A Numerical Approach for Estimating the Entropy Generation in Flat Heat Pipes

Mahesh Kumar. P*, Muraleedharan. C#
Department of Mechanical Engineering
National Institute of Technology Calicut
NIT Campus (P.O), Kerala, India. PIN- 673601

ABSTRACT

Heat Pipe is a thermodynamic device which transports heat from one location to another with a very small temperature drop. Entropy generation can be considered as a significant parameter on heat pipe performance. Major reasons for entropy generation in a heat pipe system are temperature difference between cold and hot reservoirs, frictional losses in the working fluid flows and vapor temperature/pressure drop along heat pipe. The objective of the present work is to estimate the entropy generation in a flat heat pipe. A computational model is developed for the analysis of the transient operation of a flat heat pipe. The analysis involves the solution of two dimensional continuity, momentum and energy equations in the vapor core with the transport equations for a porous medium in the wick. The entropy generation depends on both temperature and velocity variations of vapor and liquid. Alternating Direction Implicit (ADI) scheme is used to convert the partial differential equations into finite difference equations. A code is developed in C language to solve the system of linear and non linear equations. Variation in temperature, pressure and velocity fields are obtained as a function of time by solving the system of equations using the developed code. The temperature and velocity distributions in the heat pipe are employed for estimating the entropy generation rate in the system.

*nandumahesh03@yahoo.co.in # murali@nitc.ac.in

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NOMENCLATURE

C	Specific Heat Capacity at Constant Pressure, J/kgK
C_E	Ergun's Constant
D_a	Darcy's Number
d	Wick Thickness, m
h_{fg}	Latent Heat of Vaporization, J/kgK
h	Vapor Core Thickness, m
k	Thermal Conductivity, W/mK
K	Wick Permeability, m^2
L	Length of Heat Pipe, m
p	Pressure, Pa
S	Entropy, J/kgK
$S^{111}{}_{gen}$	Volumetric Entropy Generation, W/m^3K
t	Time, s
T	Temperature, K
u	Axial Velocity, m/s
v	Transverse Velocity, m/s

x Axial Distance, m
y Transverse Distance, m

Greek Symbols

β Parameter in Rayleigh's equation, $\left(1+{}^{k_s}\!/\!_{k_f}\right)\Big/\left(1-{}^{k_s}\!/\!_{k_f}\right)$

μ Dynamic Viscosity, Ns/m^2
ρ Density, kg/m^3
ε Porosity
φ Viscous Dissipation
θ Non dimensional Temperature
τ Non dimensional Time

Subscripts
a Adiabatic section
amb Ambient
c Condenser section
e Evaporator section
f Liquid
s Solid
sat Saturated

1. INTRODUCTION

A heat pipe is a simple device of very high thermal conductance. It can transmit heat at high rate over considerable distance with extremely small temperature drop [12]. The design of heat pipe is simple and it is easy to manufacture and maintain. Heat pipes have found various applications including energy conversion systems, cooling of nuclear reactors and electronic components, etc.

A conventional heat pipe has three sections: (1) the evaporator where heat is added to the system, (2) the condenser where heat is rejected from the system and (3) the adiabatic section which connects the evaporator and condenser, serving as a flow channel. The working fluid inside the heat pipe undergoes a thermodynamic cycle which generates entropy. The entropy generation in a heat pipe is due to frictional losses in the flow of working fluid and heat transfer across a finite temperature difference. The entropy generation rate can be used to quantify the irreversibility of the system which is directly related to the lost work during any process.

A large number of theoretical and experimental studies on heat pipes have been reported over the past few decades. Cotter [1] analyzed the laminar, steady, incompressible one-dimensional vapor flow in a cylindrical heat pipe. Bankston and Smith [2] presented the solutions for the axisymmetric Navier-Stokes equations for steady laminar vapor flow in circular heat pipes with various evaporator and condenser lengths. Numerical calculations of the vapor flow in a flat heat pipe were presented by Van Ooijen and Hoogendoon [3]. They also conducted experimental studies to obtain pressure profiles along the vapor channel of a flat heat pipe [4]. Numerical analysis of the vapor flow in a double walled concentric heat pipe was presented by Faghri [5]. Chen and Faghri [6] studied the overall performance of the heat pipe with single or multiple sources of heat. A transient two dimensional analysis of the vapor core and wick regions of a flat heat pipe was performed by Unnikrishnan and Sobhan [7].

Vasilev and Konev [8] presented a thermodynamic analysis based on the assumption of constant vapor pressure along the heat pipe. Rajesh and Ravindran [9] developed an optimum design of heat pipe using nonlinear programming technique. Khalkali et al. [10] presented the entropy generation in a heat pipe system. They developed a thermodynamic model of conventional heat pipe based on the second law of thermodynamics. A detailed parametric analysis was presented in which the effects of various heat pipe parameters on entropy generation were examined. A computational model for the analysis of the transient operation of a flat heat pipe was presented by Shobhan [11].

This paper aims for finding the entropy generation developed by thermodynamic irreversibility for a two dimensional flat heat pipe. It is seen that the entropy generation can be quantified in terms of velocity and temperature distributions of both liquid and vapor flows.

The major three factors causing entropy generation in a heat pipe are: (1) temperature difference between hot and cold reservoirs (attached to the evaporator and condenser outer surfaces), (2) temperature drop in the vapor flow and (3) frictional losses associated with the vapor and liquid flows.

A computational model for analyzing the transient and steady state performance of a copper-water flat heat pipe system is presented here. The wick and vapor region are analyzed by solving the appropriate governing equations to obtain temperature, velocity and pressure distributions in the heat pipe. ADI method is used for the discretization of governing equations into finite difference equations. SIMPLE algorithm is used for solving the equations. Using the obtained velocity and temperature distributions, entropy generation rate due to vapor and liquid flows are estimated.

2. ANALYSIS OF HEAT PIPE

The heat pipe consists of an evacuated chamber, the interior of which is lined by a wick saturated with a working fluid. The heat is essentially transferred as latent heat by evaporating the liquid working substance in a heating zone called evaporator and condensing the vapor in a cooling region called condenser. The circulation is completed by the return flow of the condensate to the evaporator through the wick under the driving action of capillary forces. This process will continue as long as the flow passage for the working fluid is not blocked and a sufficient capillary pressure is maintained. Due to the heat transfer and fluid flow between the reservoirs, entropy is generated. The entropy generation is formulated as follows.

2.1 Entropy generation

The volumetric rate of entropy generation in a convective heat transfer problem is given as [13],

$$S_{gen}^{III} = \left(\frac{K}{T^2}\right)(\nabla T)^2 + \left(\frac{\mu}{T}\right)\phi$$

where the first term represents the entropy generation due to heat transfer and second term represents entropy generation due to fluid flow friction.

For a heat pipe the parameters in the above equation are:

k = thermal conductivity of the working fluid (vapor or liquid form)

T = operating temperature, μ = absolute viscosity of the working fluid,

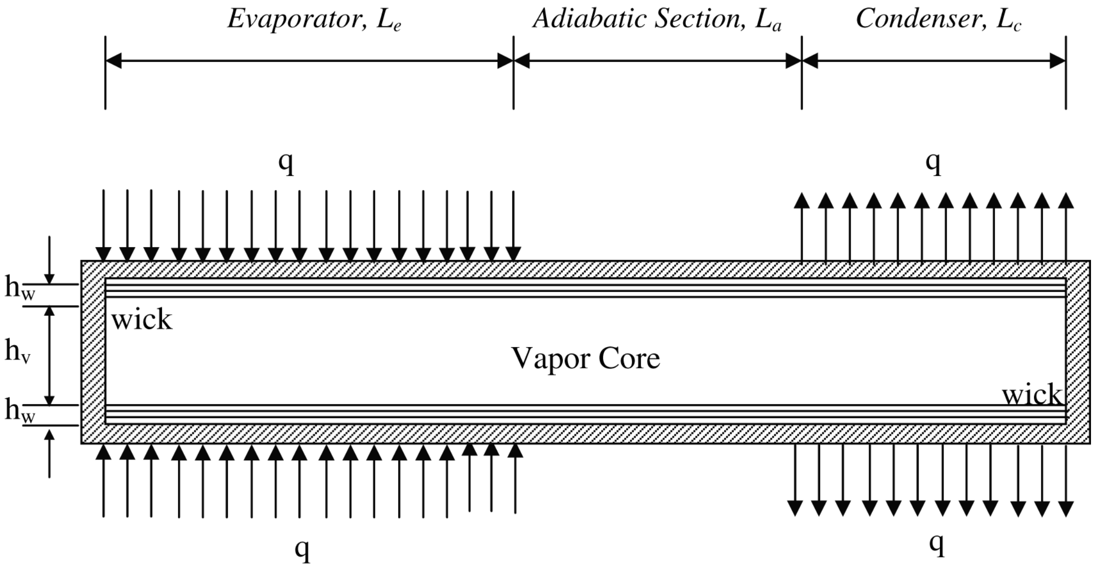
∇T = temperature gradient, ϕ = viscous dissipation factor.

For a two dimensional flow, the entropy generation equation becomes:

$$S_{gen}^{III} = \left(\frac{K}{T^2}\right)\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \left(\frac{\mu}{T}\right)\left\{2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right) + \left(\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right)\right\} \quad (1)$$

So for obtaining the entropy generation rate the velocity and temperature distributions of both vapor and liquid flows in a heat pipe are required.

3. THE PHYSICAL MODEL



Schematic Representation of a Heat Pipe System

The present analysis involves a flat heat pipe of 10 cm length (evaporator length = 4 cm, condenser length = 3 cm and adiabatic length = 3 cm). Water is chosen as the working fluid. The wick and wall are made of copper. A wick porosity of 0.65 is used in the present analysis with the voids being saturated with water. The adiabatic section is externally insulated such that there is no heat exchange between this section and the exterior.

The assumptions taken are:

1. Body forces are negligible.
2. Two dimensional unsteady, laminar flow of vapor and liquid
3. Thermo physical properties of the working fluid remain constant
4. Vapor is assumed to be saturated ideal gas.

4. MATHEMATICAL FORMULATION

4.1 Governing equations for liquid and vapor flow

1. Continuity Equation

For both vapor and liquid flows:

$$\left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial v}{\partial y}\right)=0$$

2. Momentum Equations

For vapor core, x-momentum equation is

$$\rho\left\{\left(\frac{\partial u}{\partial t}\right)+u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)\right\}=-\left(\frac{\partial p}{\partial x}\right)+\mu\left\{\left(\frac{\partial^2 u}{\partial x^2}\right)+\left(\frac{\partial^2 u}{\partial y^2}\right)\right\}$$

y-momentum equation is

$$\rho\left\{\left(\frac{\partial v}{\partial t}\right)+u\left(\frac{\partial v}{\partial x}\right)+v\left(\frac{\partial v}{\partial y}\right)\right\}=-\left(\frac{\partial p}{\partial y}\right)+\mu\left\{\left(\frac{\partial^2 v}{\partial x^2}\right)+\left(\frac{\partial^2 v}{\partial y^2}\right)\right\}$$

For liquid wick, x-momentum equation is

$$\frac{\rho}{\varepsilon}\left\{\left(\frac{\partial \mathrm{u}}{\partial \tau}\right)+u\left(\frac{\partial u}{\partial x}\right)+v\left(\frac{\partial u}{\partial y}\right)\right\}=-\left(\frac{\partial \mathrm{p}}{\partial \mathrm{x}}\right)+\left(\frac{\mu}{\varepsilon}\right)\left\{\left(\frac{\partial^2 u}{\partial x^2}\right)+\left(\frac{\partial^2 u}{\partial y^2}\right)\right\}-\frac{\mu}{\mathrm{K}}\mathrm{u}-\frac{\mathrm{C_E}}{\mathrm{K}^{0.5}}\rho \mathrm{u}\ |\mathrm{u}|$$

y-momentum equation is

$$\frac{\rho}{\varepsilon}\left\{\left(\frac{\partial \mathrm{v}}{\partial \tau}\right)+u\left(\frac{\partial v}{\partial x}\right)+v\left(\frac{\partial v}{\partial y}\right)\right\}=-\left(\frac{\partial \mathrm{p}}{\partial \mathrm{y}}\right)+\left(\frac{\mu}{\varepsilon}\right)\left\{\left(\frac{\partial^2 v}{\partial x^2}\right)+\left(\frac{\partial^2 v}{\partial y^2}\right)\right\}-\frac{\mu}{\mathrm{K}}\mathrm{v}-\frac{\mathrm{C_E}}{\mathrm{K}^{0.5}}\rho \mathrm{v}\ |\mathrm{v}|$$

3. Energy Equation

For vapor core:

$$(\rho \mathrm{C})_{\mathrm{v}}\left\{\left(\frac{\partial T}{\partial t}\right)+u\left(\frac{\partial T}{\partial x}\right)+v\left(\frac{\partial T}{\partial y}\right)\right\}=\left(\frac{\partial p}{\partial t}\right)+u\left(\frac{\partial p}{\partial x}\right)+v\left(\frac{\partial p}{\partial y}\right)+K_v\left\{\left(\frac{\partial^2 T}{\partial x^2}\right)+\left(\frac{\partial^2 T}{\partial y^2}\right)\right\}+\phi$$

where

$$\phi=2\mu\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right\}+\mu\left\{\left(\frac{\partial \mathrm{u}}{\partial \mathrm{y}}+\frac{\partial v}{\partial x}\right)^2\right\}$$

For liquid wick:

$$(\rho C)_m\left(\frac{\partial T}{\partial t}\right)+(\rho C)_f\left\{u\left(\frac{\partial T}{\partial x}\right)+v\left(\frac{\partial T}{\partial y}\right)\right\}=K_m\left\{\left(\frac{\partial^2 T}{\partial x^2}\right)+\left(\frac{\partial^2 T}{\partial y^2}\right)\right\}$$

where

$$(\rho C)_m=(1-\varepsilon)(\rho C)_s+\varepsilon(\rho C)_f \quad \text{and} \quad k_m=\left(\frac{\beta-(1-\varepsilon)}{\beta+(1-\varepsilon)}\right)k_f$$

Boundary Conditions

For vapor core:
At x = 0, y = 0 to h, u = 0, v = 0,
At x = L, y = 0 to h, u = 0, v = 0
At y = 0, x = 0 to L, u = 0, v = 0
At y = h, x = 0 to L, u = 0, v = 0
At x = 0, y = 0 to h, $T = T_h$
At x = L, y = 0 to h, $T = T_{amb}$

For liquid wick:
At y = 0, x = 0 to L, u = 0, v = 0
At y = d, x = 0 to L, u = 0, v = 0

Initial Condition

At time t = 0, $T = T_{amb}$, and in the liquid and vapour $p = p_{sat}$

4.2 Non dimensionalisation

$$X=\frac{x}{L},\ Y=\frac{y}{h},\ U=\frac{u}{U_0},\ V=\frac{v}{V_0},\ \tau=\frac{U_0 t}{h},\ P=\left(\frac{p-p_\infty}{\rho U_0^2}\right),\ \theta=\left(\frac{T-T_\infty}{T_h-T_\infty}\right),\ \mathrm{Re}=\left(\frac{\rho U_0 L}{\mu}\right),\ \mathrm{Da}=\left(\frac{K}{h^2}\right)$$

Then the governing equations becomes

1. Continuity Equation

For both vapor and liquid flow

$$\left(\frac{\partial U}{\partial X}\right)+\left(\frac{\partial V}{\partial Y}\right)=0$$

2. Momentum Equations

For vapor core, x-momentum equation is

$$\left\{\left(\frac{\partial U}{\partial \tau}\right)+U\left(\frac{\partial U}{\partial X}\right)+V\left(\frac{\partial U}{\partial Y}\right)\right\}=-\left(\frac{\partial P}{\partial X}\right)+\left(\frac{1}{\mathrm{Re}}\right)\left\{\left(\frac{\partial^2 U}{\partial X^2}\right)+\left(\frac{\partial^2 U}{\partial Y^2}\right)\right\}$$

y-momentum equation is

$$\left\{\left(\frac{\partial V}{\partial \tau}\right)+U\left(\frac{\partial V}{\partial X}\right)+V\left(\frac{\partial V}{\partial Y}\right)\right\}=-\left(\frac{\partial P}{\partial Y}\right)+\left(\frac{1}{\mathrm{Re}}\right)\left\{\left(\frac{\partial^2 V}{\partial X^2}\right)+\left(\frac{\partial^2 V}{\partial Y^2}\right)\right\}$$

For liquid wick, x-momentum equation is

$$\left(\frac{1}{\varepsilon}\right)\left\{\left(\frac{\partial U}{\partial \tau}\right)+U\left(\frac{\partial U}{\partial X}\right)+V\left(\frac{\partial U}{\partial Y}\right)\right\}=-\left(\frac{\partial P}{\partial X}\right)+\left(\frac{1}{\varepsilon\,\mathrm{Re}}\right)\left\{\left(\frac{\partial^2 U}{\partial X^2}\right)+\left(\frac{\partial^2 U}{\partial Y^2}\right)\right\}-\left(\frac{U}{\mathrm{ReDa}}\right)-\left(\frac{C_E}{\mathrm{Da}^{0.5}}\right)U|U|$$

y-momentum equation is

$$\left(\frac{1}{\varepsilon}\right)\left\{\left(\frac{\partial V}{\partial \tau}\right)+U\left(\frac{\partial V}{\partial X}\right)+V\left(\frac{\partial V}{\partial Y}\right)\right\}=-\left(\frac{\partial P}{\partial Y}\right)+\left(\frac{1}{\varepsilon\,\mathrm{Re}}\right)\left\{\left(\frac{\partial^2 V}{\partial X^2}\right)+\left(\frac{\partial^2 V}{\partial Y^2}\right)\right\}-\left(\frac{V}{\mathrm{ReDa}}\right)-\left(\frac{C_E}{\mathrm{Da}^{0.5}}\right)V|V|$$

3. Energy Equation

For vapor core:

$$\left\{\left(\frac{\partial \theta}{\partial \tau}\right)+U\left(\frac{\partial \theta}{\partial X}\right)+V\left(\frac{\partial \theta}{\partial Y}\right)\right\}=\mathrm{Ec}\left\{\left(\frac{\partial P}{\partial \tau}\right)+U\left(\frac{\partial P}{\partial X}\right)+V\left(\frac{\partial P}{\partial Y}\right)\right\}$$
$$+\frac{1}{\mathrm{RePr}}\left\{\left(\frac{\partial^2 \theta}{\partial X^2}\right)+\left(\frac{\partial^2 \theta}{\partial Y^2}\right)\right\}+2\frac{Ec}{\mathrm{Re}}\left\{\left(\frac{\partial U}{\partial X}\right)^2+\left(\frac{\partial V}{\partial Y}\right)^2\right\}+\frac{Ec}{\mathrm{Re}}\left\{\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2\right\}$$

For liquid wick

$$\left\{\left(\frac{\partial \theta}{\partial \tau}\right)+U\left(\frac{\partial \theta}{\partial X}\right)+V\left(\frac{\partial \theta}{\partial Y}\right)\right\}=\frac{1}{\mathrm{RePr}}\left\{\left(\frac{\partial^2 \theta}{\partial X^2}\right)+\left(\frac{\partial^2 \theta}{\partial Y^2}\right)\right\}$$

5. SOLUTION PROCEDURE

The governing equations which are in the partial differential form are converted into corresponding finite difference equations using ADI scheme. The obtained system of linear and non linear equations is solved using SIMPLE algorithm. The equations are solved with appropriate boundary conditions using the code developed in C $^{++}$ language.

6. RESULTS AND DISCUSSION

Transient and steady state results from the computations are discussed in this section. The important results of analysis are the distributions of the velocity components, pressure, temperature and entropy generation rate in the heat pipe. Figure 1 represents the axial distribution of longitudinal velocity component along the center-line of vapor core at different time instants. The steep increase in velocity along the evaporator section observed is due to the mass addition into the vapor core. At the adiabatic and condenser sections the velocity is found to be decreasing. The rate of decrease is less at the adiabatic section. The steep decrease in velocity at the condenser section is due to high rate of mass transfer from vapor core into the wick. Figure 2 shows the distribution of the longitudinal liquid velocity component along the centre line of the wick. Actually, the axial component of velocity shown is in the negative x-direction. The nature of the liquid velocity distribution is similar to the vapor velocity distribution at each time instant. The liquid velocity increases steeply at the section where mass addition takes place (here condenser section) and decreases at very slow rate at the adiabatic section and decreases at a fast rate where mass rejection takes place (evaporator section). As expected, the magnitude of vapor velocity is very much higher than that of liquid due to vapor-liquid density difference.

Figure 3 represents the longitudinal distribution of vapor temperature from transient state to steady state along the centre line of vapor core. The temperature is found as decreasing along the longitudinal direction. The total temperature difference at steady state between two ends of the heat pipe is found to be 5^{o}C. This temperature drop can be considered as a heat pipe performance

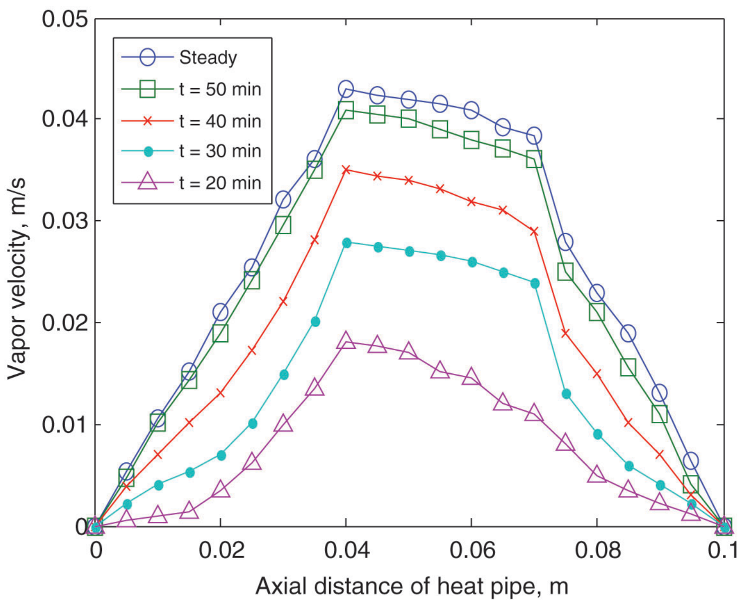


Figure 1.

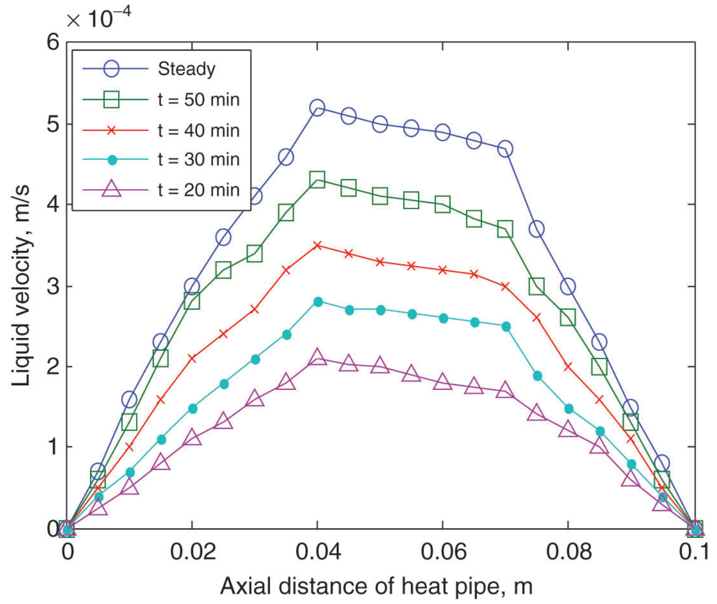


Figure 2.

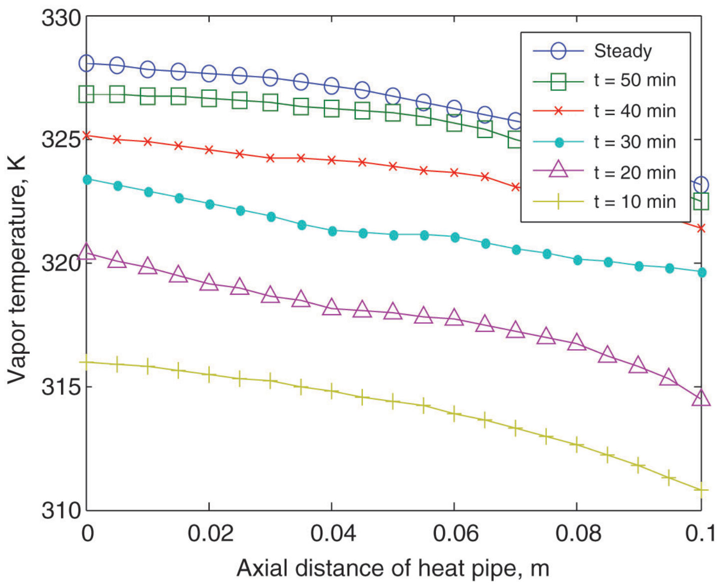


Figure 3.

parameter. Figure 4 gives the axial distributions of vapor pressure at steady state along the vapor core. The pressure distribution matches with the temperature distribution since the ideal gas equation is used to compute the pressure of vapor. The variation of entropy generation rate due to vapor flow along the axial direction of heat pipe is depicted in figure 5. It is observed that the entropy generation due to vapor flow friction is negligible comparing with the entropy generation

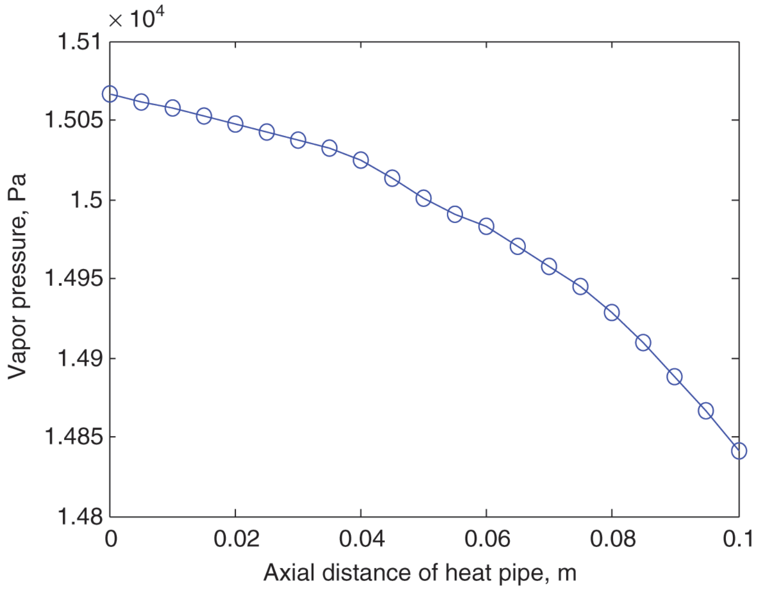


Figure 4.

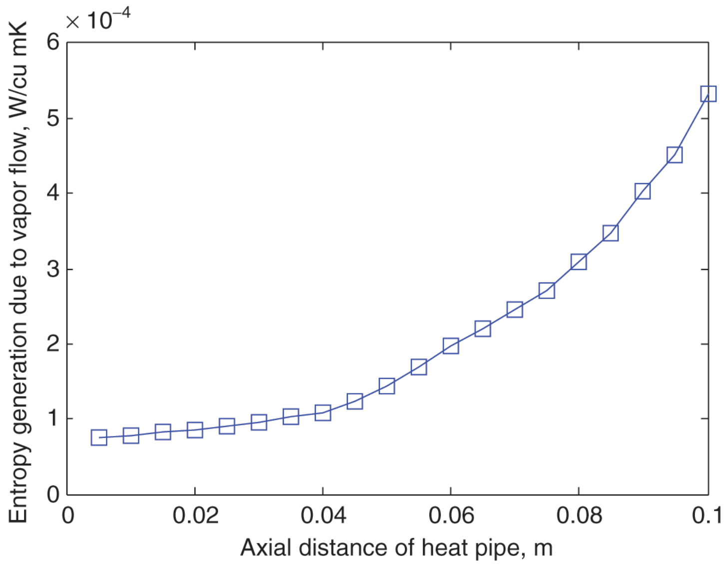


Figure 5.

due to heat transfer. This is due to very low value of vapor viscosity. The entropy generation due to the heat transfer depends on the temperature gradient. It is found that the temperature gradient increases from evaporator to condenser section. Due to this increase in temperature drop, the entropy generation rate also increases along the axial direction. Figure 6 describes the variation of entropy generation due to liquid flow through wick along the axial direction. It can be observed that

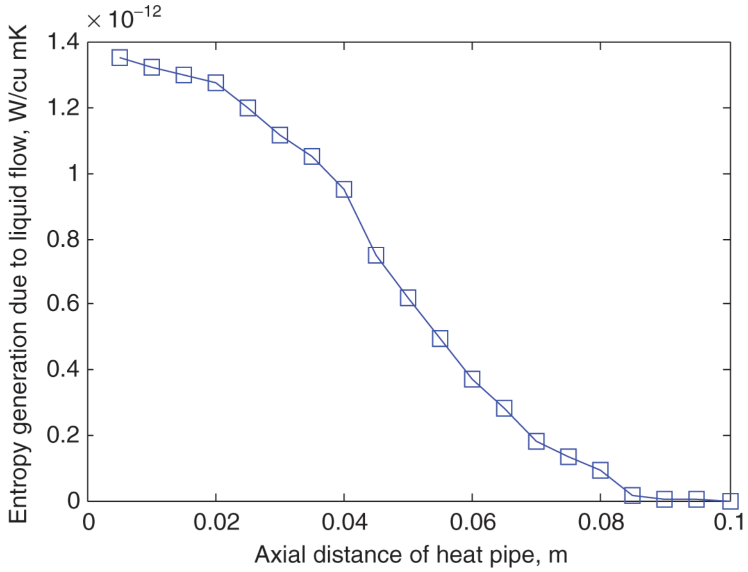


Figure 6.

in the case of entropy generation due to liquid flow, the effect of heat transfer is negligible since the temperature drop for the liquid flow is very low. So the entropy generation is only due to the flow effect, ie. because of the velocity gradient. It is observed that the velocity gradient decreases with axial direction of the heat pipe; similar nature can be observed for the entropy generation due to liquid flow. It can also be observed that the magnitude of entropy generation due to liquid flow is negligible in comparison with the entropy generation due to vapor flow. Figure 7 represents

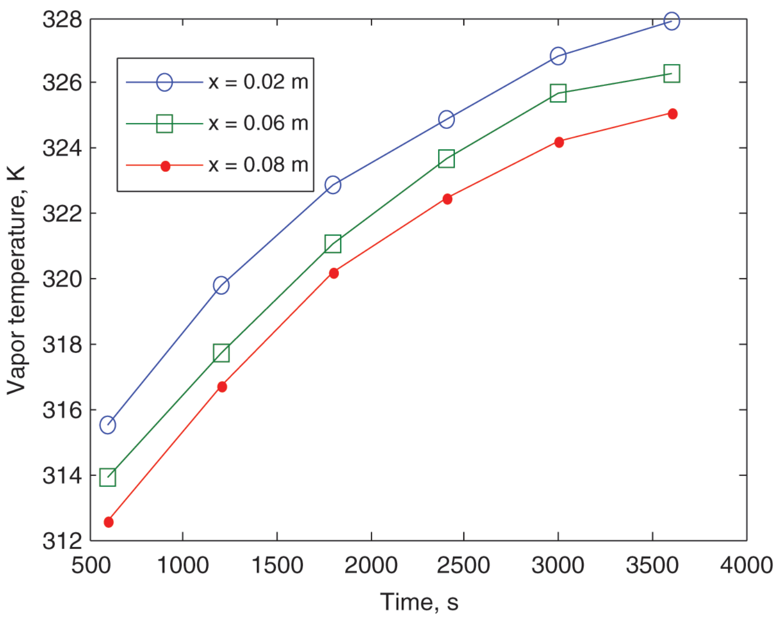


Figure 7.

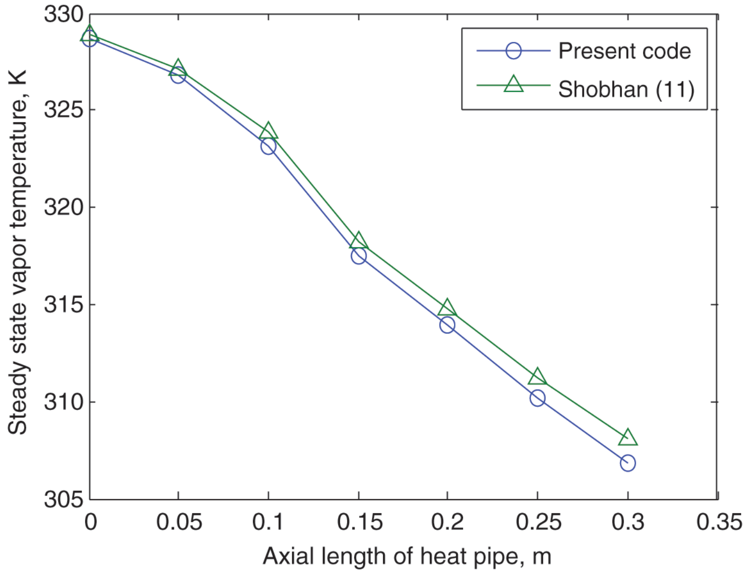


Figure 8.

variation of vapor temperature with time, until steady state is achieved at three different locations of the heat pipe, ie. at evaporator, adiabatic and condenser sections. Figures 8 and 9 shows the comparison of the steady state vapor temperature and pressure obtained by solving a physical system in the literature [11] using the developed code and results presented in the literature. It is found that the results obtained are closely agreeing with the results presented in the literature.

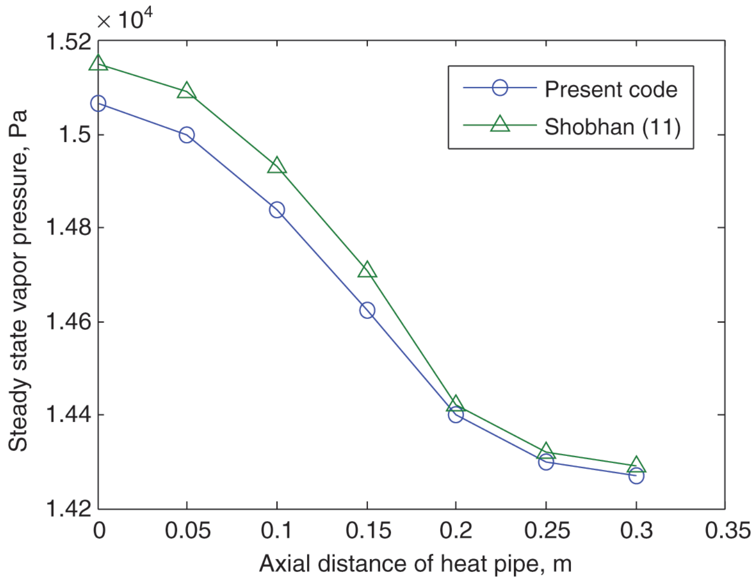


Figure 9.

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