A Multiple Model Approach to Robust Performance

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Abstract

A two degree-of-freedom controller structure is proposed in this paper for robust tracking of desired trajectories. The inputs required for these trajectories, estimated by an augmented state-space model approach, constitutes the feed-forward part that ensures output tracking for the nominal plant model, and a feedback part, via a multiple model approach, makes the overall system robust. This method is then applied to helicopter hover control.

Key words: Linear systems, Robust tracking, Multiple models

1. INTRODUCTION

Precision output tracking, a fundamental problem for control engineers, poses increasingly stringent performance requirements to be satisfied in a variety of applications, notably in the robotics and aerospace industries. Whilst perfect tracking is relatively easy to achieve in minimum phase systems, it remains a challenging problem in non-minimum phase systems due to fundamental limitations on the transient tracking performance characterised by the number and location of non-minimum phase zeros [1]. Such zeros arise, for instance, as a result of sampling of systems with relative degree greater than two [2]. In addition, the desired output trajectory in several applications is obtained through a real-life data acquisition run, and hence corrupted by measurement and/or sensor noise. For example, the reproduction of time records of accelerations and displacements obtained during test drives with prototype cars. This reproduction is made on hydraulic test-rigs that enable full car endurance tests, driving comfort assessment, etc. for prolonged periods of time thereby saving precious resources.

Several techniques have been proposed that determine the input necessary for a desired output [3]–[6]. However, these papers refrain from considering desired trajectories corrupted by noise. Asymptotic tracking of any member in a given family of signals generated by an exosystem is considered in [3,4]. By considering a dichotomic split of the system equations of a non-minimum phase plant, the use of exosystems was avoided in [5,6]. Here, pre-actuation mitigates poor transient performance in the case of non-minimum phase systems. Modification of the system dynamics is required before applying this technique to systems with zeros on the imaginary axis [7]. An attempt was made in [8] to incorporate noise by using the unknown-input decoupled observer approach of [9,10]. However, it turns out this approach is valid only for a class of minimum phase systems [8]. A different approach based on designing a Kalman filter for an associated augmented system was first outlined in [8], detailed with numerically efficient algorithms in [11], compared with other techniques in [12], and extended to nonlinear systems in [13]. This technique has also been used in [14] to help identify, from input-output measurements, the state space model of a certain class of nonlinear systems. We discuss this technique in Section 2.

The aforementioned technique to estimate the input for a given desired trajectory is based on a design, or nominal, model. As is well-known, modelling invariably involves a trade-off between simplicity and mathematical tractability of a model, and its accuracy in matching the behaviour of the plant. Whilst some models can emulate the plant with greater fidelity than others, no particular model can do so perfectly. Since the nominal model is one of the many possible ones, there is no

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guarantee that the input estimated for a desired trajectory works as well for the plant. To simultaneously satisfy conflicting objectives of output tracking and robustness we use the following controller structure: A feed-forward component, comprising the input-estimation procedure mentioned earlier, and a feedback component, dealing only with the uncertainty. This is indeed a two degree-of-freedom (DOF) structure. Such a technique was first introduced in [15]. It is well-known that a two DOF structure is necessary to meet these conflicting objectives [16]. This structure is discussed in Section 3.

However, in several applications, notably within the aerospace industry, the plant is expected to track desired trajectories despite operating in diverse conditions. Obtaining a single controller that is valid globally for all possible scenarios is a rather tedious task. On the contrary, it is easier to design controllers that are effective in local neighbourhoods. One possible way to combine these controllers is via gain scheduling. In this paper, we consider the use of multiple models to achieve this goal.

The multiple model, switching and tuning (MMST) methodology was originally introduced in [17] to cope with the problem of oscillatory response with unacceptably large amplitudes during the transient phase, particularly when there are larger errors in the initial parameter estimates. This is despite the globally stable algorithms developed for the so-called "ideal" case that result in zero steady-state tracking error [18]. MMST methodology was found necessary in several applications; for example, [19]–[22]. The stability of the overall system, and the improvement in performance has been demonstrated for both deterministic and stochastic linear time-invariant systems in [23]–[26], time-varying systems in [27], and later extended to a class of nonlinear systems in [28]. In Section 3 we use multiple models in our context, and apply this to helicopter hover control in Section 4.

2. OUTPUT TRACKING WITH NOMINAL MODELS

Let Σ be the plant whose output is expected to track a desired trajectory y_d. Suppose that the state-space representation of a nominal model Σ_{nom} of the plant is as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k, \\ y_k &= Cx_k + Du_k \end{aligned} \tag{1}$$

where u, the controllable input, is to be chosen such that the output y of the nominal model Σ_{nom} tracks a desired trajectory y_d. As mentioned earlier, the desired output signal may have been observed through measurements.

We, therefore, consider the following state-space representation:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, \\ y_k &= Cx_k + Du_k + v_k, \end{aligned} \tag{2}$$

where w_k and v_k are respectively the system and measurement noise.

Thus,

$$y(z) = T_{yw_e}(z)\,w_e(z) + T_{yu}(z)\,u(z),$$

where $w_e \overset{\Delta}{=} (w^T \;\; v^T)^T$ is the vector of disturbance signals, $T_{\vartheta_2 \vartheta_1}$ denotes the transfer function matrix from ϑ_1 to ϑ_2, and X^T denotes the transpose of a matrix X. We assume that $y_k \in \mathbb{R}^p$ *and* $u_k \in \mathbb{R}^m$.

Our objective is to obtain an estimate $\hat{u}$ of the input u given possibly corrupted measurements y of a desired output signal. We achieve this by designing a suitable filter $F(z)$ that represents the 'inverse' system Σ_{inv} as illustrated in Fig. 1. The filter $F(z)$ also yields the estimate y in a natural way. Thus, $\binom{\hat{y}}{\hat{u}} = Fy$. Clearly a filter $F(z)$ is admissible if, and only if, $F(z) \in \mathbb{IR}\mathcal{H}_\infty$, the class of real-rational asymptotically stable functions, and, given any initial state of the system (2), the estimation error $\left\|\binom{\tilde{y}}{\tilde{u}}\right\|$ is minimised, where, $\tilde{y}_k = y_k - \hat{y}_k$ $\tilde{u}_k = u_k - \hat{u}_k$ and $\|\cdot\|$ is an appropriately chosen norm. Thus,

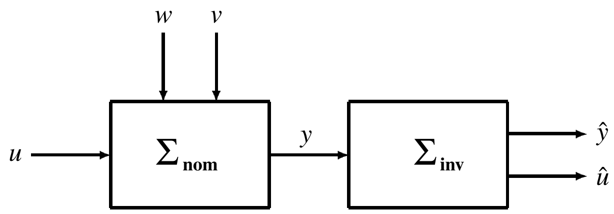

Figure 1. Input Estimation

given the model Σ_{nom} represented by (2), we choose a filter F that satisfies the following performance criterion:

$$\min_{F\in\mathcal{F},\,\delta} \| I - z^{\delta} T_{\hat{u}u p} \| \tag{3}$$

Here, $\mathcal{F}$ represents the class of admissible filters, and δ is a parameter introduced to account for the delay in the estimation of the input u. The norm p depends on the nature of the signals w_e and u. Thus if w_e and u are assumed to be unknown but with bounded energy, an appropriate choice is $p = \infty$. On the contrary, if both w_e and u are assumed to be zero mean white noise process with unit variance, then $p = 2$, which is the standard Wiener-Hopf or Kalman filter.

In a deterministic scenario with $w_e = 0$ (i.e. model (1)), we can show the following [12]:

1. The filter that achieves (3) with $\delta = 0$ is precisely $F = T_{yu}^{-1}$, provided the transfer matrix T_{yu} is invertible in $\mathbb{R}\mathcal{H}_\infty$.
2. An arbitrary sequence $\{\vartheta_i\}_{i=0}^{N}$ in $\mathbb{R}^p$ is output trackable if, and only if, $\boldsymbol{\vartheta} \in$ Im $H(N)$, the range space of the matrix $H(N)$. Here, $H(N)$ is the truncated Toeplitz matrix of the Markov parameters, and $\vartheta \triangleq \left(\vartheta_0^T \;\; \vartheta_1^T \;\; \dots \;\; \vartheta_N^T\right)^T$. For such an output sequence, the necessary input is precisely $\mathbf{u} = H^{\dagger}(N)\boldsymbol{\vartheta}$, where $\mathbf{u} \triangleq \left(u_0^T \;\; u_1^T \;\; \dots \;\; u_N^T\right)^T$ and the Moore-Penrose inverse of a matrix X denoted by $X^{\dagger}$.
3. Given an arbitrary sequence $\{\vartheta_i\}_{i=0}^{N}$ in $\mathbb{R}^p$ there exist unique sequences $\{\vartheta_i^a\}_{i=0}^{N}$ and $\{\vartheta_i^b\}_{i=0}^{N}$ such that

$$\vartheta_i = \vartheta_i^a + \vartheta_i^b, \quad 0 \le i \le N$$

   where $\{\vartheta_i\}_{i=0}^{N}$ is the largest (in the sense of the 2-norm) output trackable sequence in $\{\vartheta_i\}_{i=0}^{N}$. Moreover, $\|\boldsymbol{\vartheta}\|_2^2 = \|\boldsymbol{\vartheta}^a\|_2^2 + \|\boldsymbol{\vartheta}^b\|_2^2$ where $\boldsymbol{\vartheta}^j \triangleq \left(\vartheta_0^{j^T} \;\; \vartheta_2^{j^T} \;\; \dots \;\; \vartheta_N^{j^T}\right)^T$ *for* $j = a$ and b.
4. For a square plant ($p = m$) with a well-defined relative degree $r \triangleq (r_1\, r_2 \cdots r_2)^T$, an arbitrary sequence $\{\vartheta_i\}_{i=0}^{N}$ in $\mathbb{R}^p$ is output trackable if, and only if, $\boldsymbol{\vartheta}_{j,i} = 0$, $0 \le i \le r_j - 1$, $1 \le j \le p$, where $\vartheta_i \triangleq \left(\vartheta_{1,i} \;\; \dots \;\; \vartheta_{p,i}\right)^T$.

Comments: (i) Arbitrary sequences are not necessarily output trackable by a given system. (ii) For desired signals with a finite support, the input can be computed directly. This technique subordinates relevant issues such as the presence of non-minimum phase zeros, or the inverse of the transfer function not being causal. (iii) The delay δ in (3) is precisely r, the relative degree. In particular, for an invertible plant $r = 0$, and any arbitrary sequence is output trackable.

For the more general setting, consider the nominal model represented by (2), where we assume, without loss of generality, w_k and v_k are white noise processes with covariances Q_w and R_v respectively, uncorrelated with each other, and with the initial condition x_0. In the augmented state space model approach, the state space representation of the nominal plant model is augmented with a model for the input $u_{k+1} = u_k + \eta_k$, where η_k is white noise with covariance Q_η, and uncorrelated with w_k, v_k and the initial condition x_0. Thus,

$$\begin{aligned} x_{a,k+1} &= A_a x_{a,k} + G_a w_{a,k} \\ y_k &= C_a x_{a,k} + H_a w_{a,k} \end{aligned} \tag{4}$$

where $$x_{a,k} = \begin{pmatrix} x_k^T & u_k^T \end{pmatrix}^T, w_{a,k} = \begin{pmatrix} w_k^T & v_k^T & \eta_k^T \end{pmatrix}^T, \text{ and,}$$

$$A_a = \begin{pmatrix} A & B \\ 0 & I \end{pmatrix}, \; G_a = \begin{pmatrix} G & 0 & 0 \\ 0 & 0 & I \end{pmatrix},$$
$$C_a = (C \quad D), \; H_a = (0 \quad I \quad 0).$$

The augmented system is observable for almost all points in the complex plane, provided the original system is observable:

Lemma 1 ([12]). *Suppose the pair* (C, A) *is observable. The pair* (C_a, A_a) *is observable if, and only if,* $z = 1$ *is not a zero of the system* (A, B, C, D).

Under this condition we can set up a Kalman filter [29] to estimate the input signal:

$$\hat{x}_{a,k+1|k} = A_a \hat{x}_{a,k|k-1} - K_k (C_a \hat{x}_{a,k|k-1} - y_k)$$

where $K_k = A_a P_{k|k-1} C_a^T (C_a P_{k|k-1} C_a^T + R_v)^{-1}$ and $P_{k|k-1}$ satisfies the Riccati equation

$$P_{k+1|k} = \begin{pmatrix} GQ_w G^T & 0 \\ 0 & Q_\eta \end{pmatrix} + A_a P_{k|k-1} A_a^T - A_a P_{k|k-1} C_a^T \left(C_a P_{k|k-1} C_a^T + R_v \right)^{-1} C_a P_{k|k-1} A_a^T .$$

We can then show the following:

Theorem 2 ([12]). *The Kalman filter when applied to the augmented dynamic model (4) yields an input sequence* $\{\hat{x}_{k|k-1}\}_{k=0}^N$ *and a state sequence* $\{\hat{x}_{k|k-1}\}_{k=0}^N$ *such that* $\varepsilon\left\{\| y_k - \hat{y}_{k|k-1} \|^2\right\}$ *and* $\varepsilon\left\{\| u_k - \hat{u}_{k|k-1} \|^2\right\}$ *are minimised.*

We can easily see that $\hat{u}_{k+1|k} = (0 \quad I)\hat{x}_{a,k+1|k}$, and hence the transfer matrix from the measurements y_k to the estimate of the input $\hat{u}_{k/k-1}$ (i.e., the inverse system) is as follows:

$$\Sigma_{\text{inv}} = (0 \quad I)\,(zI - (A_a - KC_a))^{-1} K \tag{5}$$

where K denotes the steady state Kalman gain. Clearly, the poles of the inverse system are relocated

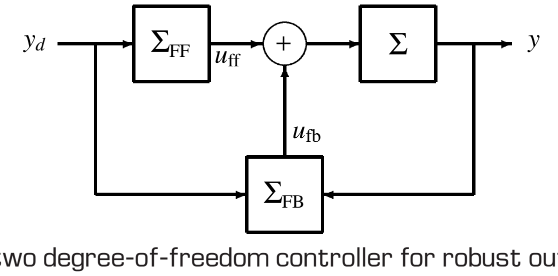

Figure 2. A two degree-of-freedom controller for robust output tracking

eigenvalues of the system matrix A_a. In addition, we can show the following:

Lemma 3 ([12]). *Every eigenvalue of the system matrix A is a zero of the inverse system* (5).

Comments: (i) Clearly, K is a stabilising matrix, and the resulting inverse system is an admissible filter. (ii) For nominal plants with no zeros at $z = 1$, the aforementioned method can handle systems with non-minimum phase zeros, and as well account for process and/or measurement noise by a suitable design of the Kalman filter. This technique, is therefore, more general than all the methods developed earlier [12]: extension of method of dichotomies, and the decoupled observer approach. (iii) Quite often, discretisation of continuous time systems using, for example, a sample and hold, lead to zeros at unity. These zeros are first dislocated numerically before designing the Kalman filter for the augmented system comprising the modified system and the input model. Although, in principle, the only offending point is $z = 1$, in practice, however, for numerical efficiency, it is recommended that any zeros located within a circle of radius $\in$ centred at unity be dislocated. Two methods for dislocating the zeros in this region and grouping them together as one function is outlined in [11]. The first method extracts an all-zero factor and assumes an *a priori* knowledge of the zeros. The other method is based on the numerically efficient method of dislocating zeros discussed in [30], and avoids the explicit computation of the zeros. This is advantageous especially when there is a cluster of zeros in this region.

3. ROBUST OUTPUT TRACKING

As outlined in Section 2, the required input for a desired output trajectory is estimated using a nominal plant model. Since there is no guarantee that this input signal yields the desired response from the actual plant Σ, we use the two DOF controller structure shown in Fig. 2. (It is well-known that with a two DOF controller, the conflicting objectives of output tracking and robustness can simultaneously be achieved [16].) The input to the plant is the sum of two signals: The first signal u_{ff} is the output of the feed-forward controller Σ_{FF}; this block represents the procedure (presented in Section 2) that estimates the input signal required for tracking a desired signal y_d given the nominal plant model Σ_{nom}. The second signal u_{fb} is the output of the feedback controller Σ_{FB} that is to be designed for robust output tracking.

One possible structure for the feedback controller is depicted in Fig. 3. Here, y_d is the desired output, u_{ff} is the feed-forward signal estimated using the nominal plant model Σ_{nom}, and H is the controller that is to be designed for robust output tracking. From Fig. 3, the control signal to be applied to the plant Σ is given by

$$u_k = u_{\text{ff},k} + u_{\text{fb},k} \tag{6}$$

The feedback signal u_{fb} may be designed in a number of ways. Let $P(z)$ and $P_{\text{nom}}(z)$ respectively be the transfer functions of the given plant Σ and the nominal plant model Σ_{nom}. From Fig. 3, observe that

$$E(z) = (I - P(z)H(z))^{-1}(Y_d(z) - P_{\text{nom}}(z)U_{\text{ff}}(z)) - (I - P(z)H(z))^{-1}\Delta P(z)U_{\text{ff}}(z), \tag{7}$$

where $P(z) = P_{\text{nom}}(z) + \Delta P(z)$, assuming additive uncertainty. By a suitable design of the Kalman filter as per the procedure given in Section 2, $Y_d(z) - P_{\text{nom}}(z)U_{\text{ff}}(z)$ is minimised in the 2-norm sense, and

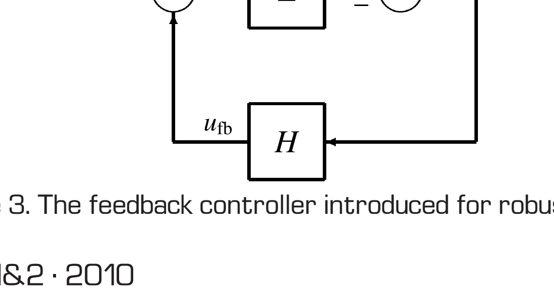

Figure 3. The feedback controller introduced for robustness

hence the first term in the above equation is minimised. The second term is a function of the perturbation $\Delta P(z)$. To deal with this perturbation, the controller $H(z)$ can be designed by a variety of techniques; for instance, simple static and dynamic output feedback controllers were designed in [12]. Thus, the scenario depicted in Fig. 3 is adequate for robust output tracking as long as the uncertainty in the plant model is sufficiently small; i.e., the feedback controller $H(z)$ can provide robust stability and performance. However, the ever-increasing demand for expanding the operating regions characterised by significantly larger levels of uncertainty places increased performance requirements on the design of control systems, notably in the aerospace industries. To deal with such situations, we propose a multiple model architecture.

As shown in Fig. 4, the chosen multiple model architecture, adopted from [17], consists of a finite number of models of the plant, denoted $\{M_i\}_{i=1}^{N}$, operating in parallel. Observe that the input signal to each of the models M_i is same as that of the given plant. The error between the outputs of a model M_i and the plant is denoted by $e_i \triangleq y_i - y$. Associated with each model M_i is a controller C_i. The control strategy is to determine that model with the least error e_i , and switch to the corresponding controller; i.e., at any instant k, one of the models, say M_j, and the corresponding controller C_j, is chosen such that

$$j = \arg\min_{i \in I_N} J_i$$

with a minimum interval $T_{min} > 0$ between switches; here, $I_N \triangleq \{1,2,\ldots,N\}$, and the performance criterion given by

$$J_i(k,e_i) = \alpha_1 \|e_{i,k}\|^2 + \alpha_2 \sum_{j=0}^{k} \|e_{i,j}\|^2 \Delta T ,$$

with α_1, $\alpha_2 \geq 0$, and ΔT, the sampling interval.

Let $P(z)$ be the transfer function of the given non-adaptive plant Σ, and $P_{\text{nom}}(z)$ be the transfer function of the chosen nominal design model Σ_{nom}. If y_d is the desired output trajectory, let the feed-forward signal u_{ff} be estimated using the nominal model P_{nom}. Let M_i, $1 \leq i \leq N$, be a set of nominal models associated the plant, and the number of such models be chosen such that the behaviour of the plant in different possible scenarios are represented. Specifically, let the models be defined as $M_i(z) \triangleq P_{\text{nom}}(z) + \Delta_i(z)$, $1 \leq i \leq N$, for some appropriately chosen additive uncertainties $\Delta_i(z)$, $1 \leq i \leq N$. Corresponding to each such model M_i, let C_i be an admissible feedback controller designed such that the effect of its uncertainty Δ_i is minimised; i.e., the second term in eqn. (7) is minimised. We then have the following result:

Proposition 4. *Given the plant Σ, let the chosen nominal models be $M_i(z)$ and the corresponding designed controllers be $C_i(z)$. Suppose that in Fig. 4, at any instant k, one of the models, say M_j, and the corresponding controller Cj, is chosen such that $J = \arg\min_{i\in I_N} J_i$ with a minimum interval $T_{min} > 0$ between switches, and $J_i(k,e_i) = \| e_{i,k} \|^2$ Then, the tracking error $e_c = y_d - y$ in Fig. 4 is bounded.*

Without loss of generality, let the model M_p be chosen at time $k = 0$. There are two possible situations: (a) The plant $P = M_i$ for some i. If $i = p$, the output of the plant y is same as the output y_p of the model M_p, leading to the minimum identification error. Therefore, no switching takes place. Since u_{ff} is estimated via an admissible filter, and the feedback part is based on an admissible controller C_i, all the signals are bounded, and so is the the tracking error e_c.

Suppose that $i \neq p$. Observe that the controller output u_p is the input to the plant and all the models; in particular, it is the input to M_i. Since $P = M_i$ the corresponding identification error $e_i = y_i - y$ is the least, and hence the model M_i is chosen after the interval T_{min}. Subsequently, the output of M_i matches that of P, and no switching takes place. Again, since u_{ff} is bounded, and C_i is admissible, the tracking error e_c is bounded.

(b) The plant $P \neq M_i$ for all i. By our assumption on the choice and number of models, P is close to M_i for some i. By an argument similar to the above, the output y_i of the model M_i is closer to that of the

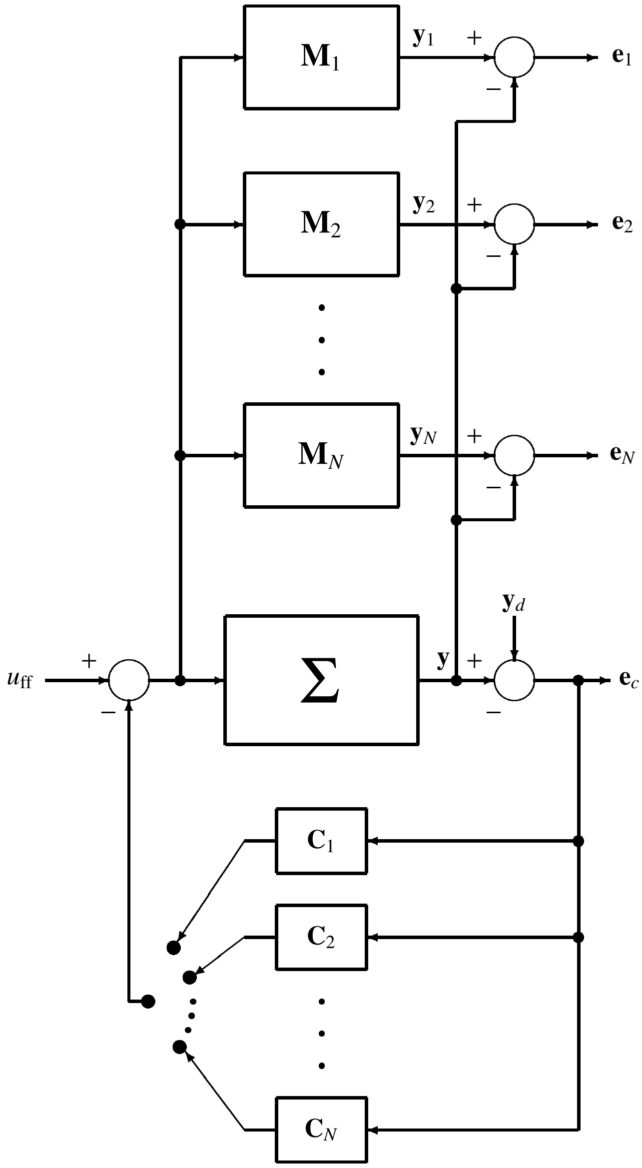

Figure 4. Multiple models for robust output tracking

plant. Hence, the instantaneous error e_i is the least. Accordingly, the model M_i is chosen at the next switching instant. Subsequently, the output M_i is closest to P, and the tracking error remains bounded.

Comments: (i) The multiple model architecture in Fig. 4 is such that the controller C_i is designed only for the uncertain part of the plant to minimise the second term in eqn. (7). In this sense, the proposed scheme in this paper is different from other possible solutions to the problem of robust tracking. (ii) In a deterministic setting, with $\Delta P(z) \equiv 0$, *and* y_d an output trackable sequence, the error in (7) is zero if the plant is invertible in $\mathbb{R}\mathcal{H}_\infty$ (iii) The stability of the overall system is an important issue; indeed, one cannot guarantee stability for arbitrary switching of models. However, in this paper, we do not consider such switching of models, and the purpose of the paper is to demonstrate the use of such an architecture for robust output tracking.

4. APPLICATION: HELICOPTER HOVER CONTROL

In this section we apply our technique to the hover control of a Bell 205 helicopter for robust output tracking. The example considered here is a case wherein the dynamics was trimmed at a nominal 5 degrees pitch attitude with a mid-range weight and a mid-position centre of gravity, and operating at near sea level [7]. The linearised state space model is an eighth order system with four inputs and four outputs. The states are forward, vertical and lateral velocities, roll, pitch and yaw rates, and, roll and pitch attitudes. The inputs to the system are collective, longitudinal and lateral cyclic, and tail rotor collective. The outputs of the system are forward, vertical and lateral velocities, and the yaw rate. The objective is to control these outputs forcing them to track certain a priori specified profiles: The forward velocity and the yaw rate are to be maintained at zero, and the desired profiles of vertical and lateral velocities are shown in Fig. 5 as solid lines.

The continuous-time model given in [7] is discretised with a sampling rate of 200 Hz and a zero order hold. As a result of the discretisation process, the zeros of the discretised model are positioned at $0.9997 \pm 0.0114i$ and $1.0000 \pm 0.0215i$, which are clearly very close to $z = 1$. These zeros are different from unity perhaps because of the numerical computations. In order to apply our procedure these zeros are dislocated; amongst these, the non-minimum phase zeros result in a small period of pre-actuation.

The application of the feed-forward control input u_{ff} estimated via the augmented state-space procedure outlined earlier to the nominal plant model is also shown in Fig. 5, with the input signals in Fig. 6. The desired signals in Fig. 5 are shown as solid lines and the actual trajectories with dotted lines. We observe satisfactory tracking for both vertical and lateral velocities. Albeit small errors are observed for both forward velocity and yaw rates, we note that these are smaller than the errors obtained in [7]. In addition, the pre-actuation that is required using our technique is only 25ms. This pre-actuation is induced by the non-minimum phase zeros in a small region around unity that have been dislocated; such zeros are the result of a numerical discretisation process. Finally, we recall that our procedure attempts to approximate the overall transfer matrix from u to $\hat{u}$ (refer Fig. 1) by a pure delay. For this

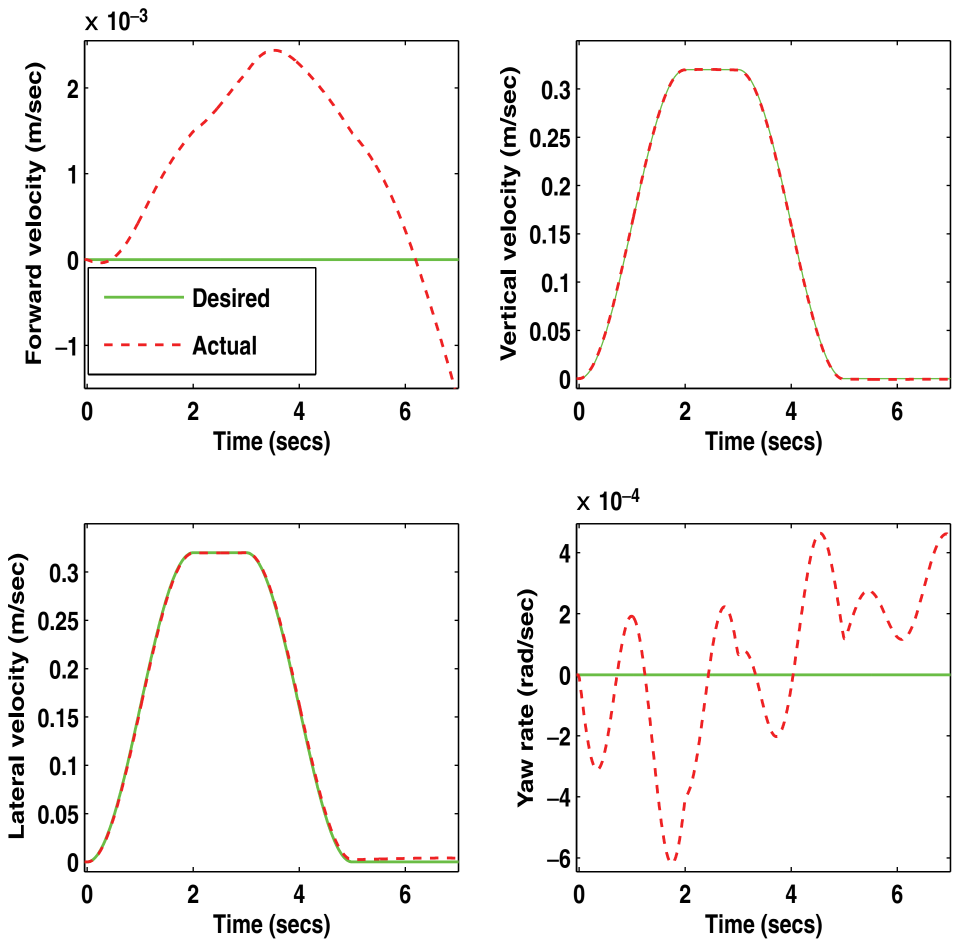

Figure 5. Feed-forward signal applied to nominal continuous time plant; -- output of system; — desired signal

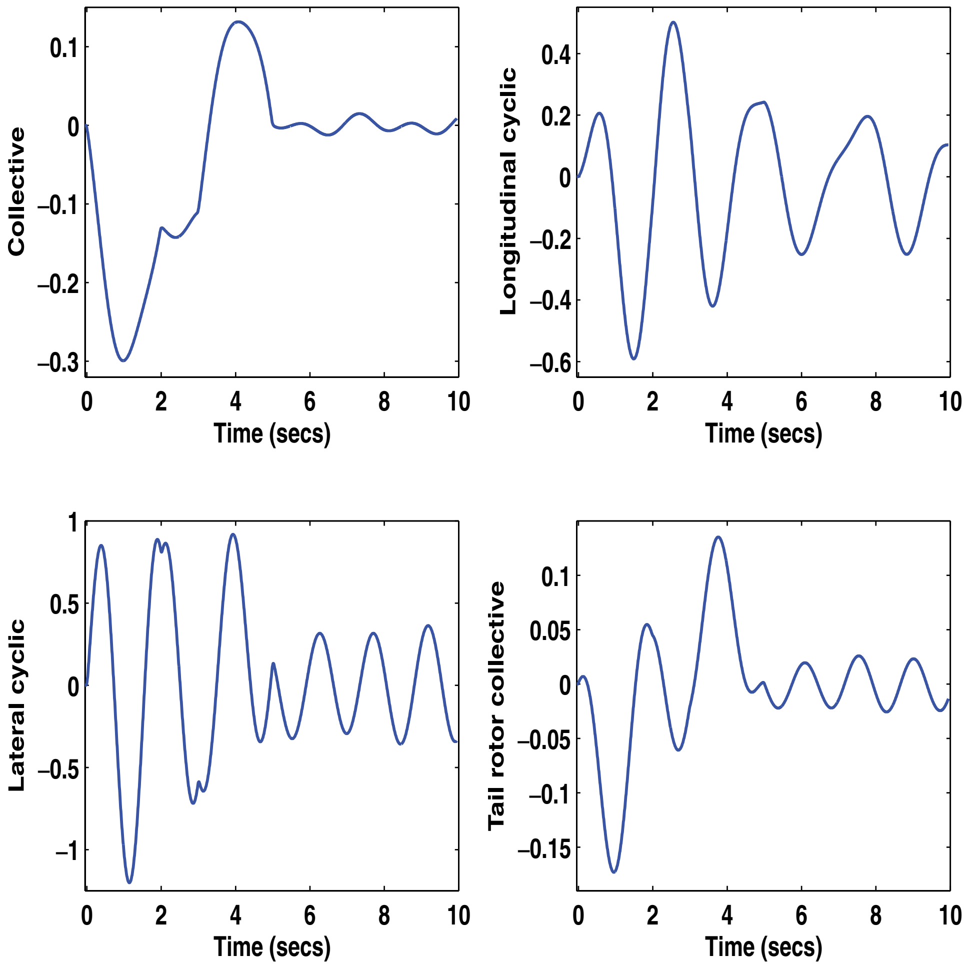

Figure 6. Estimated input signals

example, we observe a delay of 10 samples. In practical implementations, perfect tracking is obtained by applying the input after this delay, ignoring the first few samples. (We note that the method of dichotomies cannot directly be applied due to non-hyperbolic unstable internal dynamics. Also, since the relative degree is one, this method requires the knowledge of the derivatives of desired outputs. Numerically determining the derivatives of practically obtained desired trajectories is typically erroneous, especially in the presence of noise.)

As mentioned in Section 1, it is rather difficult to design a single controller that is valid globally for all operating conditions. We achieve robust output tracking via designing controllers that are effective locally, and using a multiple model architecture to implement these different controllers. In the helicopter example, we illustrate this overall procedure assuming that the uncertainty in the nominal model takes the form of structured additive uncertainty in the continuous-time state-space model:

$$x(t) = (A + \Delta_A)x(t) + (B + \Delta_B)u(t)$$

We further assume, in this paper, that the system matrix A and the input matrix B can vary rather widely, say up to 80%. For purposes of illustration, we assume that the overall uncertainty can be covered by eight different models, denoted M_1, M_2, ..., M_8, with increasing levels of uncertainty. Thus, M_1 corresponds to the 10% changes in A and B, and M_8 corresponds to 80% changes in these matrices, with the models M_2 through M_7 covering the intermediate levels of uncertainty. Corresponding to these eight different models, we design controllers C_1, C_2, ..., C_8. We use the following techniques: (a) state-feedback controllers (i.e., $u_{fb} = K(x_d - x)$ where x_d is the desired state also available from our technique described in Section 2), and (b) dynamic output feedback controllers (i.e., $u_{fb} = H(z)(y_d - y)$). However, for brevity, we present in Figures 8–10 the simulation results for only dynamic output feedback

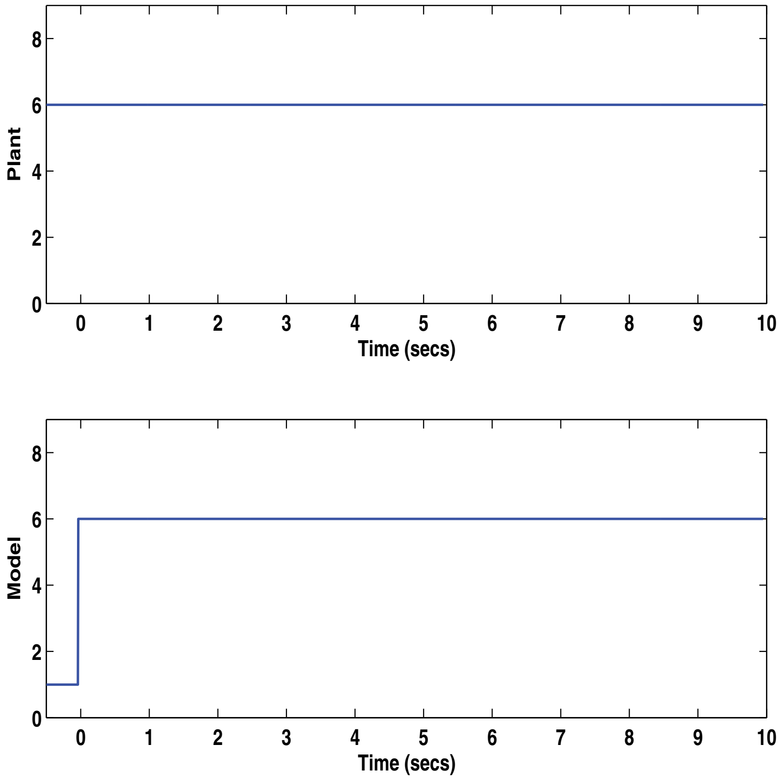

Figure 7. Model switching when the plant is exactly one of the models

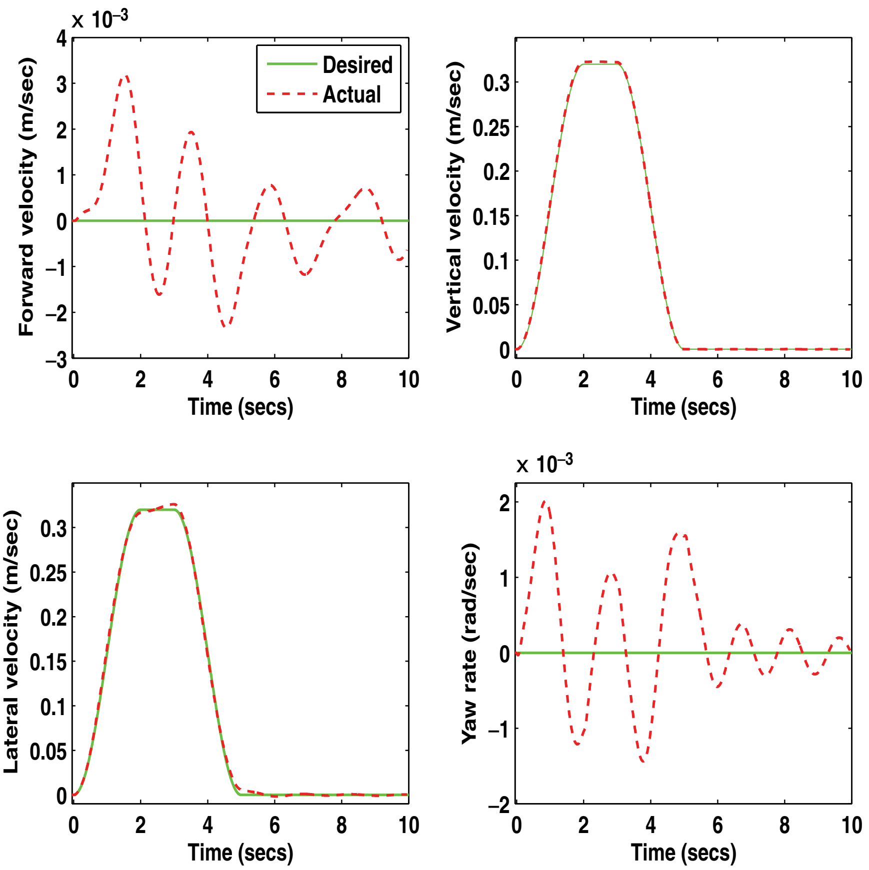

Figure 8. Desired and actual outputs when the plant is exactly one of the models. -- output of system; — desired signal

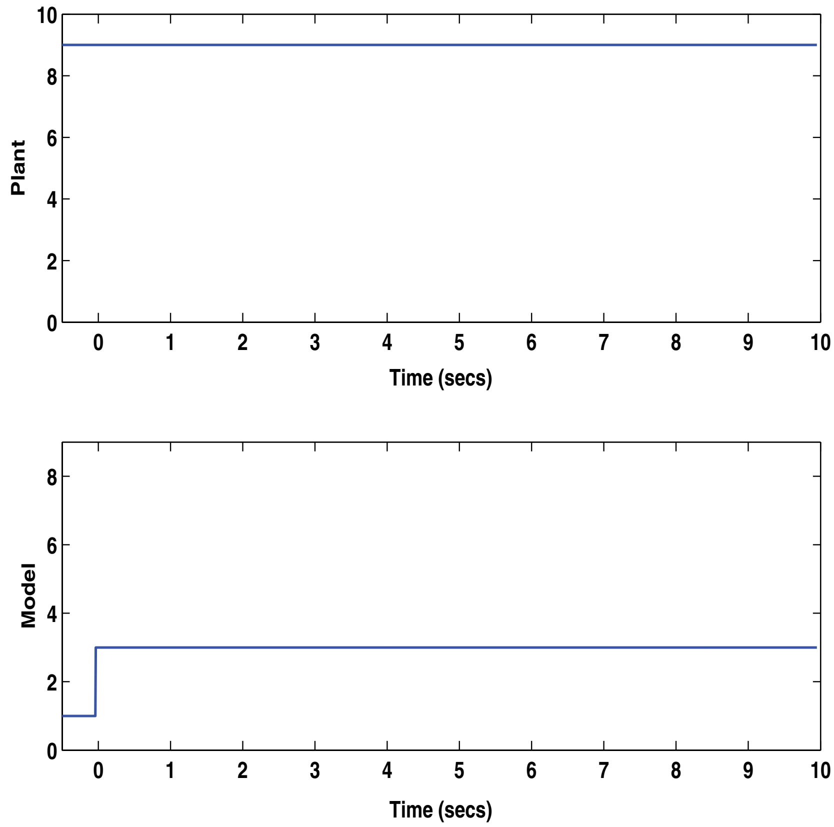

Figure 9. Switching between models when the plant is different from all the models

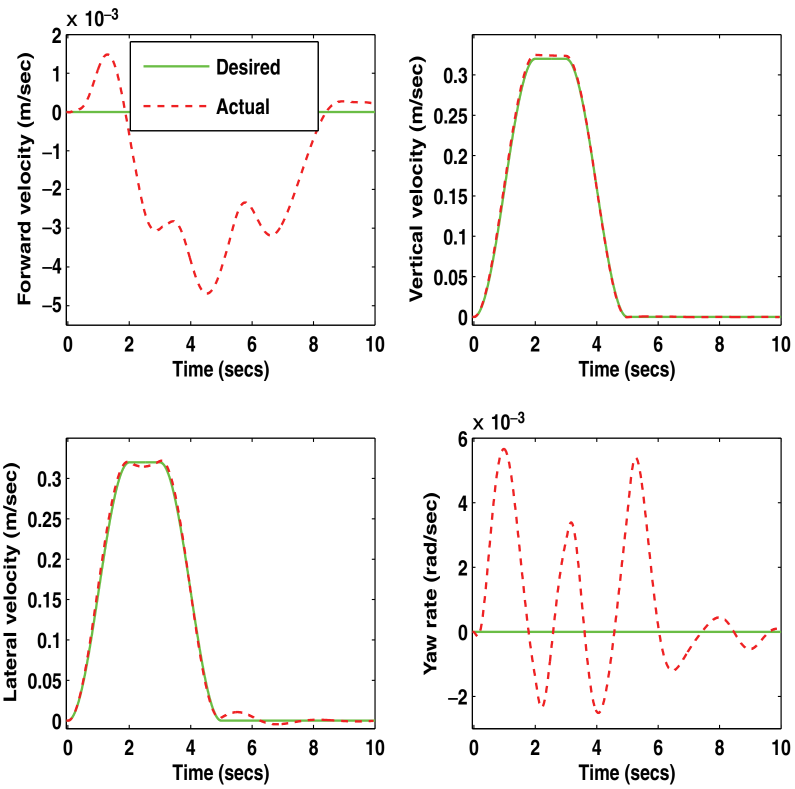

Figure 10. Desired and actual outputs when the plant is different from all of the models. -- output of system; — desired signal

controllers.

The following experiments were conducted: (a) The plant is assumed to be precisely one of the 8 models: Here, the model of the plant is assumed to M_6, and the controller does not have this knowledge a priori. Nonetheless, the controller chooses the appropriate control action, as shown in Fig. 7, and the resulting desired and output trajectories are as shown in Fig. 8. It can be observed from Fig. 7 that the plant matches the model M_6. The overall controller is such that the model M_1 has been chosen arbitrarily for time $k = 0$. Since T_{min}, the time between switches is only a few samples, the switch from model M_1 to M_6 in Fig. 7 appears to start at time $k = 0$. The tracking performance of the overall system is acceptable and comparable to the tracking performance of the nominal plant model shown in Fig. 5; the observed small levels of degradation in the performance is acceptable due to the uncertainty in the plant model.

(b) For the second experiment, we assume that the plant is different from all the eight models, and arbitrarily assigned the number 9 to emphasise that it is different from the eight models that is part of the controller architecture. In this experiment, the uncertainty level is chosen such that it is closer to the model M_3 relative to the other models. It can be observed from Fig. 9 that the controller architecture indeed chooses the model M_3, and applies the corresponding control action after an interval T_{min} (not perceptible in Fig. 9). The desired and output trajectories for this simulation exercise is depicted in Fig. 10. It is observed that the tracking performance is acceptable, and comparable to that chosen in Fig. 5. It is, however, quite natural to expect a further degradation in the performance.

(c) The uncertainty in the plant is assumed in the third experiment to increase progressively indicating that the distance between the plant and the nominal model (corresponding to normal operating condition) is increasing. In order to simplify the experiment, the uncertainty in the plant is assumed to progressively increase from 10% to 80% in eight steps (as represented by models M_1

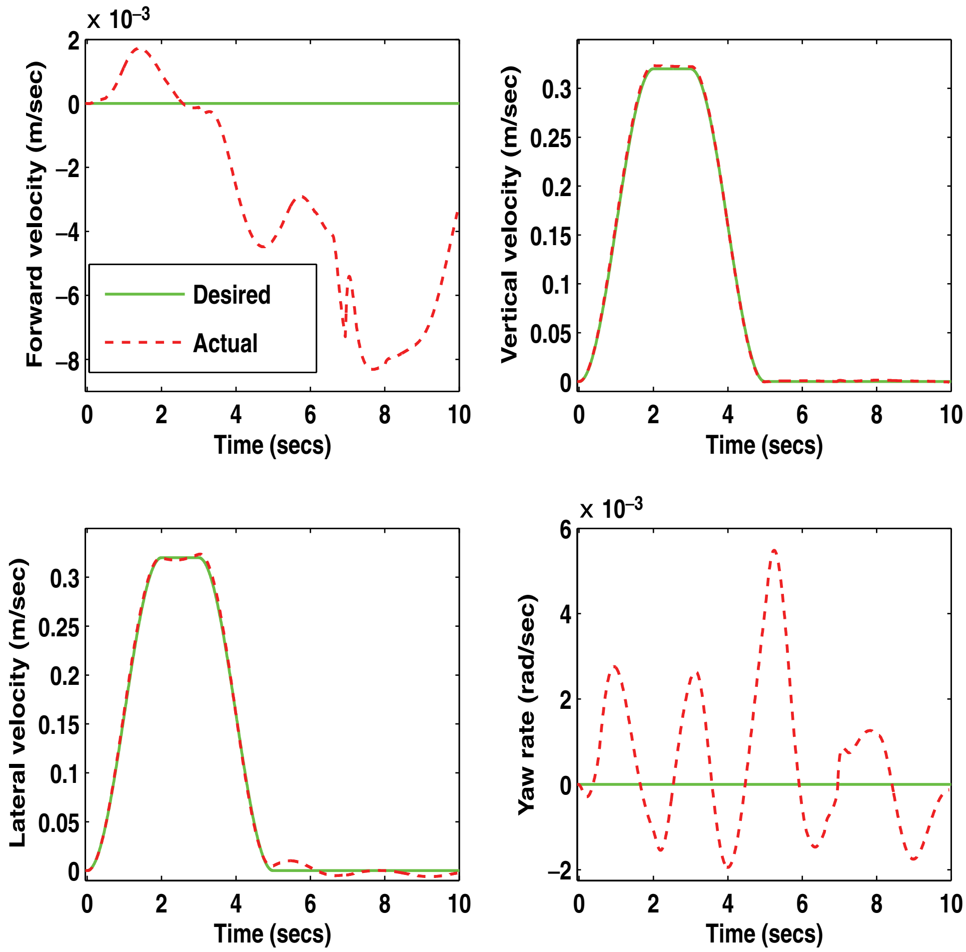

Figure 11. Desired and actual outputs when the uncertainty in the plant is increasing. -- output of system; — desired signal

through M_8), with each step lasting 2.5 seconds. The tracking performance of the overall controller when the plant is changing rather rapidly is as shown in Fig. 11. It can easily be observed that despite the fact that the uncertainty of the plant model changes widely from 10% to 80%, the overall controller is such that appropriate control action is taken to keep the tracking errors bounded and reasonable, thereby demonstrating the efficacy of the technique.

5. CONCLUSIONS

A two degree-of-freedom controller comprising a feedforward controller that estimates the necessary input for a desired output trajectory, and a feedback controller via a multiple-model approach to ensure robustness shows promise in output tracking applications wherein the plant dynamics may change widely.

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