Randomness in Elastic Properties of Composite: A Numerical Study

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Abstract
Randomness in any composite structure is inevitable. Randomness in fiber arrangement and constituent material properties for a metal matrix composite are considered, with both fiber and matrix being isotropic. Finite Element Analysis has been done with the assistance of Probabilistic Design module of ANSYS. Random samples has been generated using Latin Hyper-Cube Sampling technique. Three sets of simulation were carried out. In the first simulation, only the fiber arrangement was varied. In the second simulation, only the material properties of the constituents were varied. In the third simulation, both fiber arrangement and material property were varied. The results show that when randomness in both fiber arrangement and constituent material property were considered the elastic constants of composite show wide variation. There is appreciable uncertainty in the composite material properties and the converged effect is not a trivial extrapolation of input probability distributions. Also, various material constants of the composite are affected to differing extent.

Keywords: Representative Volume Element, Latin Hyper-Cube Sampling, Unit Cell, Boundary Conditions, ANSYS.

1. INTRODUCTION

Fiber reinforced composite materials have found wide application in aerospace and other engineering structures. These materials possess high specific strengths and stiffness, excellent fatigue and corrosion resistances and better thermal characteristics than metals. Prediction of mechanical properties of these unidirectional fiber-reinforced composites has been an active research area for the past three decades.

Several theoretical and numerical models have been proposed for the prediction of composite properties from those of the constituents i.e. fiber and matrix. Many analytic models have been developed to obtain the upper and lower bounds for elastic constants. An extensive survey of the existing micromechanical models has been carried out by Hashin [1]. Among the elastic constants predicted by these models, the longitudinal modulus and Poisson's ratio show good agreement with experiments and can be approximated with the simple rules of mixtures. Numerical methods to estimate composite properties usually involve analysis of a Representative Volume Element (RVE). A single RVE corresponds to a periodic fiber packing sequence.

Though the above two methods have both shown good agreement with experimental results for samples manufactured under controlled environment of a laboratory they have failed to show for industry standard coupons as demonstrated by Philippidis [2]. The reason is the lack of sufficient control on the manufacturing process, which leads to dispersion in material properties and microstructure. Hence, randomness in fiber arrangement and orientation, volume fractions, fiber matrix interface and curing parameters are inherent in fiber reinforced composites. These uncertainties are in turn reflected in the randomness in material constants of composites.

In conventional composite structural analysis, mean values of material constants are used. This not only misses the deviations in response due to randomness, but may not even predict the mean of the response correctly. This approach is dangerous for critical components, hence leads to employment of large ad hoc factors of safety. For accurate results, it is necessary that the analysis techniques incorporate the effect of randomness in material constants. This problem has not been fully addressed in the published literature.

In the present work, an attempt has been made to study the effect of randomness in fiber arrangement and mechanical properties of constituent materials i.e. fiber and matrix on material constants of a composite. A square array RVE is studied using proper boundary conditions by Sun and Vaidya [3]. Probabilistic design tool box in ANSYS is used to simulate randomness. The random parameters were generated using Latin Hypercube Sampling (LHS) technique.

LHS is a sampling technique in which the given sampling space is subdivided into 'n' (user defined) number of sub sample space. Single random variable is chosen from these sub sample space. This method ensures uniform sampling as opposed to clustering of samples frequently observed in Monte Carlo simulation. Due to uniform sampling LHS demonstrates faster convergence rates than other sampling technique.

2. REPRESENTATIVE VOLUME ELEMENT (RVE)

In a composite lamina the actual fiber distribution is quite random across a cross-section. For simplicity reasons, most micromechanical models consider a periodic arrangement of fibers for which a RVE or Unit Cell can be isolated. The RVE has the same volume fraction as that of the composite. The periodic fiber sequences commonly used are the square array and the hexagonal array as shown in Fig. 1. These are the most idealistic situations, non-existent in the actual microstructure. For the current study of randomness, the square array is considered with random positioning of fibers as shown for example in Fig. 2. The aim is to study randomness with the existing knowledge of RVE and to minimize the computational time. This type of RVE representation, though flawed in the assumption of periodicity and the very concept of RVE is questionable, they fulfill the equivalence of energy of heterogeneous and equivalent homogenous media constrain as defined by Sun and Vaidya [3].

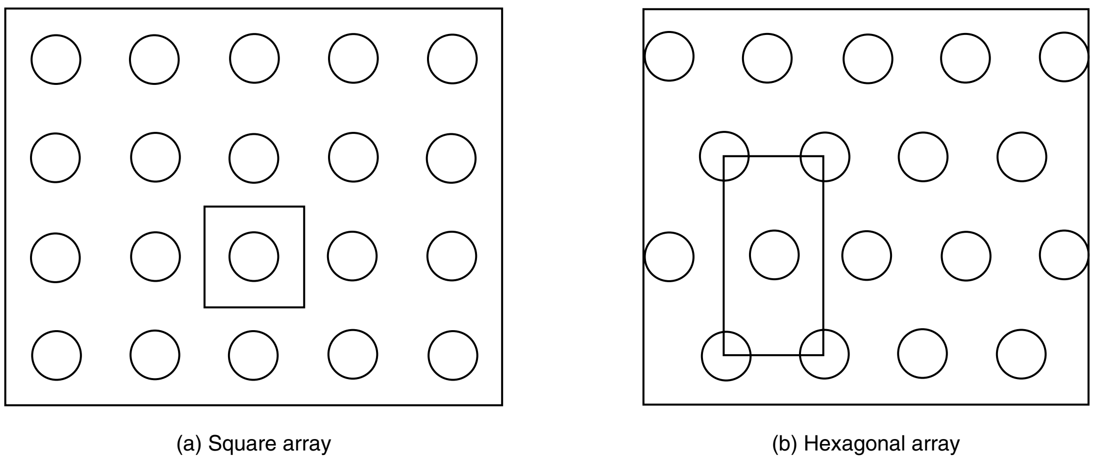

Figure 1. RVE for square and hexagonal array.

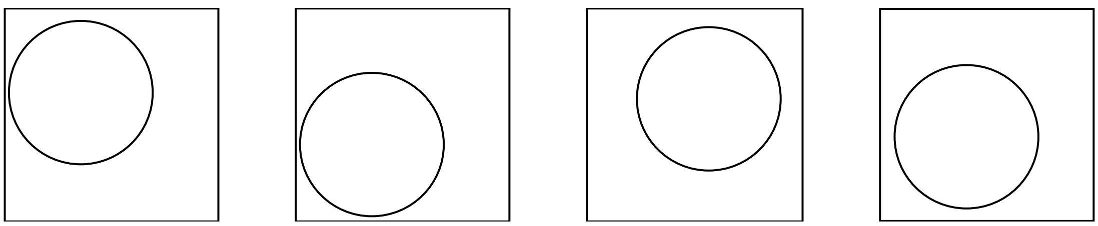

Figure 2. Random positioning of fiber illustrated.

3. FINITE ELEMENT MODELING

3.1. Normal loading

The RVE used in the analysis to model normal loads is shown in Fig. 3. The cross-section is a square i.e. $b = c$. Axial loading is modeled by a force P_1 acting on the face $x_1 = a$, while transverse loading corresponding to a force P_t acting on the face $x_2 = b$ or $x_3 = c$. The normal displacements of the boundaries are restricted to those that cause the boundary to displace only parallel to the original boundary. The displacement constraints applied on the finite element model are:

$$\begin{aligned} u(0, x_2, x_3) &= 0 \\ v(x_1, 0, x_3) &= 0 \\ w(x_1, x_2, 0) &= 0 \\ u(a, x_2, x_3) &= \delta_1 \\ v(x_1, b, x_3) &= \delta_2 \\ w(x_1, x_2, c) &= \delta_3 \end{aligned} \tag{1}$$

where u, v and w denote displacements in x_1, x_2 and x_3 directions, respectively. δ_1, δ_2 and δ_3 are solutions obtained from finite element analysis of RVE subjected to load at the boundary.
For the case of axial loading, P_l, the average longitudinal strain is

$$\bar{\varepsilon}_{11} = \frac{1}{V}\int_S u_1 n_1 dS = \frac{\delta_1}{a} \tag{2}$$

The strain energy stored within the RVE is given by:

$$U = \frac{1}{2}\bar{\sigma}_{ij}\bar{\varepsilon}_{ij}V$$

$$U = \frac{1}{2}\bar{\sigma}_{11}\bar{\varepsilon}_{11}V \tag{3}$$

since σ_{11} is the only component of stress present.
External work done on the RVE by the applied load P_l is given by

$$W = \frac{1}{2}P_l\delta_1 \tag{4}$$

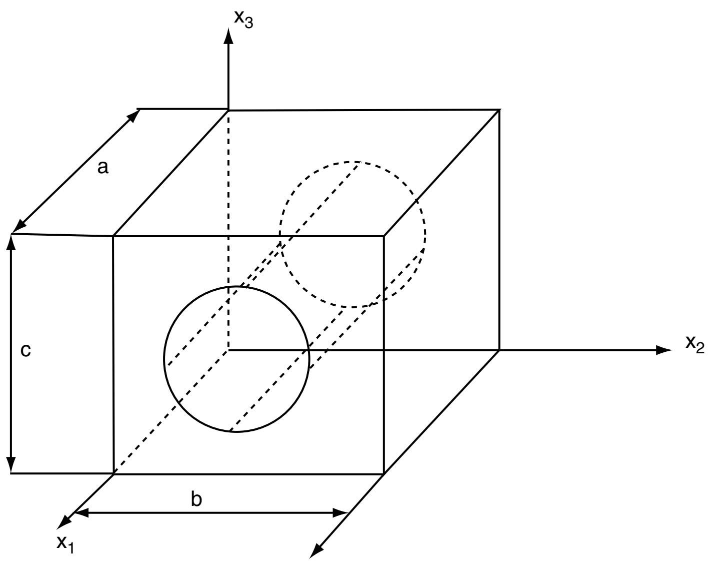

Figure 3. RVE for square arrays.

Using the equivalence of energy between stored energy and external work:

$$\frac{1}{2}P_l\delta_1 = \frac{1}{2}\sigma_{11}\varepsilon_{11}V \tag{5}$$

together with equation (1), the average stress in the RVE is obtained as

$$\bar{\sigma}_{11} = \frac{P_l}{bc} \tag{6}$$

The longitudinal modulus and Poisson's ratio are given by:

$$Y_1 = \frac{\sigma_{11}}{\varepsilon_{11}} = \frac{P_l a}{bc\delta_1} \quad \text{and} \quad \nu_{12} = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{\delta_2 a}{\delta_1 b} \tag{7}$$

Similarly, for load P_t acting transverse to the fiber direction at $y = b$:

$$Y_2 = \frac{\sigma_{22}}{\varepsilon_{22}} = \frac{P_t b}{ac\delta_2} \quad \text{and} \quad \nu_{23} = -\frac{\varepsilon_{33}}{\varepsilon_{22}} = -\frac{\delta_3 b}{\delta_2 c} \tag{8}$$

3.2. Shear loading

The stress and strain fields in a composite under a shear loading are independent of the axial coordinate x_1 (fiber direction) and are functions of x_2 and x_3 only. A two-dimensional generalized plain strain analysis can thus be used in the analysis of this type of loading. The RVE for the square array model is shown in Fig. 4 with randomly positioned fiber. Here the deformed shape is assumed to remain a parallelogram with straight edges as assumed by Naikand and Crews [4] and Brockenbrough et. al [5]. Though this is an overly restrictive constraint and the randomness in fiber arrangement causes random deformation of RVE, which should be captured for each random arrangement and then analyzed, it makes the stochastic study cumbersome and time consuming. Again since the aim is to study the overall randomness pattern and the difference between the shear module obtained from periodic boundary condition and parallelogram boundary condition is less than 5%, the parallelogram boundary condition can be used satisfactorily.

The displacement constraints applied to the finite element method are:

$$\begin{aligned} v(y,-c) &= 0 \\ w(y,\ z) &= 0 \end{aligned} \tag{9}$$

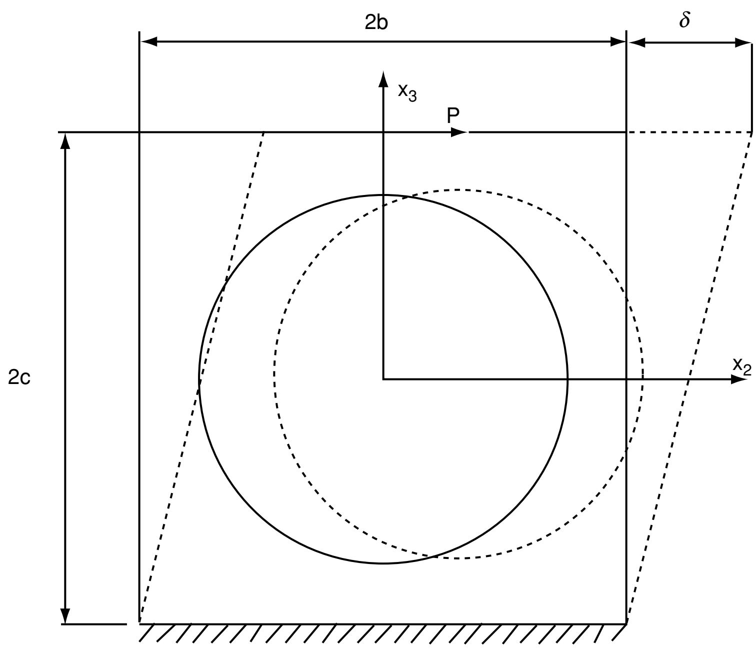

Figure 4. RVE under transverse shear loading.

The average shear strain is obtained as:

$$\bar{\gamma}_{23} = \frac{1}{V}\int_V \gamma_{23} dV = \frac{1}{4bc} \times 2\delta b = \frac{\delta}{c} \tag{10}$$

Equating external work to strain energy in the system:

$$\frac{1}{2}P\delta = \frac{1}{2}\bar{\sigma}_{23}\bar{\gamma}_{23}V \tag{11}$$

Using Eqn. (10) and (11), the expression of average transverse shear stress and shear modulus are:

$$\begin{aligned} \bar{\sigma}_{23} &= \frac{P}{2b} \\ G_{23} &= \frac{\bar{\sigma}_{23}}{\bar{\gamma}_{23}} = \frac{Pc}{b\delta} \end{aligned} \tag{12}$$

4. RESULTS AND DISCUSSION

The simulation has been carried out for boron/aluminum composite. Gaussian distribution of material properties have been considered, with 5% of the mean as the standard deviation. Details of the material properties are given in Table 1.

For the randomness in fiber arrangement the mean position of fiber center is taken at the center of the RVE. e.g., for a square array with unit side length, the mean position of center of fiber is (0.5, 0.5). Gaussian distribution is taken for fiber orientation with standard deviation as 0.03. This value is taken to facilitate the meshing of the matrix. Figs 5, 6 and 7 show the sample distribution of fiber center position and material property with Latin Hyper-Cube Sampling.

Table 1. Material property of boron and aluminum with probabilistic distribution

Material	Property	Mean value	Standard deviation	Distribution
Boron	E(GPa)	379.3	18.965	Gaussian
	v	0.1	0.005	Gaussian
Aluminum	E(GPa)	68.3	3.415	Gaussian
	v	0.3	0.015	Gaussian

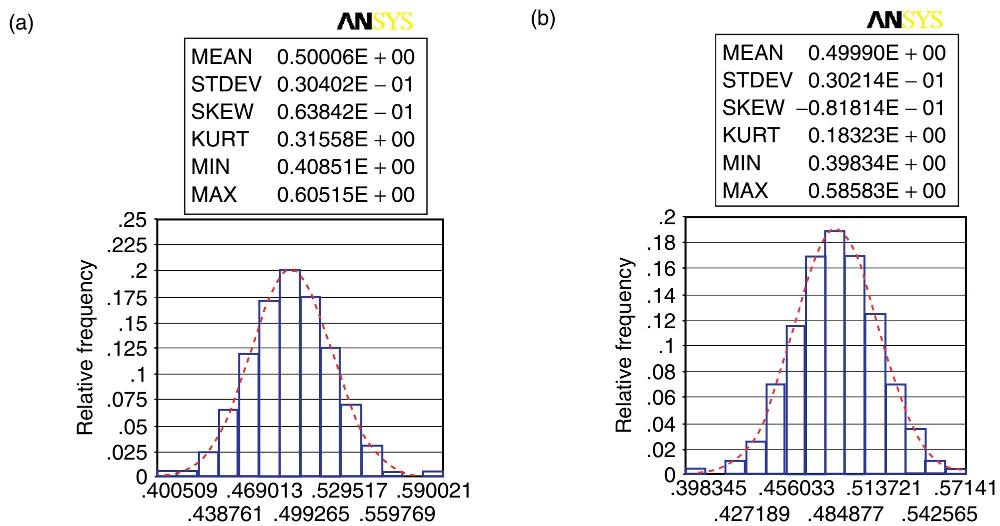

Figure 5. Variation in center of fiber a) x_2 co-ordinate and b) x_3 co-ordinate.

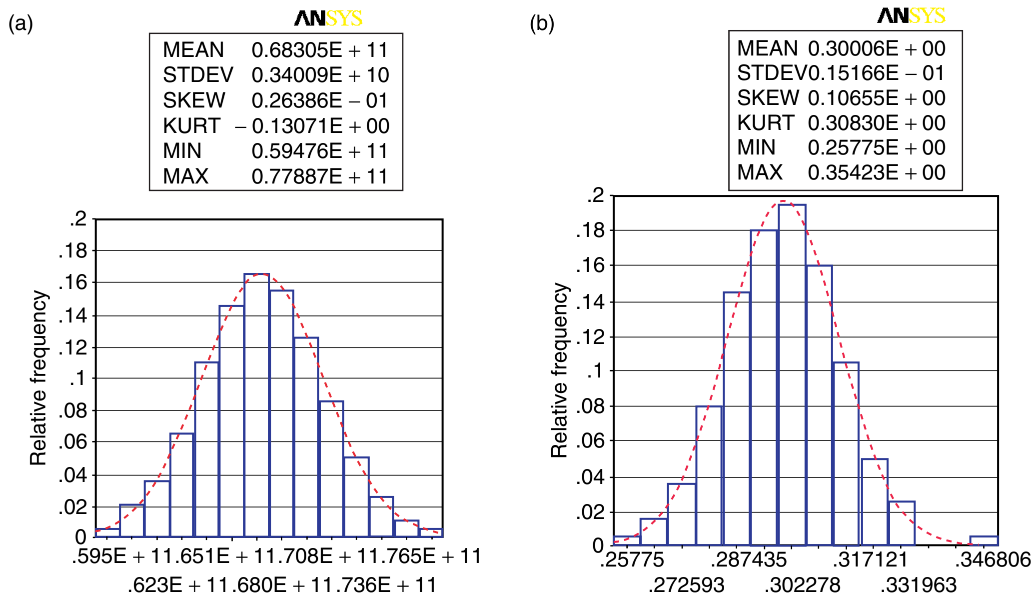

Figure 6. Variation in material properties of Aluminum: a) Young's modulus (TPa) and b) Poisson's ratio.

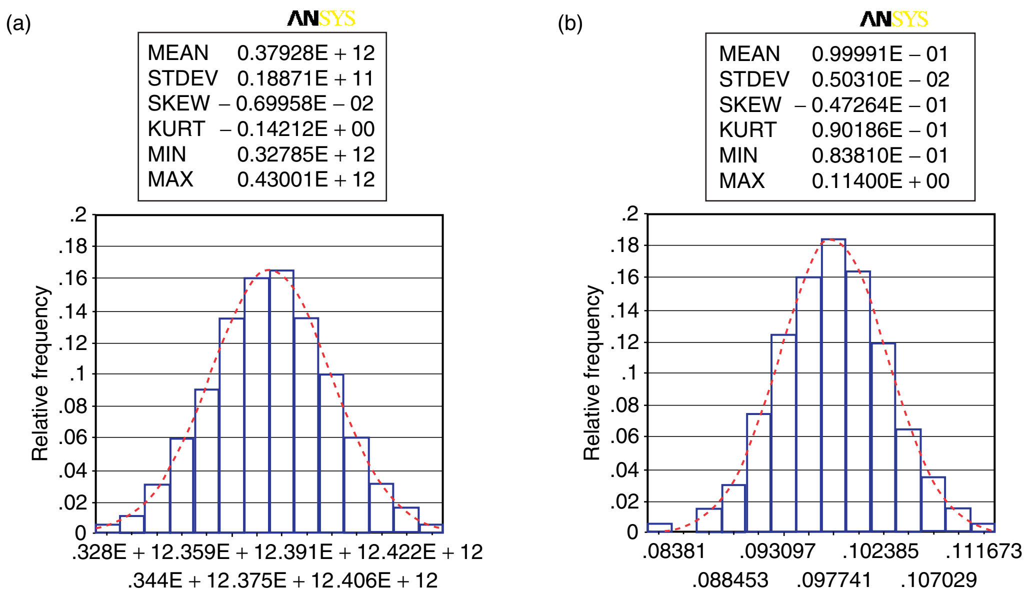

Figure 7. Variation in material properties of Boron: a) Young's modulus (TPa) and b) Poisson's ratio.

4.1. Young's modulus along fiber direction

Along the fiber direction the fibers are the primary load carrying members. Hence the Young's modulus along the fiber direction is mainly dominated by Young's modulus of the fiber. This deterministic phenomenon is also observed in this probabilistic analysis. When randomness in fiber position is considered the scatter is limited to a narrow band of 190 *GPa* to 206 *GPa* as shown in Fig. 8(a). Similar trend is also observed while considering randomness in material properties alone. The scatter is limited to the range 195 *GPa* to 215 *GPa* as shown in Fig. 8(b), which is about 10% of the mean value. When randomness in both the parameters are considered simultaneously the range becomes wider i.e. between 183 *GPa* to 223 *GPa* as shown in Fig 9. It should be noted that when randomness in both fiber

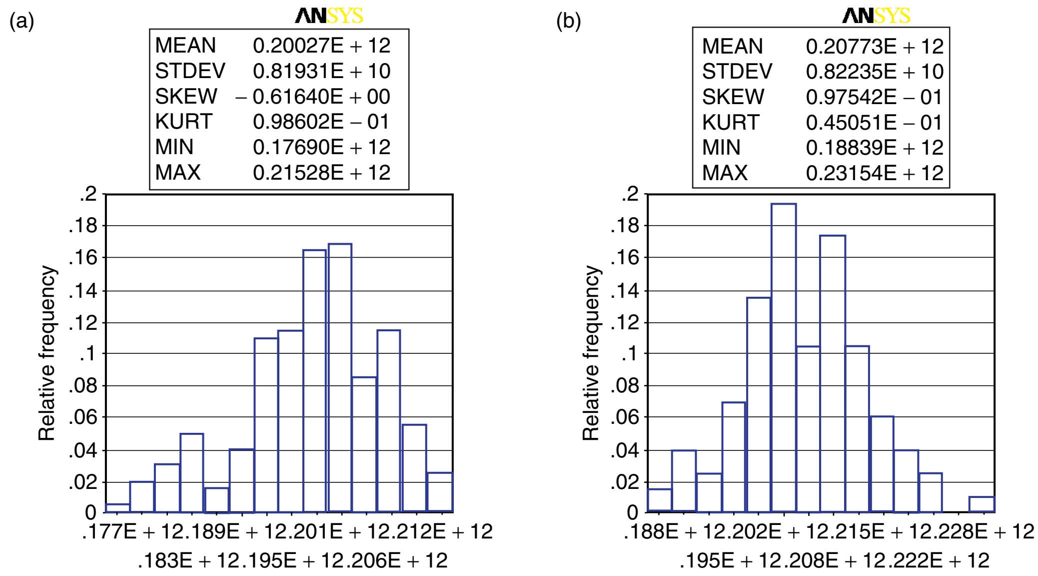

Figure 8. Variation in Young's modulus (TPa) along fiber direction due to variation in (a) fiber location only (b) material properties only.

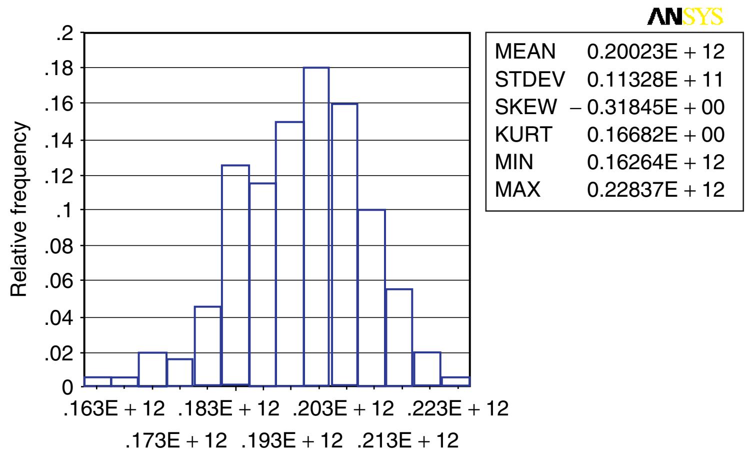

Figure 9. Variation in Young's modulus (TPa) along fiber direction due to simultaneous variation of fiber location and material properties.

distribution and material property are accounted there is a nearly uniform distribution of probability between 190 *GPa* to 213 *GPa* i.e. any value in this range is almost equally possible.

4.2. Young's modulus transverse to fiber direction

Young's modulus in transverse direction is primarily dependent on matrix and micro structural arrangement. Hence, once the micro-structure symmetry is broken the transverse Young's modulus decreases. This is what is observed in Fig. 10 (a) where the plot shows randomness in transverse Young's modulus due to randomness in location of fiber. When randomness in material properties alone are considered there is a almost uniform distribution of transverse Young's modulus in the range of 139 *GPa* to 149 *GPa* as shown in Fig. 10(b). Here it should be noted that the band of variation though only 10 *GPa* which is about 7.5% of the mean value. Once both the randomness are simultaneously incorporated, the trend remains same as that due to randomness in material property but the band widens to the range of 134 *GPa* to 151 *GPa* as seen Fig. 11.

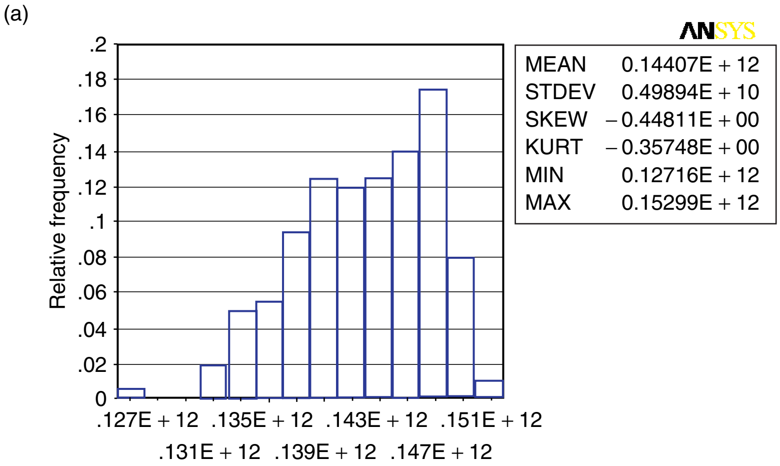

Figure 10. Variation in Young's modulus (TPa) transverse to fiber direction due to variation in (a) only fiber location and (b) only material properties.

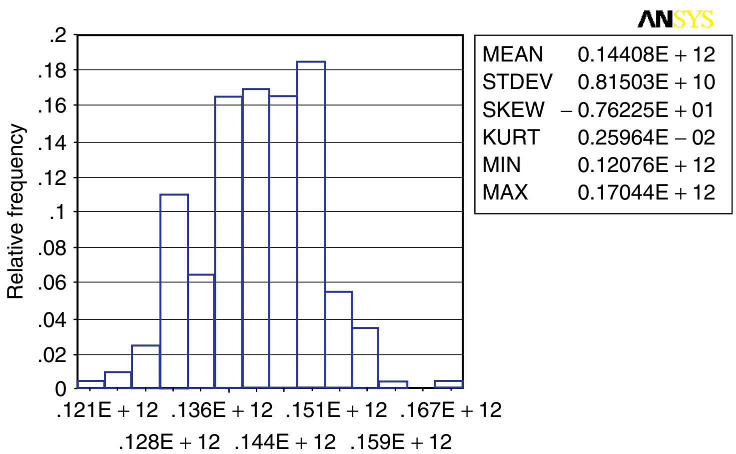

Figure 11. Variation in Young's modulus (TPa) transverse to fiber direction due to simultaneous variation of fiber location and material properties.

4.3. Shear modulus

The shear modulus is little affected due to positioning of the fiber as seen in fig. 12(a). The dispersion is limited in the range of 47.7 *GPa* to 48.2 *GPa* which is about 2% of the mean value. The scatter is prominent in the second case where randomness in material property is only considered. As seen in

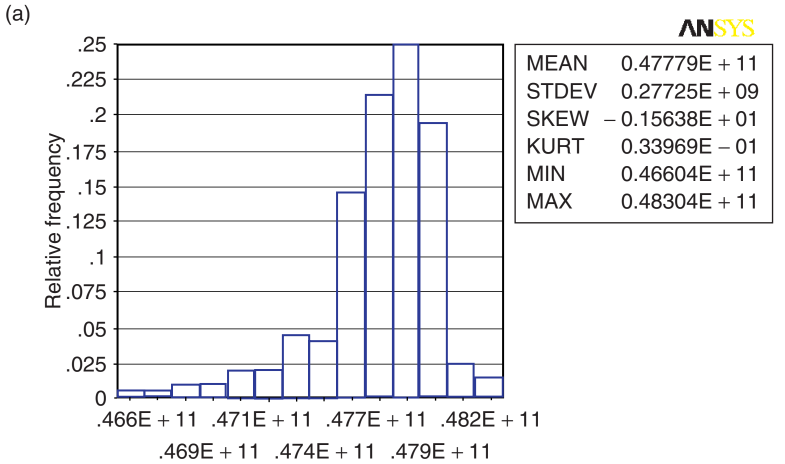

Figure 12. Variation in shear modulus (TPa) due to variation in (a) only fiber location (b) only material properties.

Fig 12(b) the range is between 46 *GPa* and 50 *GPa* with nearly flat probability distribution in the range. When randomness in both the fiber arrangement and material properties are simultaneously considered, this trend remains but the band widens to 45 *GPa* and 51 *GPa* as seen in Fig. 13.

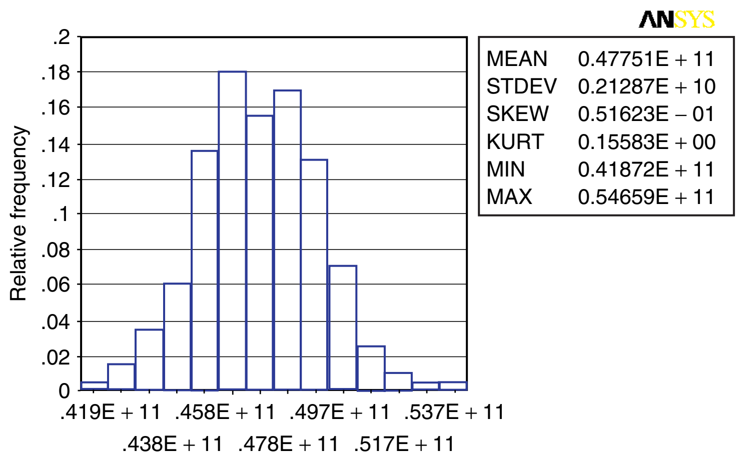

Figure 13. Variation in shear modulus(TPa) due to simultaneous variation in fiber location and material properties.

5. CONCLUSION

The critical observations of the above study are

- There is a wide scatter in the effective material property of the composite. Fiber arrangement influences to a great extend on the effective material property.
- The distribution is almost flat i.e. the true value of material constant has very high value of uncertainty in this range.
  - E_1 shows a scatter of 7.5% of the mean value
  - E_2 shows a scatter of 10% of the mean value
- Shear modulus is least affected as it is dominated by material property of matrix.

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