Extended H_∞ Estimator with Miss Distance Optimization for Ballistic Target Tracking

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Abstract
The problem considered in this paper is that of estimating the relative kinematics of a ballistic target tracked by a homing interceptor missile in a 2D scenario with the objective of minimizing the miss distance. The miss distance is the final relative range between the target and the interceptor and its minimization has traditionally been considered in guidance problems. In this work, an Extended H_∞ Filter with a miss distance minimization objective (EHF-2) has been formulated to provide accurate estimates of the relative kinematics of the target and interceptor. The performance of this filter has been compared with those of the noise optimal Extended Kalman Filter (EKF) and the (noise) robust Extended H_∞ Filter (EHF-1). It is observed that the proposed EHF-2 shows significant improvement in performance with respect to both miss distance as well as sightline rate (SLR) compared to EKF and EHF-1 over the total terminal range.

Key words: Ballistic target tracking, Miss distance optimization, Estimation, EKF, H_∞ filter.

1. INTRODUCTION

The main objective of target tracking by an interceptor missile is to achieve zero miss distance. In other words, the final relative range measured in terms of the centers of gravity of the two bodies must be such that the target is hit. As the speeding and maneuvering capabilities of the missiles improve, the performance requirement in terms of miss distance becomes more stringent. In order to achieve this objective, target tracking by a homing interceptor missile has traditionally been subdivided into three independent sub-problems, that of the estimation of the relative kinematics of the interceptor and target, guidance of the interceptor, and its control.

This is a valid simplification yielding suitable performance for a relatively low or medium speed target interception problem considering the validity of the certainty equivalence principle. However, for the case of high speed flight vehicles, the certainty equivalence principle is shown to be violated [1] since there are inherent nonlinearities in the dynamics of the system as well as additional nonlinearities due to the presence of bounded accelerations and various other saturation effects. Thus, it becomes necessary to consider the integration of any two or all three sub-problems in order to meet the requirements in the presence of the disturbances and noises inherent in the system. Recently, researchers have started looking into various problems involving integrated estimation, guidance and control [1,2,3].

For optimal guidance, the guidance subsystem tries to maintain the sightline rate (SLR) of the interceptor ideally as zero so that the seeker in the interceptor constantly points towards the target despite any target maneuvers and/or disturbances. It is to be noted that the line connecting the two centers of gravity of the target and the interceptor is called the line of sight (LOS) or sightline. In the terminal phase, the seeker provides measurements of the relative kinematics, typically those of

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range rate, LOS angles and SLRs. An accurate estimate of the sightline rate (SLR) is the main information used by the guidance subsystem. This subsystem then provides the command input to the control subsystem.

The seeker measurements are, however, highly corrupted with noise from various sources, so an estimator is used in these devices to provide the most accurate estimate of the kinematics to the guidance system. The usual choice for the estimator is the Extended Kalman Filter (EKF) which provides (noise) optimal estimates in presence of linearizable Gaussian noises. However, in order to optimize the overall interceptor performance for miss distance while still handling the various parameter variations, process and measurement noises and nonlinearities, a probable choice of estimator is the Extended H_∞ Filter (EHF) [4] or its variant since it is possible to define a suitable performance index in terms of a limiting (worst case) H_∞ norm in this filter. Several types of estimators have been used in state-space estimation problems including target tracking problems by Sayed [5], Farina et al. [6] and other researchers. It is also to be noted that these filters usually do not deal with miss distance minimization since traditionally, this aspect has been handled as part of guidance law design [7].

In this paper, the problem considered is that of estimation of the relative kinematics of a ballistic target tracked by a interceptor in a 2D scenario. In this case, there are no sudden target maneuvers or any major relative acceleration changes either in magnitude or direction. The objective of this paper is to propose a filter that provides accurate estimates of the relative kinematics of the ballistic target while also addressing the problem of achieving a minimum miss distance. Two EHF formulations have been stated in this paper of which one follows the noise robust formulation as in [4] while the other proposes to minimize the miss distance objective. The performances of these EHFs have been compared with conventional EKF in terms of root mean square error (RMSE) plots as well as terminal instant scatter plots for a 2D ballistic target tracking problem. The present paper is subdivided in 5 sections. Section 2 describes the target motion and measurement model. The EKF formulation is stated and the EHF formulations, in a form similar to the EKF using the approach in Simon [8], are developed in Section 3 on the basis of the filter and system optimization objectives. Simulation results are provided and their implications are discussed in Section 4 while Section 5 is the conclusion.

2. MATHEMATICAL MODEL OF SEEKER

This section describes the mathematical model used in representing a 2D seeker for filtering purposes. The frame of reference considered is termed as CP frame in which the state equations are in Cartesian Local Vertical (LV) frame while the measurement equations are in Polar LV frame. Consequently, in the measurement equation, all interim variables are nonlinear functions of the state variables.

The process model of the system is in the State Dependent Coefficient (SDC) form which captures the nonlinearity of the practical system with the states being the relative positions and velocities in Cartesian coordinates and the inverse of the target ballistic coefficient which is related to the target acceleration. The measurement model consists of the available measurements of relative range rate, LOS angle and SLR in the polar coordinates along the elevation. It is to be noted that relative range measurements are not available.

The system model thus consists of 5 state variables (Δx, Δz, ΔV_x, ΔV_z, $1/\beta$) in Cartesian frame of reference. The measurements considered in the filtering model are the interceptor-target relative range rate $\dot{r}$, elevation angle λ_e and sightline rate for elevation $\dot{\lambda}_e$. The control inputs to the system are the interceptor acceleration components (a_{mx}, a_{mz}) and the interceptor attitude quaternion vector (q_1, q_2, q_3, q_4) which is available from the Strap Down Inertial Navigation System (SDINS) of the interceptor.

Measurement data are generated from the relevant true relative positions and velocities as obtained from the seeker input in a typical 6DOF simulation and adapted suitably to a 2D scenario. These true states available in Cartesian are converted to obtain the corresponding polar states of range rate, elevation angle and SLR. These are then corrupted using the nominal measurement covariances. Range is considered to be unavailable as is the case in several realistic seekers.

A. Seeker State Equations Using SDC Formulation

The standard continuous-time 2D state space model of the seeker expressed using SDC formulation [4] is:

$$\begin{bmatrix} \Delta\dot{x} \\ \Delta\dot{z} \\ \dot{V}_x \\ \dot{V}_z \\ \dot{\frac{1}{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -0.5\rho VV_{tx} \\ 0 & 0 & 0 & 0 & -0.5\rho VV_{tz} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta z \\ V_x \\ V_z \\ \frac{1}{\beta} \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a_{mx} + g \\ a_{mz} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_{amx} \\ w_{amz} \\ w_{\beta} \end{bmatrix} \tag{2.1}$$

It is to be noted that due to the SDC formulation, the states of the nonlinear system can be expressed without the calculation of any Jacobians.

B. Measurement Equations

Since range is unavailable, so only three sets of measurements are available. The measurement equations are generated in polar LV frame from the Cartesian states and considering the respective additive measurement noises, having normal distribution of zero mean and standard deviation of $\sigma_{\dot{r}}$, σ_{λ_e}, $\sigma_{\dot{\lambda}_e}$ respectively.

$$\dot{r}_m = \frac{\Delta x\,\Delta\dot{x} + \Delta z\,\Delta\dot{z}}{\sqrt{\Delta x^2 + \Delta z^2}} + N(0, \sigma_{\dot{r}}) \tag{2.2}$$

$$\lambda_{em} = \tan^{-1}\frac{\Delta x}{\Delta z} + N(0, \sigma_{\lambda_e}); \tag{2.3}$$

$$\dot{\lambda}_{em} = \frac{(\Delta\dot{x}\cdot\Delta z - \Delta x\cdot\Delta\dot{z})}{\left(\Delta x^2 + \Delta z^2\right)} + N(0, \sigma_{\dot{\lambda}_e}); \tag{2.4}$$

The continuous-time state and measurement equations are converted using standard procedure in discrete time for use in the filter algorithms as

$$x_{k+1} = f\left(x_k, u_k, w_k\right) = F\,x_k + G\,u_k + w_k \tag{2.5}$$

$$y_k = h\left(x_k, v_k\right) \tag{2.6}$$

where x_k denotes the states, u_k the input, w_k denotes the process noise and v_k denotes the measurement noise. The discrete-time state matrices are obtained by standard procedure using a realistic sampling time T_s. The process noise covariance and the measurement noise covariance matrices are considered as $Q_k = E[w_k w_k^T]$ and $R_k = E[v_k v_k^T]$. The estimated state uncertainty matrix is denoted as P_k. It is to be noted that the allowable errors in measurements are quantified in the measurement uncertainty matrix R while the design parameters are the initial state estimate x_0, the initial state uncertainty P_0 and the process noise covariance Q_k which are chosen so that the best Kalman filter performance is obtained for nominal system parameters.

3. EXTENDED KALMAN FILTER AND EXTENDED H$_\infty$ FILTER FORMULATIONS

The first estimator formulation is that of the conventional Extended Kalman Filter (EKF) [4, 8] and uses the standard apriori and aposteriori updates of the states and state covariances with the measurement Jacobian matrix being calculated. No Jacobian needs to be calculated while representing the state matrix and hence no approximations are used. Instead, an input Gu_k is added at each time update from the available measurements. The details of the formulation are stated hereafter.

Apriori Update:

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} + Gu_k \tag{3.1}$$

$$\hat{P}_{k+1|} = F_k \hat{P}_{k|k} F_k^T + Q_k \tag{3.2}$$

where

$$F_k = \begin{bmatrix} 1 & 0 & T_s & 0 & 0 \\ 0 & 1 & 0 & T_s & 0 \\ 0 & 0 & 1 & 0 & -0.5\rho V V_{tx} T_s \\ 0 & 0 & 0 & 1 & -0.5\rho V V_{tz} T_s \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The estimated apriori measurement estimates are thus:

$$\hat{y}_{k+1|k} = h(\hat{x}_{k+1|k}) \tag{3.3}$$

The Jacobian matrix for measurements H_k has to be calculated at each instant using instantaneous value of measurements y_k and apriori estimated states $\hat{x}_{k+1|k}$ for use in the filter update equations and is given as:

$$H_k = \begin{bmatrix} \frac{\partial \dot{r}_m}{\partial \Delta x} & \frac{\partial \dot{r}_m}{\partial \Delta z} & \frac{\partial \dot{r}_m}{\partial \Delta V_x} & \frac{\partial \dot{r}_m}{\partial \Delta V_z} & \frac{\partial \dot{r}_m}{\partial (1/\beta)} \\ \frac{\partial \lambda_{em}}{\partial \Delta x} & \frac{\partial \lambda_{em}}{\partial \Delta z} & \frac{\partial \lambda_{em}}{\partial \Delta V_x} & \frac{\partial \lambda_{em}}{\partial \Delta V_z} & \frac{\partial \lambda_{em}}{\partial (1/\beta)} \\ \frac{\partial \dot{\lambda}_{em}}{\partial \Delta x} & \frac{\partial \dot{\lambda}_{em}}{\partial \Delta z} & \frac{\partial \dot{\lambda}_{em}}{\partial \Delta V_x} & \frac{\partial \dot{\lambda}_{em}}{\partial \Delta V_z} & \frac{\partial \dot{\lambda}_{em}}{\partial (1/\beta)} \end{bmatrix} \tag{3.4}$$

For y_{k+1} being the measurements available at time step $(k + 1)$ from the sensor, the measurement update of states and state error covariances are given as:

$$M_{k+1} = \left(I + H_K^T R_k^{-1} H_k \hat{P}_{k+1|k}\right)^{-1} \tag{3.5}$$

$$K_{k+1} = \hat{P}_{k+1|k} M_{k\,+\,1} H_k^T R_k^{-1} \tag{3.6}$$

$$\hat{P}_{k+1|k+1} = (I - K_{k+1} H_k) \hat{P}_{k+1|k} \tag{3.7}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1} - \hat{y}_{k+1|k}) \tag{3.8}$$

where the initial state and its uncertainty are initialized as $\hat{x}_{1|1} = x_0, \hat{P}_{1|1} = P_0$.

As mentioned earlier, another estimator considered in this paper is the Extended H$_\infty$ filter (EHF). The EHF algorithm has been expressed in this paper in a manner analogous to the EKF formulations using the approach outlined in Simon[8] in which the apriori filter has been considered.

The performance objective in this filter formulation is expressed in terms of the aposteriori energy cost function J as defined hereafter which has to be minimized.

$$J = \frac{\sum_{k=0}^{N} \left\| z_k - \hat{z}_k \right\|^2_{R_{k1}^{-1}}}{\left\| x_0 - \hat{x}_0 \right\|^2_{P_0^{-1}} + \sum_{k=0}^{N} \left(\left\| w_k \right\|^2_{Q_k^{-1}} + \left\| v_k \right\|^2_{R_k^{-1}} \right)} < \gamma \tag{3.9}$$

It must be noted that P_0, Q_k and R_k in the cost function J are not necessarily the covariances as defined in the case of EKF but instead are symmetric, positive definite weighting matrices chosen by the engineer based on the performance requirements in the particular problem. However, the initial choice for these weighting matrices is usually the covariance matrices itself with $P_0 = \hat{P}_{0|0}$. The parameter γ is often referred to as the robustness bound. This is the upper limit of the cost function and is specified by the designer in order to quantify the worst case performance of the EHF or in other words, the robustness of the desired outputs to disturbances and uncertainties. R_{k1} is a suitable weighting matrix for the robustness bound and is specified by the engineer depending on the chosen L_k and γ. The numerator of the cost function is in terms of a new state variable $\hat{z}_k = L_k \hat{x}_k$, which is defined as the desired output and can be formulated as a suitable combination of the available states. The matrix L_k defines that combination.

Given a chosen γ, it is necessary to calculate the value of the cost function J at the start of each iteration to ensure the validity of J < γ. This is equivalent to satisfying a necessary and sufficient condition for the existence of the EHF at each time step. This condition can be stated in terms of the robustness bound γ as follows:

$$\hat{P}_{k/k}^{-1} + H_k^T R_k^{-1} H_k - (1/\gamma) L_k^T R_{k1}^{-1} L_k > 0 \tag{3.10}$$

Considering the states and initial state error covariance matrix to be initialized as $\hat{x}_{0|0} = \hat{x}_0, \hat{P}_{0|0} = P_0$, the apriori update equations are calculated as

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} + G u_k \tag{3.11}$$

$$\hat{P}_{k+1|k} = F_k \hat{P}_{k/k} F_k^T + Q_k \tag{3.12}$$

The measurement Jacobian matrix H_k for state estimates $\hat{x}_{k+1|k}$ and measurements y_k are used to calculate the filter gain as follows:

$$\begin{aligned} M_{k+1} &= \left(I - \frac{1}{\gamma} L_k^T R_{k1}^{-1} L_k \hat{P}_{k+1|k} + H_k^T R_k^{-1} H_k \hat{P}_{k+1|k} \right)^{-1} \\ K_{k+1} &= \hat{P}_{k+1|k} M_{k+1} H_k^T R_k^{-1} \end{aligned} \tag{3.13}$$

The aposteriori update equations for the states and the state error covariances are thus calculated as

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \left(y_{k+1} - H_k \hat{x}_{k+1|k} \right) \tag{3.14}$$

$$\hat{P}_{k+1|k+1} = \hat{P}_{k+1|k} M_{k+1} \tag{3.15}$$

In the conventional formulation [4] of the EHF, the desired output $\hat{z}_k = \hat{y}_k = H_k \hat{x}_k$. Thus the EHF is expressed in a form similar to that of the EKF with the major performance objective being robustness to noise. It is to be noted that in this filter formulation, $L = H$ as is done for the standard Kalman filter. The major difference from the EKF in this case is in considering the robustness bound γ= 350. The corresponding weighting matrix R_{k1} is suitably configured as the unit matrix. This filter formulation will hereafter be referred to as the EHF-1 filter.

As discussed earlier, minimizing the miss distance is of prime significance in the target tracking problem. Hence, a second choice of L was considered for another H_∞ filter formulation (EHF-2). In this filter, L considers only the estimated range with the robustness bound given as γ= 200. The weighting matrix R_{k1} in this case uses a weight equal to the inverse square of iteration time in the relevant terms in order to reduce the tolerance to error in relative range as time to go decreases.

4. SIMULATION RESULTS AND OBSERVATIONS

4.1 Parameter Values

The filter tuning elements as used for testing the nominal performances of EKF, EHF-1, EHF-2 in the 2D seeker model is stated hereafter in Table-1, Table-2, and Table-3 in terms of the covariances of the relevant states and/or the measurements.

Table 1. Filter tuning parameter P_0 for 2D model [4]

$\sigma^2_{\Delta x}\,(m^2)$	$\sigma^2_{\Delta z}\,(m^2)$	$\sigma^2_{\Delta V_x}\,(m/s^2)$	$\sigma^2_{\Delta V_z}\,(m/s^2)$	σ^2_{β}
1×10^2	1×10^2	10	10	0.00008

Table 2. Filter tuning parameter Q_k for 2D model [4]

$\sigma^2_{\Delta x}\,(m^2)$	$\sigma^2_{\Delta z}\,(m^2)$	$\sigma^2_{\Delta V_x}\,(m/s^2)$	$\sigma^2_{\Delta V_z}\,(m/s^2)$	σ^2_{β}
0.01	0.01	0.01	0.01	10^{-11}

Table 3. Filter tuning parameter R for 2D model [4]

$\sigma^2_{r}(m/s)$	$\sigma^2_{\lambda_e}(deg)$	$\sigma^2_{\dot{\lambda}_e}\,(deg/s)$
15.0	0.3	5.0

The choices of the other tuning parameters for the filters EHF-1 and EHF-2 as discussed in the previous section are stated in Table-4.

Table 4. Filter tuning parameters of EHF-1 and EHF-2 for 2D model

	L_k	γ	R_{k1}
EHF-1	H_k	350	Unit matrix
EHF-2	$\begin{bmatrix} \dfrac{\partial \hat{r}}{\partial \Delta \hat{x}} & \dfrac{\partial \hat{r}}{\partial \Delta \hat{z}} & 0 & 0 & 0 \end{bmatrix}$	200	Non-zero elements = $1/k^2$ where k denotes iteration

4.2. Results and Discussions

The EKF, conventional EHF (EHF-1), and modified objective based EHF (EHF-2) have been considered for solving this problem and the comparative performance of the filters have been analyzed. For this purpose, the RMSE plots for estimate range, range rate, elevation angle and corresponding SLR for all these three filters have been plotted for 1000 MC runs. The final iteration time is k = 301.

The RMSE plot for the estimated range in Fig.1 shows that EHF-1 performance is not smooth over time but has spiky nature throughout the time span, whereas EKF and EHF-2 performances are smooth and quite comparative. But, at the terminal stage, EHF-2 outperforms both EKF and EHF-1 reaching the lowest value of 0.15m at k = 301 as evident from Table 5 in which the RMSE for the 3 filters are given for all the parameters of interest.

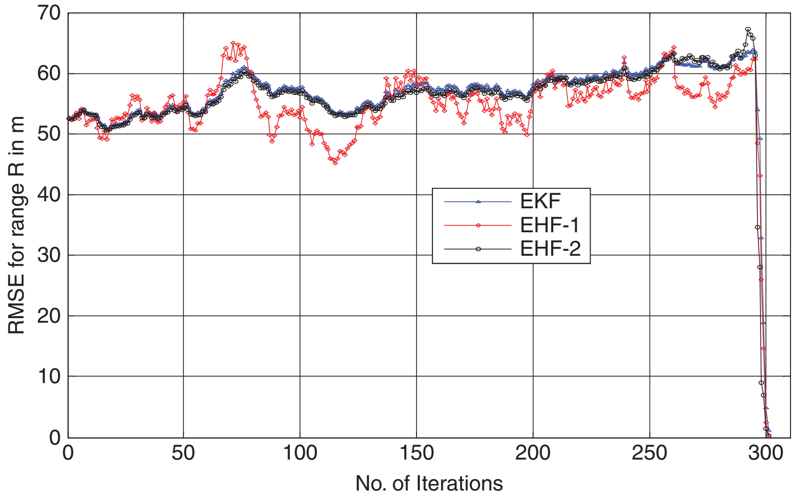


Figure 1. RMSE plot for Range R in metre

Table 5. RMSE of range, range rate, elevation angle and SLR at terminal instant (k = 301) for 1000 MC runs

	Range (m)	Range rate (m/s)	λ_e (deg)	$\dot{\lambda}_e$ (deg/s)
EKF	1.15	20.08	0.0277	4.50
EHF-1	0.33	11.67	0.0099	1.47
EHF-2	0.15	2.57	0.0028	0.26

Fig. 2. shows the plots of the RMSE of range rate where it is seen that beyond k = 296, the error increases sharply for EKF and also for EHF-1 whereas for EHF-2 it remains almost constant and quite low. In this case also, the EHF-1 performance is quite spiky unlike EKF and EHF-2. The RMSE at terminal instant for range rate is 2.57 m/s for EHF-2 as stated in Table 5.

Fig. 3 and Fig. 4 are the RMSE plots for elevation angle and the corresponding SLR respectively. For both the cases, it is observed that the errors increase sharply for EKF, is lesser for EHF-1 whereas EHF-2 is able to limit the error appreciably.

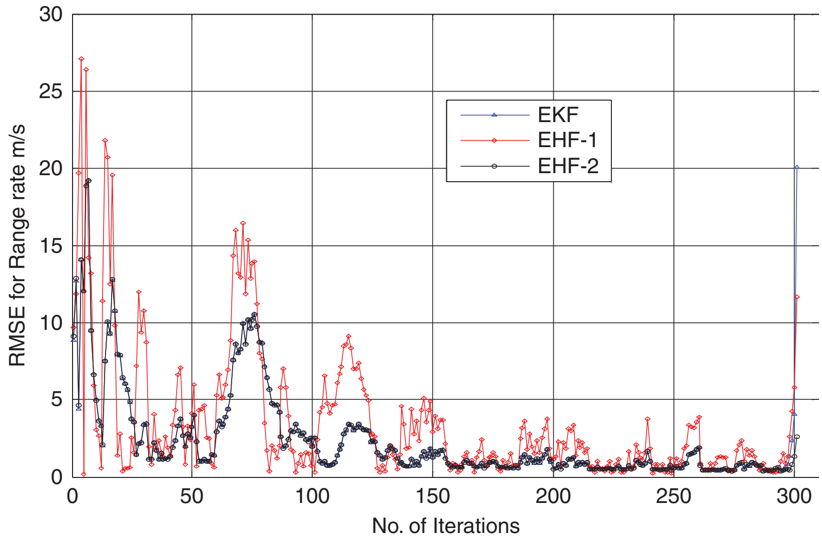


Figure 2. RMSE plot for Range rate, m/s

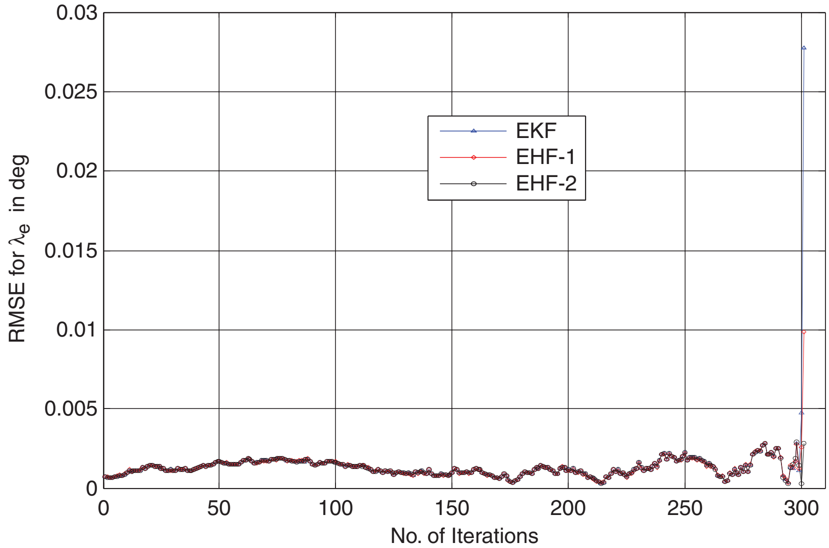


Figure 3. RMSE plot for λ_e in deg

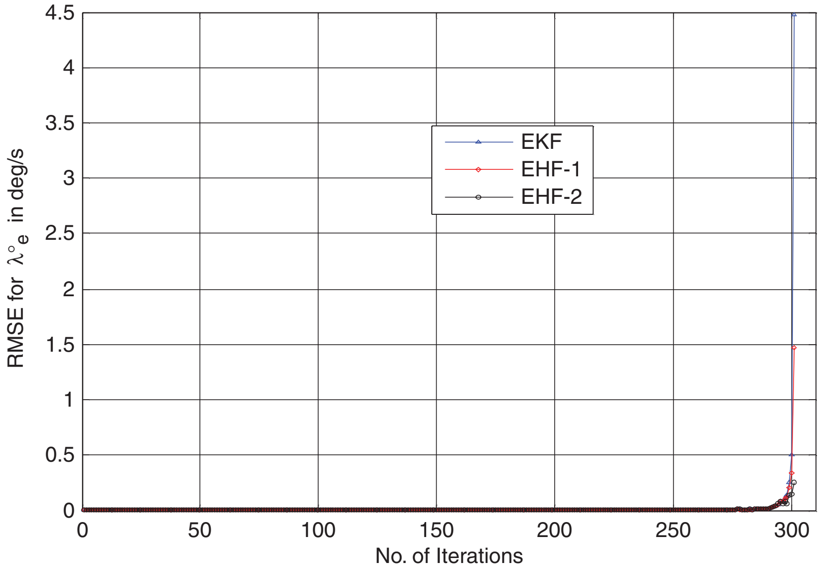


Figure 4. RMSE plot for $\dot{\lambda}_e$ in deg/s

Table 6 Mean and SD of Miss Distance (MD) and SLR at terminal instants

	Miss Distance (m)		$\dot{\lambda}_e$ (deg/s) at k = 296		$\dot{\lambda}_e$ (deg/s) at k = 298		$\dot{\lambda}_e$ (deg/s) at k = 299		$\dot{\lambda}_e$ (deg/s) at k = 301	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
EKF	0.89735	0.71645	4.6991	0.5414	7.8996	1.1304	14.273	1.9372	219.81	132.23
EHF-1	0.2511	0.20811	4.3742	0.5703	6.5908	1.1573	11.714	1.6919	62.453	56.684
EHF-2	0.135	0.0686	3.605	0.3377	3.7553	0.5773	7.910	0.4207	12.42	8.004

At terminal time, the RMSE for elevation angle and SLR are minimum for EHF-2 at 0.0028 degree and 0.26 deg/s respectively. Fig. 5 shows the estimation error time history for SLR for all filters in a single run for the total duration prior to interception (that is, leaving the last 10 instants). As is to be expected, all the filters show comparable performance in this regime.

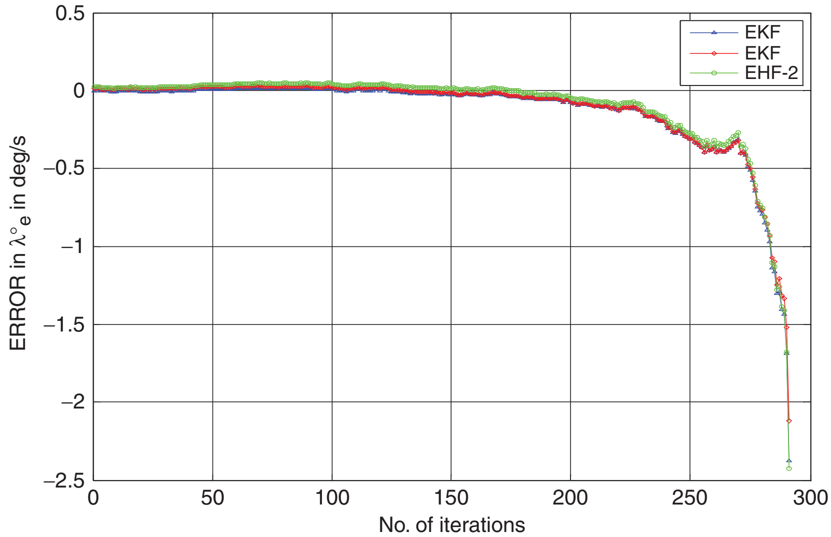


Figure 5. Single run Estimation error for SLR

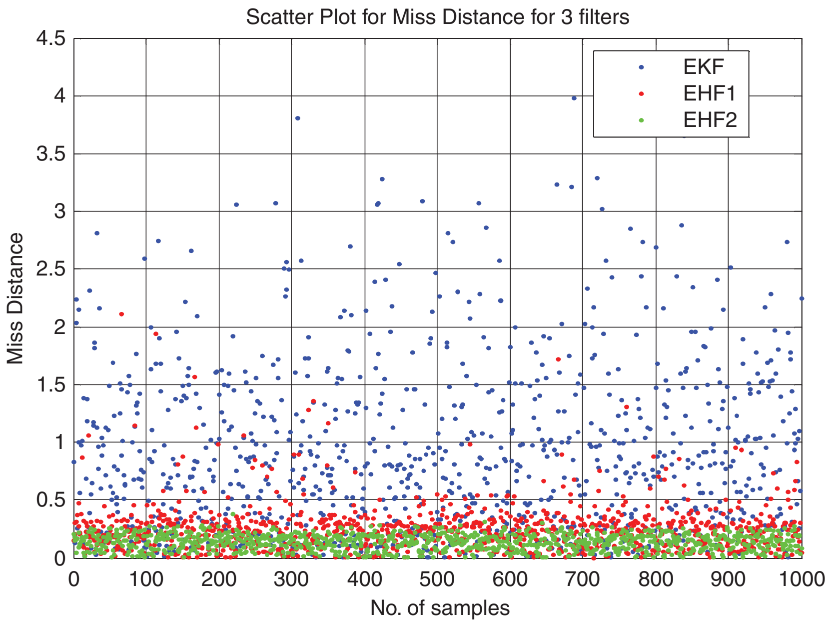
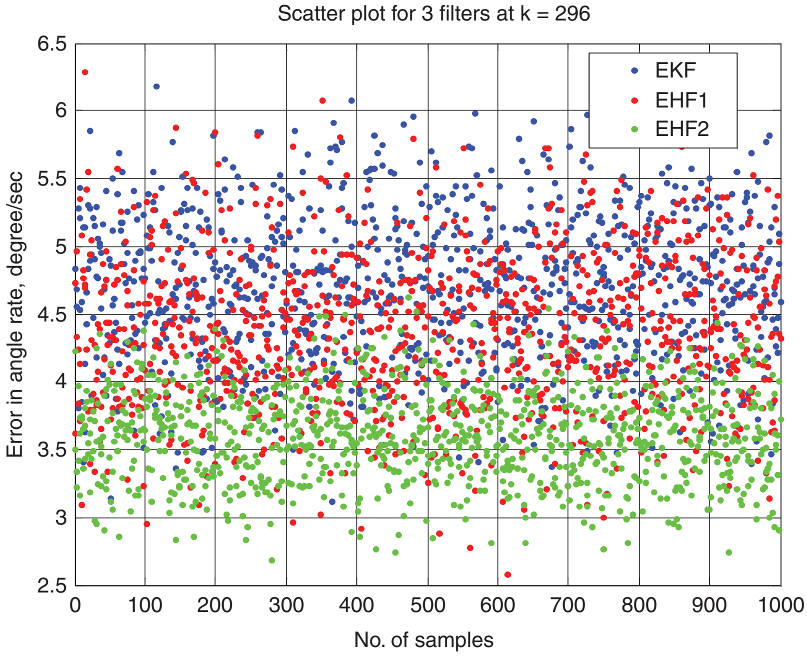
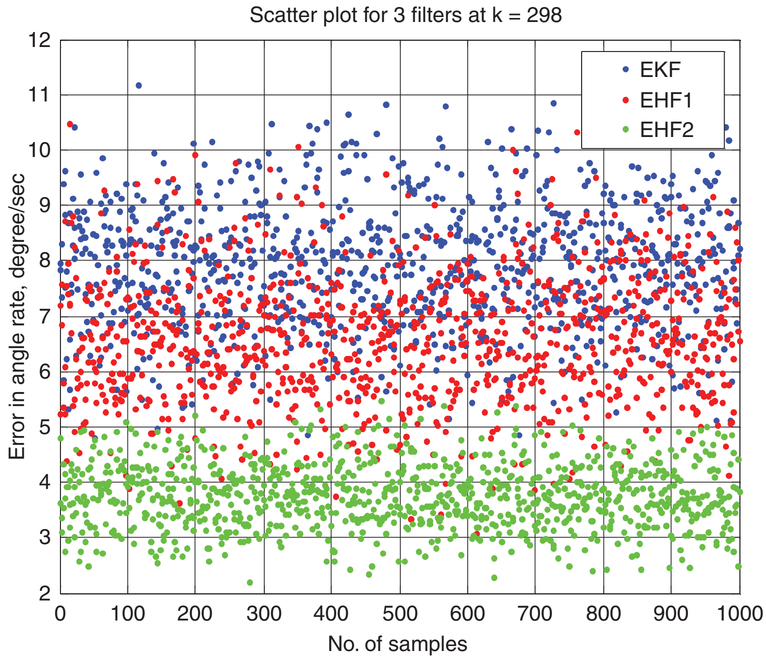


Figure 6. Scatter plot for miss distance for different filters

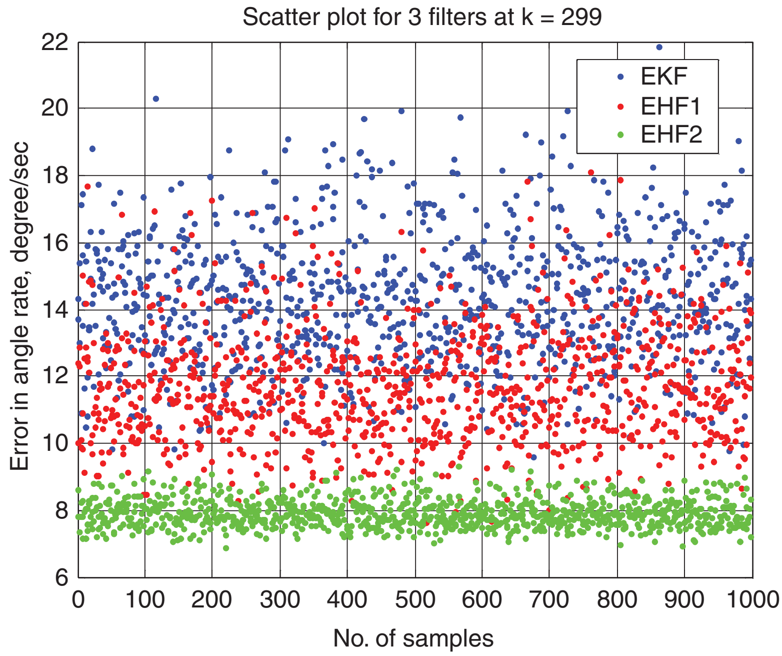
It is thus observed that for the ballistic target tracking scenario, the EHF-2 performs the best out of all the three filter formulations, both in terms of miss distance optimization as well as keeping the SLR close to zero enabling better performance of the optimal guidance scheme. As also expected, the EHF-1 provides some robustness in these performances compared to the noise optimal EKF for the same defined output, specially at the terminal stage although the performance of EKF or EHF-2 is much smoother than EHF-1 at earlier stages.



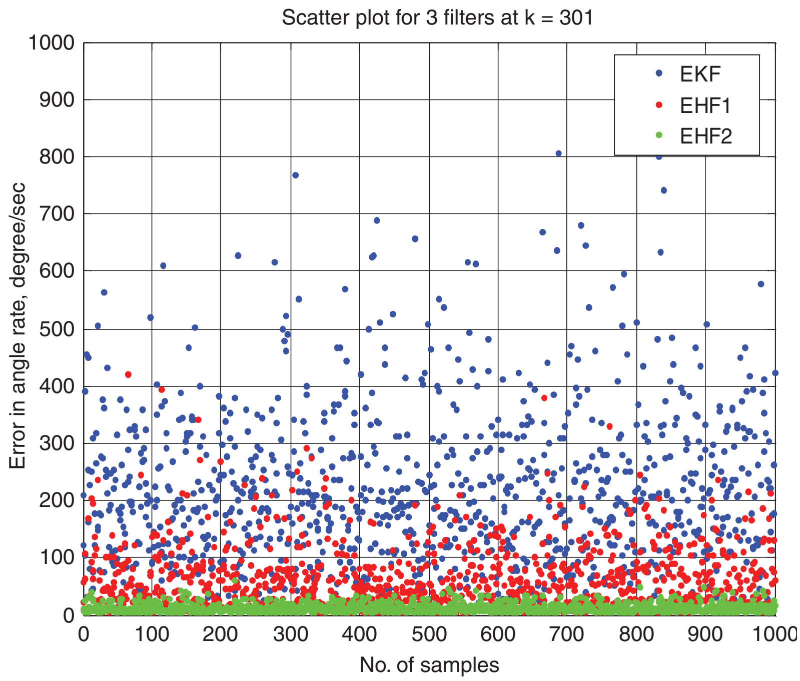
(a) Error in SLR at k = 296



(b) Error in SLR at k = 298



(c) Error in SLR at k = 299



(d) Error in SLR at k = 301

Figure 7. Scatter plot for error in SLR $\dot{\lambda}_e$ for different filters at different instants

5. CONCLUSION

The present paper deals with the case of non-maneuvering ballistic target tracking with the objective of miss distance optimization. It is found that an Extended H_∞ filter (EHF) with the performance index formulated to minimize miss distance (EHF-2) yields better results in comparison to standard Extended Kalman filter (EKF) or an EHF filter with the performance index with desired output being the same as actual output (EHF-1) for both miss distance as well as error in sightline rate (SLR). The RMSE plots signify that EHF-2 provides lowest RMSE error for estimated range, range rate, elevation angle as well as SLR. The scatter plot for miss distance shows that EHF-2 provides a mean of 0.135m with standard deviation (SD) of 0.0686m in the present realistic 2D case compared to means of 0.897m and 0.25m and SDs of 0.716m and 0.208m respectively by the EKF and EHF-1 filters. In addition, EHF-2 also restricts the SLR to a final mean of 12.42 degrees/s compared to 219.81 degrees/s and 62.453 degrees/s by the EKF and EHF-1 filters respectively. This approach could possibly be extended to 6DOF cases and also scenarios with target accelerations and suitable estimator formulations could be used to obtain better results in such cases also.

6. ACKNOWLEDGMENT

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