two-layered: (0°/90°) (b) eight-layered: $(0°/90°)_4$



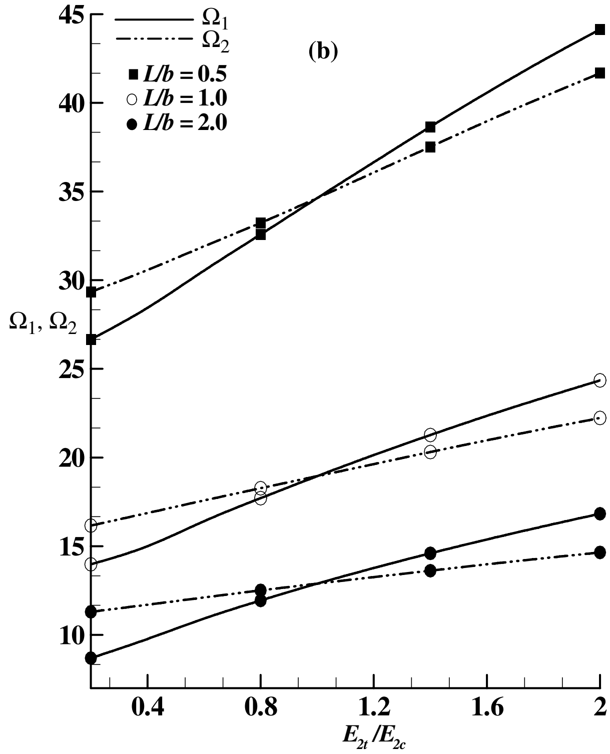


Figure 5. Variation of positive and negative half cycle frequencies for various *L/b* ratios with E_{2t}/E_{2c} of cross-ply panels ($r_1/h = 50$, $\Psi = 45°$, $b/h = 10$) (a) two-layered: (0°/90°) (b) eight-layered: $(0°/90°)_4$

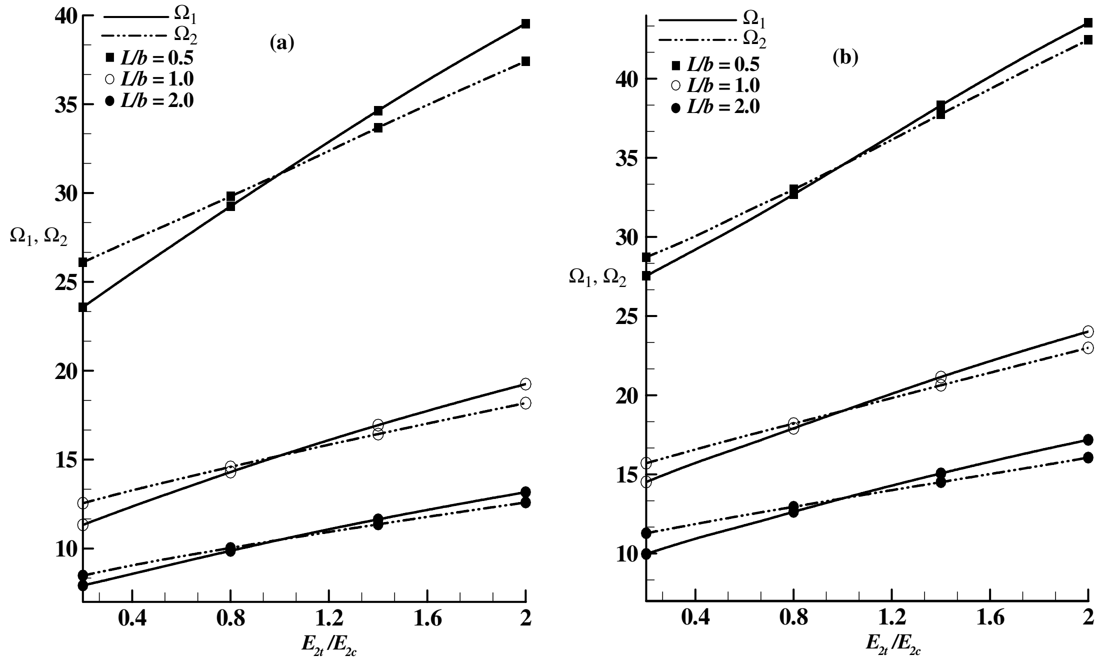


Figure 6. Variation of positive and negative half cycle frequencies for various L/b ratios with E_{2t}/E_{2c} of cross-ply panels ($r_1/h = 100$, $\Psi = 45°$, $b/h = 10$) (a) two-layered: (0°/90°) (b) eight-layered: $(0°/90°)_4$

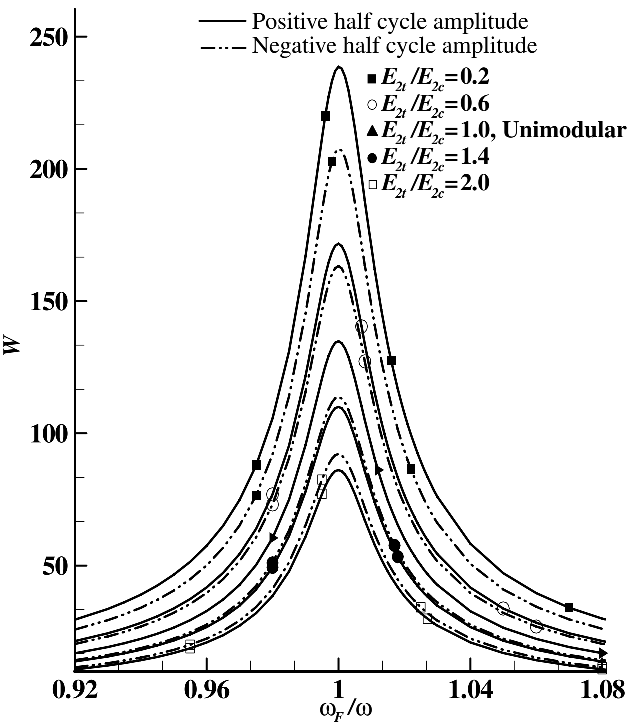


Figure 7. Frequency response of CCCC two layered cross-ply panels (0°/90°, $r_1/h = 50$, $L/b = 0.5$, $b/h = 10$, $\Psi = 15°$)

It can be seen that the bimodularity ratio has significant effect on the frequency response which is quite different from unimodular case.

The non-dimensional fiber direction stresses at the centre for r_1/h = 50, L/b = 0.5, b/h = 10, $(0°/90°)_4$, Ψ = 15° at z/h = 0.5 are plotted for ω_F/ω = 0.99,1.0 for E_{2t}/E_{2c}= 0.2, 1.0, 2.0 in Figs 8-10. The maximum tensile and compressive stress for E_{2t}/E_{2c}=0.2, 2.0 are not same as E_{2t}/E_{2c}=1.0 (unimodular case) and maximum compressive /tensile stress are greater for ω_F/ω = 1.0 compared to ω_F/ω = 0.99.

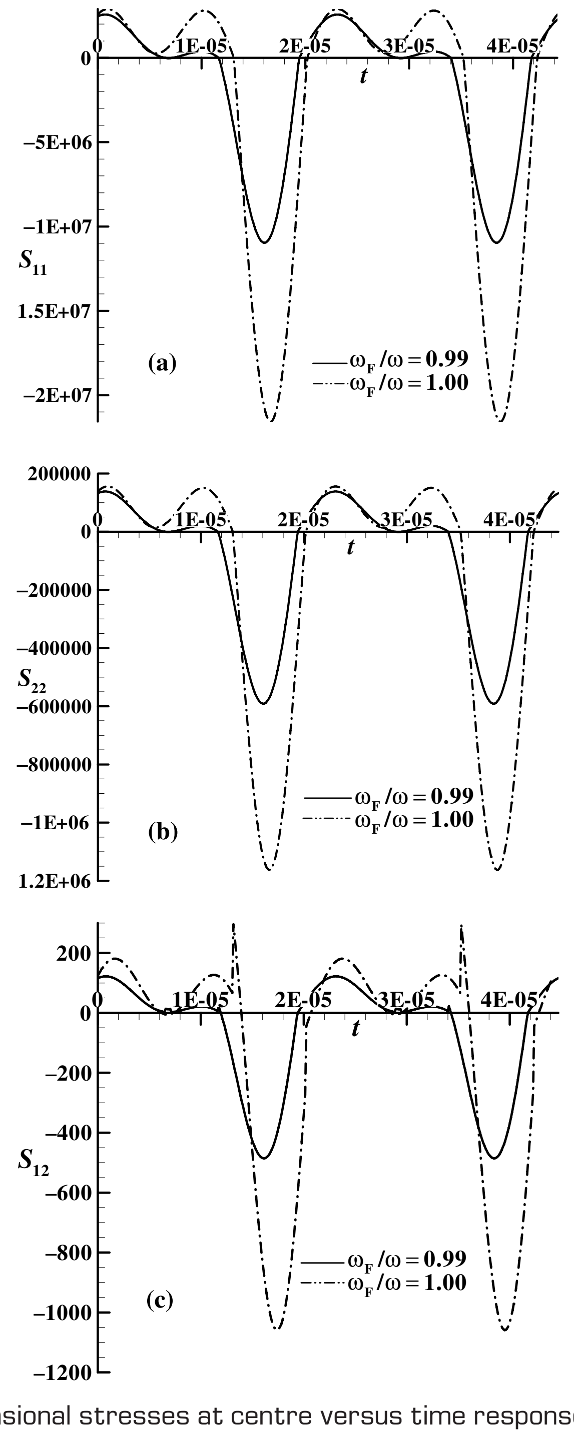


Figure 8. Nondimensional stresses at centre versus time response for E_{2t}/E_{2c} = 0.2 of conical panel ($(0°/90°)_4$, r_1/h = 50, b/h = 10, L/b = 0.5 Ψ= 15°) (a) fiber direction stress (S_{11}) (b) transverse to fiber direction stress (S_{22}) (c) in-plane shear stress (S_{12}) at z/h = 0.5

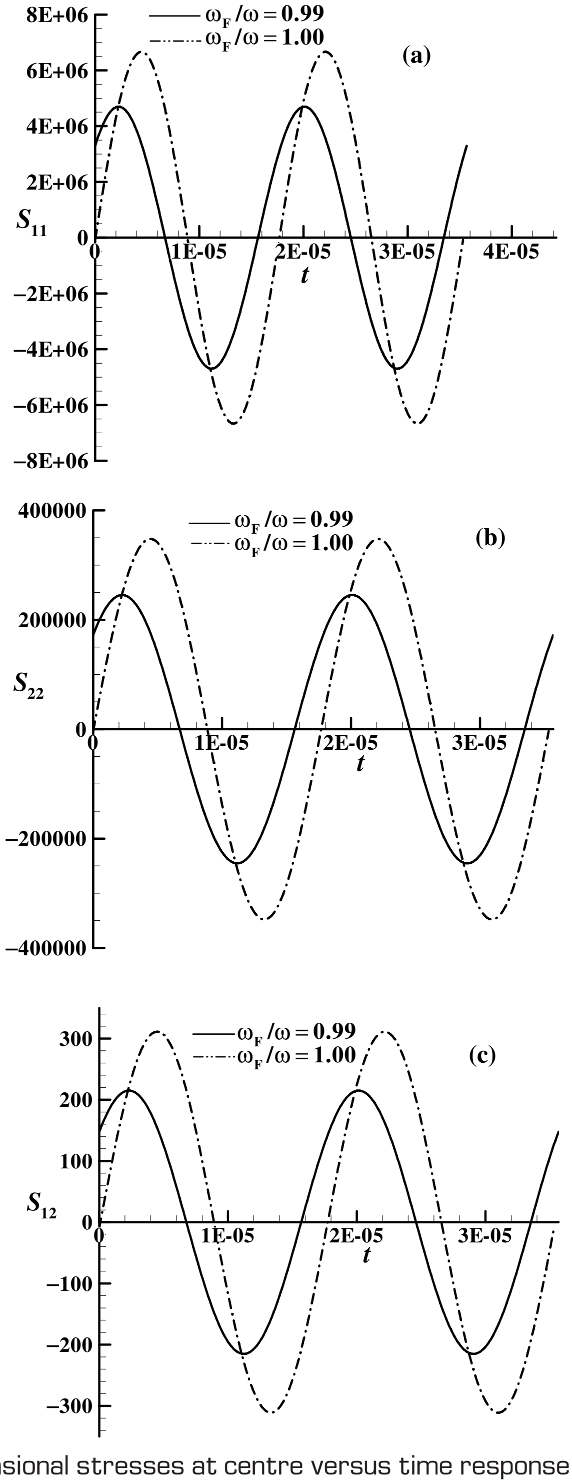


Figure 9. Nondimensional stresses at centre versus time response for $E_{2t}/E_{2c} = 1.0$ of conical panel $((0°/90°)_4$, $r_1/h = 50$, $b/h = 10$, $L/b = 0.5$ $\Psi = 15°)$ (a) fiber direction stress (S_{11}) (b) transverse to fiber direction stress (S_{22}) (c) in-plane shear stress (S_{12}) at $z/h = 0.5$

The point, for which stresses are plotted, is under tension for positive half cycle and under compression in negative half cycle. For E_{2t}/E_{2c}=0.2, the compressive properties are greater than the tensile properties resulting in greater amplitude of S_{11}, S_{22}, and S_{12} in compression compared to tension.

For E_{2t}/E_{2c}=2.0 the tensile properties are greater than the compression properties resulting in greater amplitude of S_{11}, S_{22} and S_{12} in tension compared to compression.

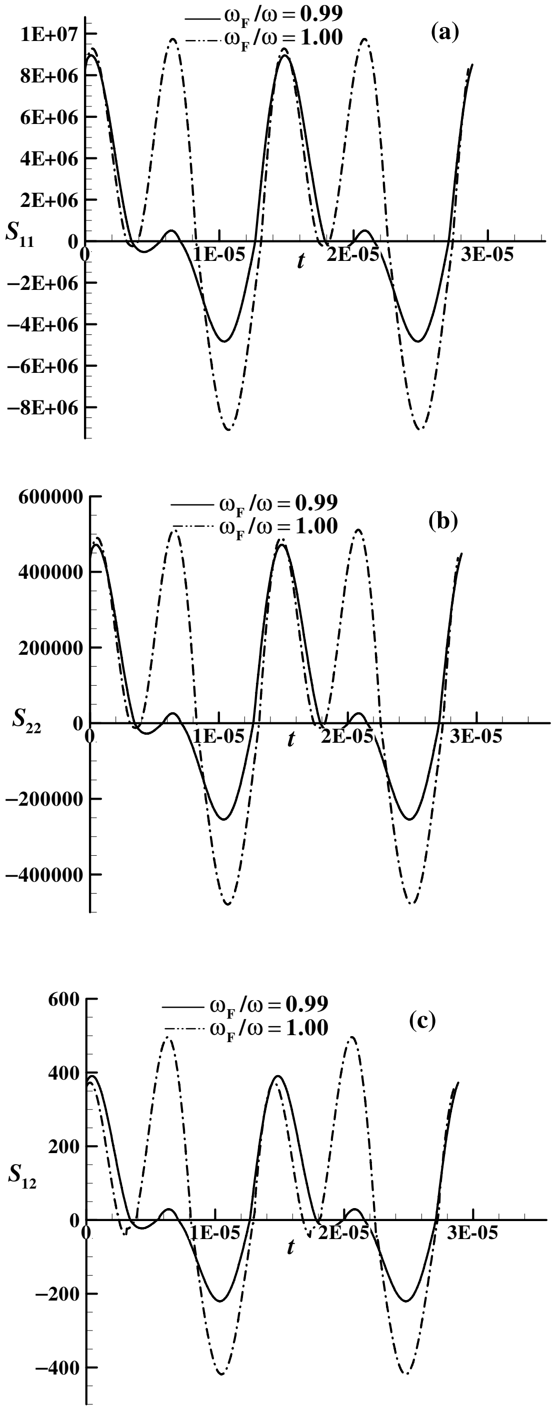


Figure 10. Nondimensional stresses at centre versus time response for E_{2t}/E_{2c} = 2.0 of conical panel ((0°/90°)$_4$, r_1/h = 50, b/h = 10, L/b = 0.5 Ψ= 15°) (a) fiber direction stress (S_{11}) (b) transverse to fiber direction stress (S_{22}) (c) in-plane shear stress (S_{12}) at z/h = 0.5

The fiber direction non-dimensional stress (S_{11}) for conical panel (r_1/h = 50, L/b = 0.5, b/h = 10, (0°/90°)$_4$ Ψ = 15°) for E_{2t}/E_{2c} = 0.2, 1.0, 2.0 for various ω_F/ω ratio is shown in Fig 11. The amplitude of compressive stress is greater for E_{2t}/E_{2c} = 0.2 and smaller for E_{2t}/E_{2c} = 2.0 compared to tensile stress.

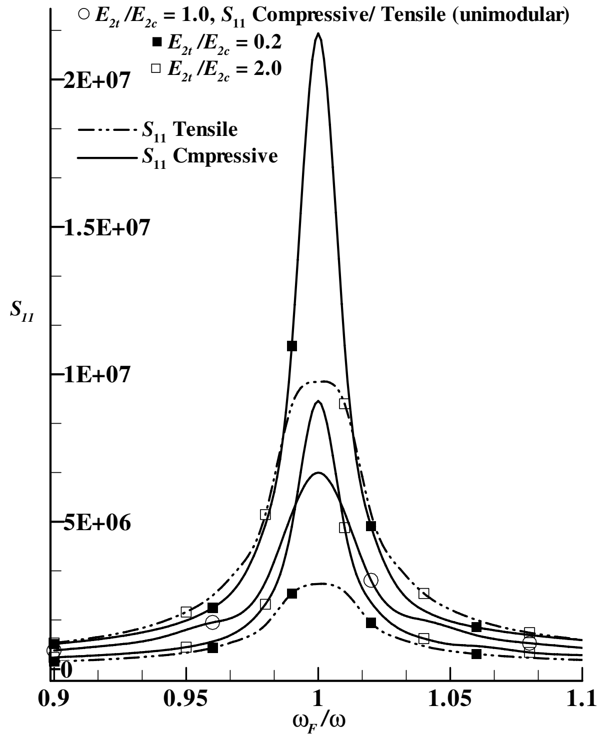


Figure 11. Amplitude of S_{11} versus ω_F/ω curves for various E_{2t}/E_{2c} ratio ($r_1/h = 50$, $L/b = 0.5$, $b/h = 10$, $(0°/90°)_4$, $\Psi = 15°$)

4. CONCLUSIONS

1) The positive and negative half cycle frequencies are different for $E_{2t}/E_{2c} \neq 1$.
2) The positive half cycle frequency is smaller than negative half cycle frequency for $E_{2t}/E_{2c} < 1$ and is greater for $E_{2t}/E_{2c} > 1$.
3) The r_1/h ratio has a little effect on frequency parameter but L/b ratio has significant effect on frequency parameters.
4) The stress distribution is significantly different in positive and negative half cycles for bimodular panels unlike unimodular panels.

5. ACKNOWLEDGEMENT

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6. REFERENCES

[1] C.W. Bert, V.S. Reddy, Cylindrical shells of bimodulus composite material, *Journal of Engineering Mechanics Division*, ASCE 108:675-688, 1982.

[2] C.W. Bert, M. Kumar, Vibration of cylindrical shells of bimodulus composite material, *Journal of Sound and Vibration* 81:107-121, 1982.

[3] K. Khan, B.P. Patel, Y. Nath, Vibration analysis of bimodulus laminated cylindrical panels, *Journal of Sound and Vibration*, 321:166-183, 2009.

[4] K. Khan, B.P. Patel, Y. Nath, Free vibration of bimodulus laminated cross-ply conical panels, Proceedings of the 9th Biennial ASME conference on Engineering System Design and Analysis ESDA08, July7-9, 2008, Haifa, Israel.

[5] K. Khan, B.P. Patel, Y. Nath, Frequency response of bimodular cross-ply laminated cylindrical panels, *Journal of Sound and Vibration*, 327:55-69, 2009.

[6] J.N. Reddy, Transient response of laminated, bimodular-material, composite rectangular plates, *Journal of Composite Materials* 16:139-152, 1982.