

An approach to manufacture a hetero-bipolar transistors in thin film structures. On the method of optimization

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In this paper we introduce an approach to manufacture more thin heterobipolar transistors due to diffusion and/or implantation of ion of dopants in heterostructure with introduced in this paper configuration and optimization of annealing of dopant and/or radiation defects. At the same time we introduce an analytical approach to model technological processes.

INTRODUCTION

One of intensively solving questions of the solid state electronic is decreasing of dimensions of elements of integrated circuits (p-n-junctions, transistors, ...) [1-12]. As a results of the decreasing it is necessary to decrease dimensions of elements of integrated circuits. To manufacture the elements one can use different approaches: dopant diffusion or implantation of ions of dopants in homogenous sample or heterostructure, epitaxial growth [8-12]. To decrease dimensions of elements of integrated circuits it could be used inhomogenous of temperature distribution, which was generated by laser or microwave type of annealing [13-17]. Another way to decrease these dimensions is using of inhomogeneity of doping structure and optimization of regime of annealing [18-21]. It is attracted an interest presents a distribution of defects in doping structure. One way to produce the distribution is radiation processing of materials [19,20,22]. Another way to obtain the same result is generation of dislocations of discrepancy [23]. Framework this paper we consider a heterostructure, which consist of a substrate and an epitaxial layer with several sections (see Figs. 1 and 2). Some dopants have been infused or implanted in the sections to generate n or p types of conductivity as it is shown in Figs. 1 and 2. Farther annealing of dopant and/or radiation defects has been done. Main aims of the present paper is modeling of redistributions of dopant and radiation defects during the above annealing and formulation conditions to maximal decreasing of dimensions of heterobipolar transistor.

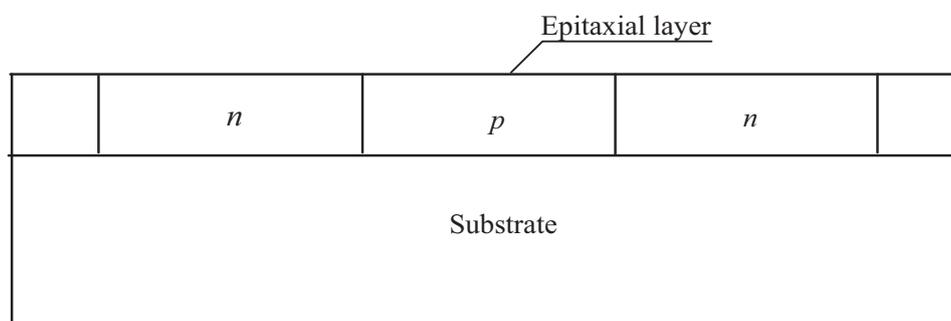


Fig. 1. Heterostructure, which consist of a substrate and an epitaxial layer with several sections. View from one side

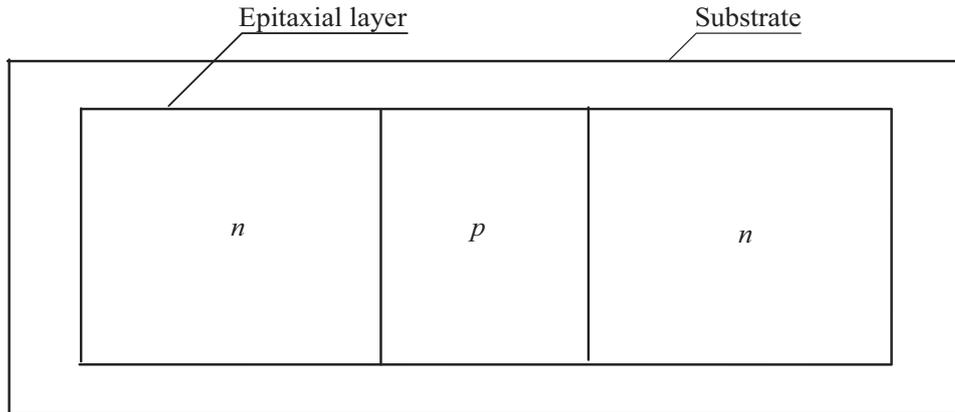


Fig. 2. Heterostructure, which consist of a substrate and an epitaxial layer with several sections. View from top

METHOD OF SOLUTION

To solve our aims let us determine spatio-temporal distributions of dopants. We calculate the distributions by solving the second Fick's law [8-12]

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_c \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_c \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_c \frac{\partial C(x, y, z, t)}{\partial z} \right] \quad (1)$$

with boundary and initial conditions

$$\left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad (2)$$

$$\left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad C(x, y, z, 0) = f(x, y, z).$$

Here $C(x, y, z, t)$ is the spatio-temporal distribution of concentration of dopant, T is the temperature of annealing, D_c is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials in heterostructure, speed of heating and cooling of heterostructure (with account of Arrhenius law). Dependences of dopant diffusion coefficient on parameters could be approximated by the following function [24-26]

$$D_c = D_L(x, y, z, T) \left[1 + \xi \frac{C^y(x, y, z, t)}{P^r(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right], \quad (3)$$

where $D_L(x, y, z, T)$ is the spatial (due to inhomogeneity of heterostructure) and temperature (framework Arrhenius law) dependences of dopant diffusion coefficient; $P(x, y, z, T)$ is the limit of solubility of

dopant; parameter γ depends on properties of materials and could be integer in the interval $\gamma \in [1,3]$ [9]; $V(x, y, z, t)$ is the spatio-temporal distribution of concentration of vacancies; V^* is the equilibrium distribution of vacancies. Concentrational dependence of diffusion coefficient has been described in details in [9]. It should be noted, that in the case of diffusion doping radiation damages are absent and $\zeta_1 = \zeta_2 = 0$. Spatio-temporal distributions of concentrations of radiation defects we determine by solving the following system of equations [25,26]

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - \\ & - k_{I,I}(x, y, z, T) I^2(x, y, z, t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial V(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - \\ & - k_{V,V}(x, y, z, T) V^2(x, y, z, t) \end{aligned}$$

with initial

$$\rho(x, y, z, 0) = f_\rho(x, y, z) \quad (5a)$$

and boundary conditions

$$\begin{aligned} \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \\ \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0. \end{aligned} \quad (5b)$$

Here $\rho = I, V$; $I(x, y, z, t)$ is the spatio-temporal distribution of concentration of interstitials; $D_\rho(x, y, z, T)$ are the diffusion coefficients of vacancies and interstitials; terms $V^2(x, y, z, t)$ and $I^2(x, y, z, t)$ corresponds to generation of divacancies and analogous complexes of interstitials; $k_{I,V}(x, y, z, T)$, $k_{I,I}(x, y, z, T)$ and $k_{V,V}(x, y, z, T)$ are parameters of recombination of point radiation defects and generation their complexes.

Farther we determine spatio-temporal distributions of concentrations of divacancies $\phi_v(x, y, z, t)$ and diinterstitials $\phi_i(x, y, z, t)$ by solving of the following system of equations [25,26]

$$\frac{\partial \Phi_I(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t) +$$

$$\begin{aligned}
& + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] - \\
& - k_I(x, y, z, T) I(x, y, z, t)
\end{aligned} \tag{6}$$

$$\begin{aligned}
\frac{\partial \Phi_V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t) + \\
& + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] - \\
& - k_V(x, y, z, T) V(x, y, z, t)
\end{aligned}$$

with boundary and initial conditions

$$\begin{aligned}
\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \\
\left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} &= 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \\
\Phi_I(x, y, z, 0) &= f_{\Phi_I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\Phi_V}(x, y, z).
\end{aligned} \tag{7}$$

Here $D_{\Phi_I}(x, y, z, T)$ and $D_{\Phi_V}(x, y, z, T)$ are the diffusion coefficients of complexes of point defects; $k_I(x, y, z, T)$ and $k_V(x, y, z, T)$ are the parameters of decay of complexes of point defects.

It has been recently shown, that to manufacture of more sharper p-n-junctions and more shallow devices (p-n-junctions, transistors, ...) it is attracted an interest microwave annealing of dopant and/or radiation defects [19,27]. Using microwave annealing leads to production inhomogeneous distribution of temperature. The inhomogeneous distribution leads to increase sharpness of p-n-junctions [19,27]. We described distribution of temperature by the second law of Fourier [28]

$$\begin{aligned}
c(T) \frac{\partial T(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[\lambda(x, y, z, T) \frac{\partial T(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\lambda(x, y, z, T) \frac{\partial T(x, y, z, t)}{\partial y} \right] + \\
& + \frac{\partial}{\partial z} \left[\lambda(x, y, z, T) \frac{\partial T(x, y, z, t)}{\partial z} \right] + p(x, y, z, t),
\end{aligned} \tag{8}$$

with boundary and initial conditions

$$\begin{aligned}
\left. \frac{\partial T(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial T(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial T(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \\
\left. \frac{\partial T(x, y, z, t)}{\partial y} \right|_{y=L_y} &= 0, \quad \left. \frac{\partial T(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial T(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad T(x, y, z, 0) = f_T(x, y, z),
\end{aligned} \tag{9}$$

where $T(x,y,z,t)$ is the spatio-temporal distribution of temperature; $c(T)=c_{\text{ass}} [1-\eta\exp(-T(x,y,z,t)/T_d)]$ is the heat capacitance (in the most interest limiting case, when current temperature is larger, than Debye temperature T_d , we can assume $c(T) \approx c_{\text{ass}}$) [28]; λ is the heat conduction coefficient. Value of heat conduction coefficient depends on properties of materials and temperature. Temperature dependence of heat conduction coefficient in most interest interval of temperature could be approximated by the following relation: $\lambda(x,y,z,T)=\lambda_{\text{ass}}(x,y,z) [1+\mu (T_d/T(x,y,z,t))^\theta]$ (see, for example, [28]). $p(x,y,z,t)$ is the volumetric density of power, which have been generated in heterostructure. $\alpha(x,y,z,T) = \lambda(x,y,z,T)/c(T)$ is the thermal conductivity.

Farther we determine spatio-temporal distribution of concentration of dopant by using method of averaging of function correction [19-21,23,27,29] with decreased quantity of iteration steps [30]. Framework the approach we used solutions of the second laws of Fick's and Fourier with averaged values of diffusion coefficients and thermal conductivity $D_{0L}, D_{0I}, D_{0V}, D_{0\Phi}, D_{0\Phi_V}, \alpha_0$ and zero values of parameters of recombination of defects, generation and decay of their complexes as zero-order approximations of real solutions of the above laws. The above zero-order approximations could be written as

$$C_1(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t),$$

$$I_1(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nI} c_n(x) c_n(y) c_n(z) e_{nI}(t),$$

$$V_1(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nV} c_n(x) c_n(y) c_n(z) e_{nV}(t),$$

$$\Phi_{I1}(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_I} c_n(x) c_n(y) c_n(z) e_{n\Phi_I}(t),$$

$$\Phi_{V1}(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_V} c_n(x) c_n(y) c_n(z) e_{n\Phi_V}(t),$$

$$T_1(x,y,z,t) = \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nT} c_n(x) c_n(y) c_n(z) e_{nT}(t),$$

where $F_{np} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_p(u,v,w) dw dv du$, $\text{cn}(\chi) = \cos(\pi n \chi/L_\chi)$,

$$e_{n\rho}(t) = \exp\left[-\pi^2 n^2 D_{0\rho} t \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)\right], e_{nT}(t) = \exp\left[-\pi^2 n^2 \alpha_0 t \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)\right].$$

We determine approximations of the second and higher orders framework standard iterative procedure of method of averaging of function corrections [19-21,23,27, 29,30]. Framework the approach to determine n-th order approximations of concentrations of dopant and radiation defects we replace the required

functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_1(x,y,z,t)$, $\Phi_V(x,y,z,t)$ and $T(x,y,z,t)$ on the sum of average value of the n -th order approximations and approximations of the $n-1$ -th orders, i.e. $\alpha_{n\rho} + \rho_{n-1}(x,y,z,t)$. The replacement gives us possibility to obtain the following relations of the second-order approximations

$$\begin{aligned} \frac{\partial C_2(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_L(x,y,z,T) \left[1 + \zeta_1 \frac{V(x,y,z,t)}{V^*} + \zeta_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \frac{\partial C_1(x,y,z,t)}{\partial x} \right] \times \\ &\times \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)} \right\} + \frac{\partial}{\partial y} \left[D_L(x,y,z,T) \left[1 + \zeta_1 \frac{V(x,y,z,t)}{V^*} + \zeta_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \right] \times \\ &\times \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)} \right\} \frac{\partial C_1(x,y,z,t)}{\partial y} + \frac{\partial}{\partial z} \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)} \right\} \times \\ &\times D_L(x,y,z,T) \left[1 + \zeta_1 \frac{V(x,y,z,t)}{V^*} + \zeta_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \frac{\partial C_1(x,y,z,t)}{\partial z} \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial I_2(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x,y,z,T) \frac{\partial I_1(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x,y,z,T) \times \right. \\ &\times \left. \frac{\partial I_1(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_I(x,y,z,T) \frac{\partial I_1(x,y,z,t)}{\partial z} \right] - [\alpha_{2I} + I_1(x,y,z,t)] \times \\ &\times k_{I,V}(x,y,z,T) [\alpha_{2V} + V_1(x,y,z,t)] - k_{I,I}(x,y,z,T) [\alpha_{2I} + I_1(x,y,z,t)]^2 \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial V_2(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x,y,z,T) \frac{\partial V_1(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x,y,z,T) \times \right. \\ &\times \left. \frac{\partial V_1(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_V(x,y,z,T) \frac{\partial V_1(x,y,z,t)}{\partial z} \right] - [\alpha_{2I} + I_1(x,y,z,t)] \times \\ &\times [\alpha_{2V} + V_1(x,y,z,t)] k_{I,V}(x,y,z,T) - k_{V,V}(x,y,z,T) [\alpha_{2V} + V_1(x,y,z,t)]^2 \end{aligned}$$

$$\frac{\partial \Phi_{I2}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x} \left[D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_{I1}(x,y,z,t)}{\partial x} \right] + k_{I,I}(x,y,z,T) I^2(x,y,z,t) +$$

$$-k_I(x, y, z, T)I(x, y, z, t) \tag{12}$$

$$\begin{aligned} \frac{\partial \Phi_{V2}(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial x} \right] + k_{v,V}(x, y, z, T)V^2(x, y, z, t) + \\ &+ \frac{\partial}{\partial y} \left[D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial z} \right] - \\ &- k_V(x, y, z, T)V(x, y, z, t) \end{aligned}$$

$$\begin{aligned} c \frac{\partial T(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left(\lambda_{ass}(x, y, z) \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, t)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, t)]}{\partial x} \right) + \\ &+ \frac{\partial}{\partial y} \left(\lambda_{ass}(x, y, z) \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, t)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, t)]}{\partial y} \right) + \\ &+ \frac{\partial}{\partial z} \left(\lambda_{ass}(x, y, z) \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, t)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, t)]}{\partial z} \right) + p(x, y, z, t) \cdot \end{aligned} \tag{13}$$

Integration of the left and right sides of equations (10)-(13) gives us possibility to obtain relations of the second-order approximations of the required functions in the following form

$$\begin{aligned} C_2(x, y, z, t) &= \frac{\partial}{\partial x} \left(\int_0^t D_L(x, y, z, T) \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \frac{\partial C_1(x, y, z, \tau)}{\partial x} \times \right. \\ &\times \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} d\tau + \frac{\partial}{\partial y} \left(\int_0^t \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \right. \\ &\times D_L(x, y, z, T) \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial y} d\tau \right) + \frac{\partial}{\partial z} \left(\int_0^t D_L(x, y, z, T) \times \right. \\ &\times \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} d\tau + \frac{\partial}{\partial y} \left(\int_0^t \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \right. \\ &\times \left. \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial y} d\tau \right) + \end{aligned}$$

$$+f_C(x, y, z) \quad (10a)$$

$$\begin{aligned} I_2(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial y} d\tau \right] + \\ & + \frac{\partial}{\partial z} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{I,I}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)]^2 d\tau - \\ & - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau + f_I(x, y, z) \end{aligned} \quad (11a)$$

$$\begin{aligned} V_2(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial y} d\tau \right] + \\ & + \frac{\partial}{\partial z} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{V,V}(x, y, z, T) [\alpha_{2I} + V_1(x, y, z, \tau)]^2 d\tau - \\ & - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau + f_V(x, y, z) \end{aligned}$$

$$\begin{aligned} \Phi_{I2}(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_{\Phi I}(x, y, z, T) \times \right. \\ & \times \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial y} d\tau \left. \right] + \frac{\partial}{\partial z} \left[\int_0^t D_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial z} d\tau \right] + f_{\Phi I}(x, y, z) - \\ & - \int_0^t k_I(x, y, z, T) I(x, y, z, \tau) d\tau + \int_0^t k_{I,I}(x, y, z, T) I^2(x, y, z, \tau) d\tau \end{aligned} \quad (12a)$$

$$\begin{aligned} \Phi_{IV}(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_{\Phi V}(x, y, z, T) \times \right. \\ & \times \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial y} d\tau \left. \right] + \frac{\partial}{\partial z} \left[\int_0^t D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial z} d\tau \right] + f_{\Phi V}(x, y, z) - \\ & - \int_0^t k_V(x, y, z, T) V(x, y, z, \tau) d\tau + \int_0^t k_{V,V}(x, y, z, T) V^2(x, y, z, \tau) d\tau \end{aligned}$$

$$cT(x, y, z, t) = \frac{\partial}{\partial x} \left(\lambda_{ass}(x, y, z) \int_0^t \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, \tau)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, \tau)]}{\partial x} d\tau \right) +$$

$$\begin{aligned}
 &+ f_T(x, y, z) + \frac{\partial}{\partial y} \left(\lambda_{ass}(x, y, z) \int_0^t \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, \tau)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, \tau)]}{\partial y} d\tau \right) + \\
 &+ \frac{\partial}{\partial z} \left(\lambda_{ass}(x, y, z) \int_0^t \left\{ 1 + \mu \left[\frac{T_d}{T(x, y, z, \tau)} \right]^\phi \right\} \frac{\partial [\alpha_{2T} + T_1(x, y, z, \tau)]}{\partial z} d\tau \right) + \\
 &+ \int_0^t p(x, y, z, \tau) d\tau .
 \end{aligned} \tag{13a}$$

We determine average values of the second-orders approximations by the standard relation [19-21,23,27,29,30]

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [\rho_2(x, y, z, t) - \rho_1(x, y, z, t)] dz dy dx dt . \tag{14}$$

Substitution of the relations (10a) – (13a) into the relation (14) gives us possibility to obtain the required relations for the values $\alpha_{2\rho}$

$$\alpha_{2f} = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_C(x, y, z) dz dy dx , \tag{15}$$

$$\begin{cases}
 \alpha_{2f} = \frac{1}{2 A_{II00}} \left\{ (1 + A_{IV01} + A_{II10} + \alpha_{2V} A_{IV00})^2 - 4 A_{II00} [\alpha_{2V} A_{IV10} - A_{II20} + A_{IV11} - \right. \\
 \left. - \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx \right\}^{\frac{1}{2}} - \frac{1 + A_{IV01} + A_{II10} + \alpha_{2V} A_{IV00}}{2 A_{II00}} \\
 \alpha_{2V} = \frac{1}{2 B_4} \sqrt{\frac{(B_3 + A)^2}{4} - 4 B_4 \left(y + \frac{B_3 y - B_1}{A} \right)} - \frac{B_3 + A}{4 B_4} ,
 \end{cases} \tag{16}$$

where $A_{abij} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{a,b}(x, y, z, T) I_1^i(x, y, z, t) V_1^j(x, y, z, t) dz dy dx dt ,$

$$\begin{aligned}
 &B_4 = A_{IV00}^2 A_{IV00}^2 - 2 (A_{IV00}^2 - A_{II00} A_{IV00})^2 , B_3 = A_{IV00} A_{IV00}^2 + A_{IV01} A_{IV00}^3 + A_{IV00} A_{II10} A_{IV00}^2 - \\
 &- 4 (A_{IV00}^2 - A_{II00} A_{IV00}) [2 A_{IV01} A_{IV00} + 2 A_{IV00} (1 + A_{IV01} + A_{II10}) - 2 A_{II00} (A_{IV10} + A_{IV10} + 1)] - \\
 &- 4 A_{IV10} A_{II00} A_{IV00}^2 + 2 A_{IV00} A_{IV01} A_{IV00}^2 , B_2 = A_{IV00}^2 \left\{ (1 + A_{IV01} + A_{II10})^2 + A_{IV00}^2 A_{IV01}^2 - 4 A_{II00} \times \right. \\
 &\times \left[A_{IV11} - A_{II20} - \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx \right] + 2 A_{IV00} A_{IV01} (A_{IV00} A_{IV01} + A_{IV00} A_{II10} - \\
 &- 4 A_{IV10} A_{II00} + A_{IV00}) \left. \left[[2 A_{IV01} A_{IV00} + 2 A_{IV00} (1 + A_{IV01} + A_{II10}) - 2 A_{II00} (A_{IV10} + A_{IV10} + 1)]^2 + \right. \right.
 \end{aligned}$$

results of numerical simulation.

It should be noted, that we used fully analytical approach to model redistribution of dopant and radiation defects with account nonlinearity of this processes. At the same time we take into account spatial (due to inhomogeneity of heterostructure) and temperature (due to Arrhenius law) dependences of parameters (dopant and defect's diffusion coefficients, parameters of recombination of point radiation defects, et al). In the case of nonstationary annealing the parameters varying in time. In the present time we did not find in literature so common analytical approach to model physical processes. Framework this paper we make approbation of this modification of this version of method of solution. Analytical approaches of modeling give us possibility to determine functional dependences of required physical values (concentrations of dopant and radiation defects, temperature of annealing, et al). Numerical approaches of simulation of physical processes give us possibility to obtain series of numerical values of physical characteristics. In this situation analytical approaches give us possibility to analyzed physical processes more demonstrate form.

DISCUSSION

In this section based on obtained in the previous section relations we analyzed dynamics of redistribution of dopant and radiation defects during their annealing. We also analyzed dynamics of temperature. Figs. 3 and 4 show typical distributions of infused and implanted dopants in direction, which is perpendicular to interface between sections in epitaxial layers, respectively. All curves were constructed based on the findings in the previous section of analytical relations. The distributions have been calculated for the case, when dopant diffusion coefficient in the average section is smaller. The figures show that manufacturing the interface between layers of heterostructure (i.e. manufacturing inhomogeneity of epitaxial layer due to using some materials at one time to manufacture of the layer) gives us possibility to increase sharpness of p-n-junctions (the increasing of sharpness of p-n-junctions correspond to increasing of absolute value of dopant concentration on coordinate after enriched by the dopant area; at the same time the derivative remaining negative) and at the same time to increase homogeneity of dopant distribution in enriched by the considered dopant area. The increasing of sharpness gives us possibility to decrease area of p-n-junctions, which included into the considered bipolar transistor. At the same time switching time of p-n-junctions decreases. Increasing of homogeneity of distribution of concentration of dopant gives us possibility to decrease local overheat in the doped area or to decrease dimensions of p-n-junctions with fixed maximal value of local overheat. Decreasing of dimensions of one or more p-n-junctions framework bipolar transistor leads to decreasing of total dimensions of bipolar transistor. It should be noted, that concentration of dopant in the average section in epitaxial layer must be smaller, than into another sections. Contrary one can obtain inversion of type of conductivity into other insertions into epitaxial layer near interfaces between the sections. It should be also noted, that substrate must be as better isolator as it possible, i.e. dopant diffusion coefficients in the substrate should be as smaller as it possible. The condition gives us possibility to obtain bipolar transistor with smaller thickness. In the considered situation planar transistor will be manufactured with smaller thickness in comparison with vertical one. On the other hand the same idea could be used to manufacture and decreasing of dimensions of vertical transistor. However thickness of such vertical transistor will be larger in comparison with thickness of planar transistor. Farther we analyzed dependences of concentration of dopant on coordinate, which is perpendicular to interface between layers of heterostructure. The analysis shown the required distributions as similar to distribution, which are shown in Fig. 3.

Further we optimized annealing time. To make the optimization we used recently introduced criterion [18-21,23,30]. Framework this criterion we approximate real distributions of dopant by step-wise function (see Fig. 5). We determine optimal value of annealing time by minimization the following mean-squared error

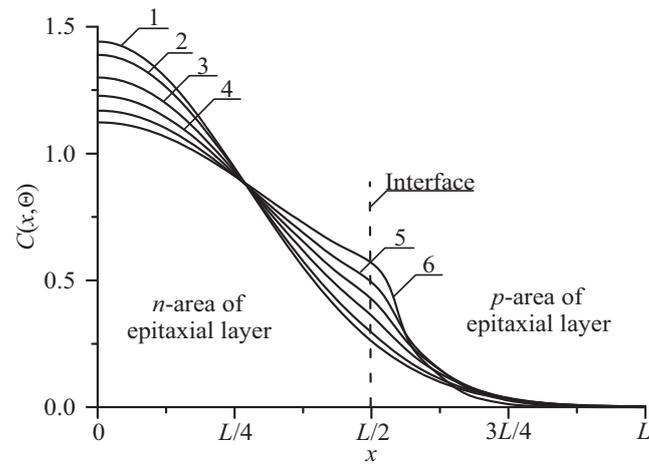


Fig.3. Distributions of infused dopant distributions in left section into epitaxial layer from Figs. 1 and 2 in direction, which is parallel to interface between layers of heterostructure. Increasing of number of curve corresponds to increasing of difference between dopant diffusion coefficients for the case, when value of dopant diffusion coefficient in this section is larger, than in average section

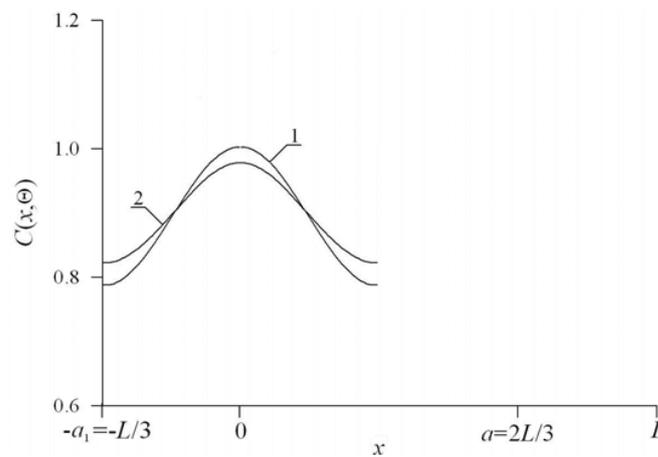


Fig.4. Distributions of infused dopant distributions in average section into epitaxial layer from Figs. 1 and 2 in direction, which is parallel to interface between layers of heterostructure. Increasing of number of curve corresponds to increasing of difference between dopant diffusion coefficients, when value of dopant diffusion coefficient in this section is smaller, than in another section

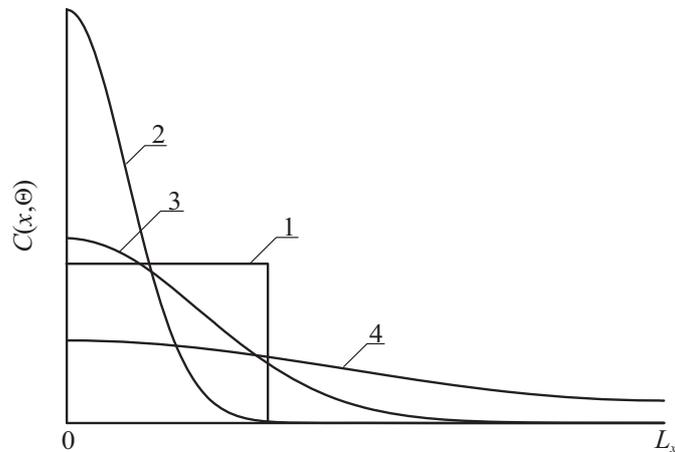


Fig. 5. Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)] dz dy dx, \tag{18}$$

where $\psi(x,y,z)$ is the approximation function, which presented as the curve 1 in Fig. 5. Dependences of optimal annealing time on several parameters are presented on Fig. 6. It should be noted, that using implantation of ions of dopant optimal annealing time is smaller, than analogous time for doping by diffusion. Reason of the difference is necessity of annealing of radiation defects [8-12,24,26,28].

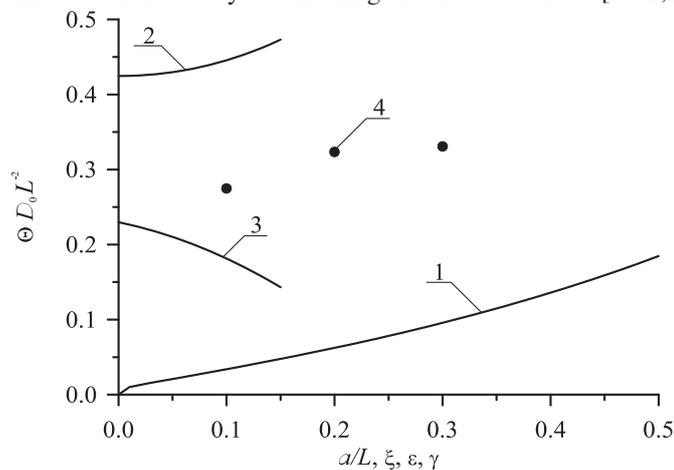


Fig. 6. Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for $a/L=1/2$ and $\varepsilon = \xi = 0$

To compare the considered approach of manufacturing of transistors we shall to note, that manufacture of heterobipolar transistor it is usually epitaxial growth [3-7]. Dopant diffusion and ion implantation are usually using to manufacture transistors in homogenous sample or in epitaxial layer [12,31]. In this paper we consider manufacturing heterobipolar transistors by diffusion or ion implantation. Inhomogeneity of heterostructure under considered in this paper conditions gives us possibility to decrease dimensions of transistors. At the same time we described the approach to optimize of dopant for minimization of dimensions of transistors.

CONCLUSION

In this paper we introduce an approach to manufacture a heterobipolar transistor in thin film structure. Framework the approach one can obtain more thin bipolar transistors with higher sharpness of p-n-junctions.

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