

Using porous layers to decrease quantity of radiation defects, generated during ion implantation

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Abstract

It has been recently shown that manufacturing of an implanted-junction rectifier in a semiconductor heterostructure for optimal relationship between energy of implanted ions, materials and thicknesses of layers of the heterostructure after annealing of radiation defects gives us possibility to increase sharpness of p - n - junction and at the same time to increase homogeneity of dopant distribution in doped area [1,2]. In this paper we consider a possibility to decrease quantity of radiation defects, which were generated during ion implantation, using porous epitaxial layers of the heterostructure.

Keywords: Implanted-junction rectifier, Decreasing of quantity of radiation defects, Heterostructure with porous layers.

INTRODUCTION

In the present time intensive refinement elements of integrated circuits (IS) is in progress. One of the most refined elements is p - n -junctions and their systems (bipolar transistors and thyristors) [3-5]. An interest increasing sharpness of p - n -junctions and at the same time increasing homogeneity of dopant distribution in enriched by the doped area is attracted an interest. To increase the sharpness of p - n -junction laser [6] or microwave [7] types of annealing of dopant (for diffusion- junction rectifiers) or radiation defects (for implanted-junction rectifiers) and inhomogenous distribution of defects in doped sample or heterostructure (H) could be used. In the Refs. [1,2] we consider an alternative approach to increase sharpness of implanted-junction rectifiers and at the same time to increase homogeneity of dopant distribution in doped area. Let us consider a H, which consist of an epitaxial layer (EL) and a substrate (S) within the approach. The H is presented in the Fig. 1. Type of conductivity in S (p or n) is known. We consider implantation of ions of dopant to produce the second type of conductivity in EL (n or p). During annealing of radiation defects, spreading of dopant distribution could be obtained. If energy of ions, thickness of EL and materials of H are chosen optimally, the dopant could achieved interface between layers of H due to the spreading. Due to the achievement one could obtain increasing of sharpness of p - n -junction and at the same time one could obtain increasing of homogeneity of dopant distribution in doped area [1,2]. In this paper we consider porous EL. Porosity of EL gives us possibility to increase sharpness of p - n - junction and homogeneity of dopant distribution in doped area [8]. Main aim of the present paper is analysis of influence of porosity of EL on quantity of radiation defects in H. Accompanying aim of the present paper is development of mathematical approach for modeling of modification of porosity during annealing of radiation defects, because the mathematical approach that has been used in [8], gives us possibility to describe only final stage of modification of porosity.

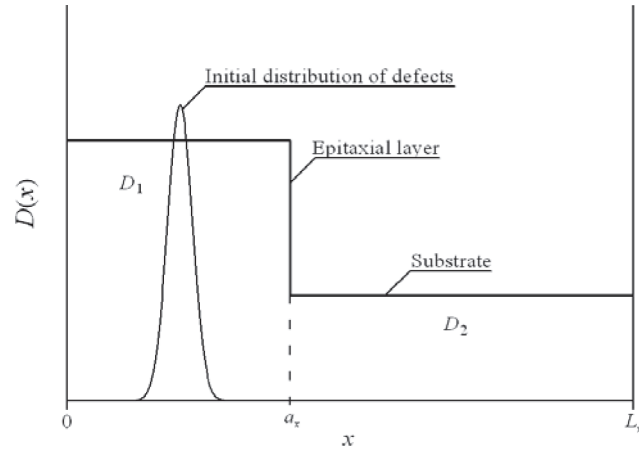


Fig. 1. Heterostructure with the epitaxial layer ($x \in [0, a_x]$ and diffusion coefficient D_1) and the substrate ($x \in [a_x, L_x]$ and diffusion coefficient D_2). The figure also illustrates initial (before starting of annealing) distribution of radiation defects.

METHOD OF SOLUTION

In this Section we determine spatiotemporal distributions of dopant and radiation defects concentrations. Analysis of the distributions gives us possibility to analyze influence of porosity of EL on quantity of radiation defects in H and at the same time to increase sharpness of p - n -junction and homogeneity of dopant distribution in doped area. Distribution of dopant has been determined by solving the second Fick's law [3-5,8,9]

$$\begin{aligned} \frac{\partial C(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_C \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_C \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D_C \frac{\partial C(x, y, z, t)}{\partial z} \right] \\ & + \frac{\partial}{\partial x} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial z} \right] \end{aligned} \quad (1)$$

with boundary and initial conditions

$$\begin{aligned} \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad C(x, y, z, 0) = f_C(x, y, z). \end{aligned} \quad (2)$$

Here $C(x, y, z, t)$ is the spatiotemporal distribution of dopant concentration; k is the Boltzmann constant; T is the annealing temperature; $\mu(x, y, z, t)$ is the chemical potential; $\bar{V}$ is the molar volume; D_C and D_{CS} are the coefficients of volumetric and surface dopant diffusion. Value of dopant diffusion coefficients depends on properties of materials of layers in H, on rate of heating and cooling of H and on spatiotemporal distribution of dopant concentration. It has been shown in [5], that in high-doped materials interaction between dopant atoms and point defects increases. If the point defects have non-

zero charge γe with e an elementary charge, then the interaction leads to concentrational dependence of the diffusion coefficient. Parameter γ depends on properties of materials of H and could be integer usually in the interval $\gamma \in [1,3]$ [5]. The parameter could be larger, than 3, but probability of the case $\gamma > 3$ is substantially smaller, than probability of the case $\gamma \in [1,3]$. Dependence of dopant diffusion coefficient on concentration of dopant could be approximated by the following functions [5]

$$\begin{aligned} D_C &= \tilde{D}_L(x, T) \beta(x, y, z, t) \left[1 + \xi_V \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, T)} \right] \\ D_{CS} &= \tilde{D}_{LS}(x, T) \beta(x, y, z, t) \left[1 + \xi_S \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, T)} \right]. \end{aligned} \quad (3)$$

Here $P(x, T)$ is the limit of solubility of dopant in H; $D_L(x, T) = \tilde{D}_L(x, T) \beta(x, y, z, t)$ and $D_{LS}(x, T) = \tilde{D}_{LS}(x, T) \beta(x, y, z, t)$ are the diffusion coefficients for low-level of doping; $V(x, y, z, t)$ is the spatiotemporal distribution of concentration of vacancies; $\beta(x, y, z, t) = 1 + [\xi_1 V(x, y, z, t)/V^*] + [\xi_2 V^2(x, y, z, t)/(V^*)^2]$. Spatiotemporal distributions of point defects (both vacancies and interstitials) we determine by solving the following system of equations [8,9,11]

$$\begin{aligned} \frac{\partial \rho(x, y, z, t)}{\partial t} &= \text{div} \left\{ D_\rho(x, T) \text{grad} [\rho(x, y, z, t)] \right\} + \text{div} \left\{ \frac{D_{\rho S}(x, T)}{V k T} \text{grad} [\mu(x, y, z, t)] \right\} \\ &\quad - k_{I,V}(x, T) I(x, y, z, t) V(x, y, z, t) - k_{\rho, \rho}(x, T) \rho^2(x, y, z, t) \end{aligned} \quad (4)$$

with boundary and initial conditions

$$\begin{aligned} \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=0} &= 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=x_1} = 0, \quad \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=y_1} = 0, \\ \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=z_1} &= 0, \quad V(x_1 + V_n t, y_1 + V_n t, z_1 + V_n t) = V^* \left(1 + \frac{2 \ell \omega}{k T \sqrt{x_1^2 + y_1^2 + z_1^2}} \right), \quad \rho(x, y, z, 0) = f_\rho(x, y, z). \end{aligned} \quad (5)$$

Here $\rho = I, V$; $I(x, y, z, t)$ is the spatiotemporal distribution of concentration of interstitials; $D_\rho(x, T)$ are the diffusion coefficients of interstitials and vacancies; $k_{I,V}(x, T)$ is the parameter of recombination of point radiation defects; terms $V^2(x, y, z, t)$ and $I^2(x, y, z, t)$ correspond to generation of divacancies and analogous complexes of interstitials (see, for example [8] and appropriate references in this paper); $k_{I,V}(x, T)$ and $k_{\rho, \rho}(x, T)$ are the parameters of recombination of point defects and generation of their complexes, respectively; k is the Boltzmann constant; V^* is the equilibrium distribution of vacancies, $\omega = a^3$, a is the atomic spacing; ℓ is the specific surface energy. To take into account porosity we assume, that porous are approximately cylindrical with average dimensions $r = \sqrt{x_1^2 + y_1^2}$ and z_1 [13]. The average size of the pores has been taken into account in boundary conditions on the pores in Eqs. (5). With time small pores decompose into vacancies. The vacancies are absorbed by large pores [14].

With time the large pores take spherical form during the absorbtion of vacancies from small pores [14]. It was assumed that the pores are distributed initially homogenous with the appropriate concentration of vacancies, described the next relation (i.e., we determined distribution of concentration of vacancies, which was formed due to porosity, by summing overall pores)

$$V(x, y, z, t) = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n V_p(x + i\alpha, y + j\beta, z + k\chi, t).$$

Here $R = \sqrt{x^2 + y^2 + z^2}$; α , β and χ are averaged distances between centers of pores in x , y and z directions, respectively; l , m and n are quantities of pores in the same directions; i , j , k are current numbers of pores.

Spatiotemporal distribution of concentrations of divacancies $\Phi_V(x, y, z, t)$ and analogous complexes of interstitials $\Phi_I(x, y, z, t)$ could be determine by the following system of equations [9,12,15]

$$\begin{aligned} \frac{\partial \Phi_\rho(x, y, z, t)}{\partial t} = & \operatorname{div} \left\{ D_{\phi\rho}(x, T) \cdot \operatorname{grad} [\Phi_\rho(x, y, z, t)] \right\} + \operatorname{div} \left\{ \frac{D_{\phi\rho s}(x, T)}{V k T} \cdot \operatorname{grad} [\mu(x, y, z, t)] \right\} \\ & + k_{\rho,\rho}(x, T) \rho^2(x, y, z, t) - k_I(x, T) \rho(x, y, z, t) \end{aligned} \quad (6)$$

with boundary and initial conditions

$$\begin{aligned} \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad \Phi_I(x, y, z, 0) = f_{\phi I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\phi V}(x, y, z). \end{aligned} \quad (7)$$

Here $D_{\phi I}(x, T)$ and $D_{\phi V}(x, T)$ are the diffusion coefficients of complexes of point defects; $k_I(x, T)$ and $k_V(x, T)$ are parameters of decay of complexes of point defects.

To determine spatiotemporal distributions of point defects in pursuance of Refs. [16-18] we transform the approximations of diffusion coefficients of the defects in the following form: $D_\rho(x, T) = D_{0\rho} [1 + \varepsilon_{\rho\rho} g_\rho(x, T)]$. In the same form we transform approximations of parameters of recombination of point defects and generation of their complexes: $k_{I,V}(x, T) = k_{0I,V} [1 + \varepsilon_{I,V} g_{I,V}(x, T)]$, $k_{\rho,\rho}(x, T) = k_{0\rho,\rho} [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(x, T)]$. Let us introduce the following dimensionless variables: $\mathcal{G} = \frac{t \sqrt{D_{0I} D_{0V}}}{L_x^2 + L_y^2 + L_z^2}$, $\mathcal{X} = \frac{x}{L_x}$,

$$\eta = \frac{y}{L_y}, \quad \phi = \frac{z}{L_z}, \quad \omega = \left(L_x^2 + L_y^2 + L_z^2 \right) \frac{I^* V^* k_{0I,V}}{\sqrt{D_{0I} D_{0V}}}, \quad b_x = 1 + \frac{L_y^2}{L_x^2} + \frac{L_z^2}{L_x^2}, \quad b_y = 1 + \frac{L_x^2}{L_y^2} + \frac{L_z^2}{L_y^2}, \quad b_z = 1 + \frac{L_x^2}{L_z^2} + \frac{L_y^2}{L_z^2},$$

$\Omega_\rho = (\rho^*)^2 k_{0\rho,\rho} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}$, $\mathcal{I}(x, y, z, t) = \frac{I(x, y, z, t)}{I^*}$, $\mathcal{V}(x, y, z, t) = \frac{V(x, y, z, t)}{V^*}$. The change of variables gives us possibility to transform the Eq. (4) and conditions Eqs.(5) to the form

$$\begin{aligned}
& \left\{ \frac{\partial \tilde{T}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = b_x \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi} \left\{ [1 + \varepsilon_I g_I(\chi, T)] \frac{\partial \tilde{T}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right\} + b_y \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \eta} \left\{ [1 + \varepsilon_I g_I(\chi, T)] \times \right. \right. \\
& \times \frac{\partial \tilde{T}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \left. \right\} + \frac{b_x}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \chi} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_z \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \phi} \left\{ [1 + \varepsilon_I g_I(\chi, T)] \times \right. \\
& \times \frac{\partial \tilde{T}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \left. \right\} + \frac{b_y}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \eta} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \frac{\partial}{\partial \phi} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \times \\
& \times \frac{b_z}{\sqrt{D_{0I} D_{0V}}} - \Omega_I [1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] \tilde{T}^2(\chi, \eta, \phi, \vartheta) - \omega \tilde{T}(\chi, \eta, \phi, \vartheta) \tilde{V}(\chi, \eta, \phi, \vartheta) [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \\
& \left. \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} = b_x \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi} \left\{ [1 + \varepsilon_V g_V(\chi, T)] \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right\} + b_y \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \eta} \left\{ \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \times \right. \right. \\
& \times [1 + \varepsilon_V g_V(\chi, T)] \left. \right\} + b_z \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \phi} \left\{ [1 + \varepsilon_V g_V(\chi, T)] \frac{\partial \tilde{V}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right\} + \frac{\partial}{\partial \chi} \left[\frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \chi} \times \right. \\
& \times \frac{D_{VS}(\chi, T)}{V k T} \left. \right] \frac{b_x}{\sqrt{D_{0I} D_{0V}}} + \frac{b_y}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \eta} \left[\frac{D_{VS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \frac{b_z}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \phi} \left[\frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \phi} \times \right. \\
& \times \frac{D_{VS}(\chi, T)}{V k T} \left. \right] \frac{1}{\sqrt{D_{0V}}} - \Omega_V [1 + \varepsilon_{V,V} g_{V,V}(\chi, T)] \tilde{V}^2(\chi, \eta, \phi, \vartheta) - \omega \tilde{T}(\chi, \eta, \phi, \vartheta) [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \times \\
& \times \tilde{V}(\chi, \eta, \phi, \vartheta), \quad (8)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right|_{\chi=0} = \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right|_{\chi=1} = 0, \quad \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right|_{\eta=0} = \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right|_{\eta=1} = 0, \\
& \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right|_{\phi=0} = \left. \frac{\partial \tilde{\rho}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right|_{\phi=1} = 0, \quad \tilde{T}(\chi, \eta, \phi, \vartheta) = \frac{f_I(\chi, \eta, \phi, \vartheta)}{I^*}, \\
& \tilde{V}(\chi, \eta, \phi, \vartheta) = \frac{f_V(\chi, \eta, \phi, \vartheta)}{V^*}. \quad (9)
\end{aligned}$$

Let us determine solutions of Eqs.(8) with conditions Eqs.(9), in pursuance of Refs. [15-17], as the following power series

$$\tilde{\rho}(\chi, \eta, \phi, \vartheta) = \sum_{i=0}^{\infty} \varepsilon_{\rho}^i \sum_{j=0}^{\infty} \omega^j \sum_{k=0}^{\infty} \Omega_{\rho}^k \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta). \quad (10)$$

Substitution of the series Eq.(10) into Eqs.(8) and conditions Eqs.(9) gives us possibility to obtain equations for zeroth-order approximations of point defects concentrations $\tilde{T}_{000}(\chi, \eta, \phi, \vartheta)$ and $\tilde{V}_{000}(\chi, \eta, \phi, \vartheta)$ and corrections to the functions $\tilde{T}_{ijk}(\chi, \eta, \phi, \vartheta)$ and $\tilde{V}_{ijk}(\chi, \eta, \phi, \vartheta)$, $i \geq 1, j \geq 1, k \geq 1$.

The equations and conditions are presented in the Appendix. Equations of the system (11) could be solved by standard approaches of the mathematical physics (see, for example, Refs. [19]). The solutions are presented in the Appendix.

Further we determine spatiotemporal distributions of complexes of radiation defects. To determine the distributions we transform the diffusion coefficients in the following form: $D_{\Phi_I}(x, y, z, T) = D_{0\Phi_I} [1 + \varepsilon_{\Phi_I} g_{\Phi_I}(x, y, z, T)]$ and $D_{\Phi_V}(x, y, z, T) = D_{0\Phi_V} [1 + \varepsilon_{\Phi_V} g_{\Phi_V}(x, y, z, T)]$. In this situation the Eqs.(6) takes the form

$$\begin{aligned} \frac{\partial \Phi_{\rho}(x, y, z, t)}{\partial t} = & \left(\frac{\partial}{\partial x} \left\{ [1 + \varepsilon_{\Phi_{\rho}} g_{\Phi_{\rho}}(x, T)] \frac{\partial \Phi_{\rho}(x, y, z, t)}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ [1 + \varepsilon_{\Phi_{\rho}} g_{\Phi_{\rho}}(x, T)] \frac{\partial \Phi_{\rho}(x, y, z, t)}{\partial y} \right\} + \right. \\ & \left. + \frac{\partial}{\partial z} \left\{ [1 + \varepsilon_{\Phi_{\rho}} g_{\Phi_{\rho}}(x, T)] \frac{\partial \Phi_{\rho}(x, y, z, t)}{\partial z} \right\} \right) D_{0\Phi_{\rho}} + \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{\rho}s}(x, T)}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial x} \right] - \rho(x, y, z, t) \times \\ & \times k_{\rho}(x, T) + \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{\rho}s}(x, T)}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{\Phi_{\rho}s}(x, T)}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial z} \right] + k_{\rho,\rho}(x, T) \rho^2(x, y, z, t). \end{aligned}$$

Let us determine solution of the equations as the power series

$$\Phi_{\rho}(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_{\Phi_{\rho}}^i \Phi_{\rho i}(x, y, z, t), \quad (11)$$

Substitution of the series Eqs.(11) into Eqs.(6) and appropriate boundary and initial conditions give us possibility to zero-order approximations of concentrations of complexes of radiation defects, corrections and conditions for them. The equations, conditions for them and their solutions are presented in the Appendix.

Let us determine spatiotemporal distribution of dopant concentration using the same approach as for determination of spatiotemporal distribution of radiation defects concentration. We transform approximation of dopant diffusion coefficient to the following form: $D_L(x, T) = D_{0L} [1 + \varepsilon_L g_L(x, T)]$.

Further we determine solution of the Eq.(1) as the following power series

$$C(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=1}^{\infty} \xi^j C_{ij}(x, y, z, t).$$

Substitution of the series in the Eqs.(1) and (2) gives us possibility to obtain equations for zero-order approximation of dopant concentration $C_{00}(x, y, z, t)$, corrections to it $C_{ij}(x, y, z, t)$ and boundary and initial conditions to them. The equations, conditions for them and their solutions are presented in the Appendix.

Analysis of spatiotemporal distributions of dopant and radiation defect concentrations has been done analytically using the second-order approximation of dopant concentration. Farther the distribution has been amended numerically.

DISCUSSION

In the previous section we obtain relation to describe spatiotemporal distributions of radiation defects and dopant concentrations. It has been recently shown, that implantation of ions of dopant in a H gives us possibility to increase sharpness of p - n -junction and to increase homogeneity of dopant distribution in doped area [1,2,8]. To obtain the both effects at one time it is necessary to optimize annealing time θ . We approximate real spatial distributions of dopant and minimizing the following mean-squared error

$$U = \frac{1}{L} \int_0^L [C(x,t) - \psi(x)]^2 dx \quad (12)$$

to make the optimization. Here $C(x,t)$ is the real spatiotemporal distribution of dopant concentration, $\psi(x)$ is the step-wise approximation of the concentration. The optimization procedure has been described in several previous works and (at the same time the optimization is not the main result of the work) will not be presented in the paper. The result of optimization is illustrated by Fig. 2. In this figure it has been compared spatial distributions of dopant in homogenous sample and in H with experimental one.

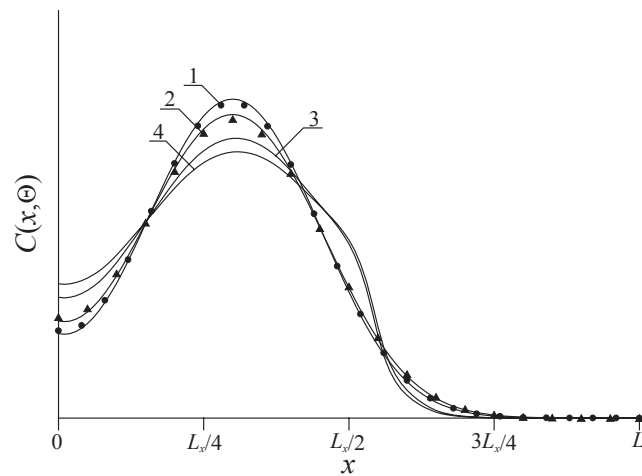


Fig.2. Distributions of dopant concentration after annealing of radiation defects with continuance $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2)/D_{0L}$ (curves 1 and 3) and $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2)/D_{0L}$ (curves 2 and 4), where D_{0L} is average value of diffusion coefficient. Circles and triangles are experimental data from [20,21]. Interface between layers of H has coordinate $a_x = L_x/2$.

The main aim of the present paper is analysis of the possibility to decrease quantity of radiation defects. To decrease the quantity of radiation defects some approaches (such as different types of annealing [3,5-7,20,21], annealing in combination with multistage ion implantation [22], et al) could be used. In this paper we consider a H, which consist of substrate and porous epitaxial layer.

In this paper we analyzed spatiotemporal distributions of concentrations of radiation defects and dopant using modified method of small parameter (we used main idea of method of small parameter, but parameters, which used in appropriate power series could correspond to enough large variations of diffusion coefficients and parameters of recombination of defects, generation of complexes of defects and decay of complexes of defects) [16-18,22]. We obtained, that porosity of EL gives us possibility to decrease quantity of radiation defects. In Fig. 3 we compare distributions of concentrations of radiation defects in porous EL and in non-porous EL. Probably, radiation defects leave to the pores from their neighborhoods.

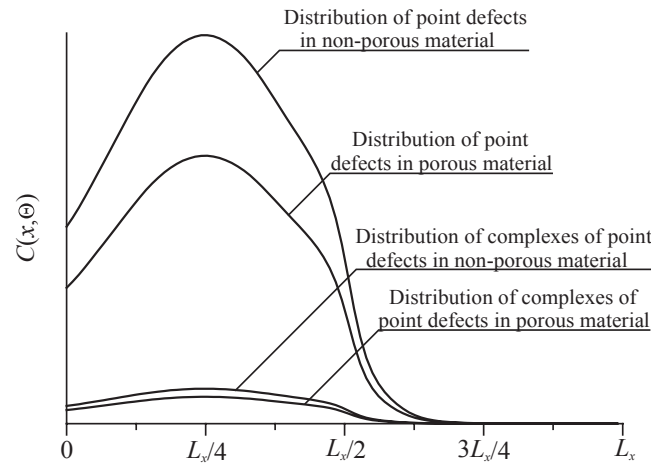


Fig.3. Distributions of concentrations of point radiation defects and their complexes after annealing with time $\Theta = 0.005(L_x^2 + L_y^2 + L_z^2)/D_{0L}$. Curves for complexes of defects have been increased into 50 times. Interface between layers of H has coordinate $a_x = L_x/2$.

In our model we take into account diffusion of dopant and radiation defects, recombination of point radiation defects, generation and decay of their simplest complexes (interstitials and divacancies). We also take into account non-linearity of dopant diffusion for high-doped materials.

CONCLUSION

In this paper we consider an approach to increase sharpness of the implanted-junction rectifier in a semiconductor heterostructure, which consist of two layers (substrate and epitaxial layer). At the same time with increase of the sharpness homogeneity of dopant distribution in doped area increases. We obtain that maximal compromise between the effects could be obtained, when p - n - junction has been fabricated near interface between layers of the heterostructure. In this paper we introduce an approach to decrease quantity of radiation defects by using porosity of epitaxial layer. In this case the radiation defects, probably, leave to the pores from their neighborhoods.

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APPENDIX

Equations for the functions $\tilde{T}_{ijk}(\chi, \eta, \phi, \vartheta)$, $\tilde{V}_{ijk}(\chi, \eta, \phi, \vartheta)$, ($i \geq 0, j \geq 0, k \geq 0$), and boundary and initial conditions for them could be written as

$$\begin{aligned} \frac{\partial \tilde{T}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \frac{1}{\sqrt{D_{0I} D_{0V}}} \times \\ &\times \left\{ b_x \frac{\partial}{\partial \chi} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + b_z \frac{\partial}{\partial \phi} \left[\frac{D_{IS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \right\}; \\ \frac{\partial \tilde{V}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[b_x \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \frac{1}{\sqrt{D_{0I} D_{0V}}} \times \\ &+ \frac{1}{\sqrt{D_{0I} D_{0V}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[\frac{D_{VS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[\frac{D_{VS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right. \\ &\quad \left. + b_z \frac{\partial}{\partial \phi} \left[\frac{D_{VS}(\chi, T)}{V k T} \frac{\partial \mu(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \right\}; \\ \frac{\partial \tilde{T}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\ &+ \sqrt{\frac{D_{0I}}{D_{0V}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[g_I(\chi, T) \frac{\partial \tilde{T}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_I(\chi, T) \frac{\partial \tilde{T}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right. \\ &\quad \left. + b_z \frac{\partial}{\partial \phi} \left[g_I(\chi, T) \frac{\partial \tilde{T}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \right\}, i \geq 1; \\ \frac{\partial \tilde{V}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[b_x \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\ &+ \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[g_V(\chi, T) \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_V(\chi, T) \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right. \\ &\quad \left. + b_z \frac{\partial}{\partial \phi} \left[g_V(\chi, T) \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \right\}, i \geq 1; \\ \frac{\partial \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\ &- [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \tilde{T}_{000}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta); \end{aligned}$$

$$\begin{aligned}
\frac{\partial \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \tilde{T}_{000}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \tilde{T}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] [\tilde{T}_{010}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta) + \tilde{T}_{000}(\chi, \eta, \phi, \vartheta) \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)]; \\
\frac{\partial \tilde{V}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [\tilde{T}_{010}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta) + \tilde{T}_{000}(\chi, \eta, \phi, \vartheta) \tilde{V}_{010}(\chi, \eta, \phi, \vartheta)] [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \phi, T)]; \\
\frac{\partial \tilde{T}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] \tilde{T}_{000}^2(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \tilde{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{V,V} g_{V,V}(\chi, T)] \tilde{V}_{000}^2(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \tilde{T}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] \tilde{T}_{001}(\chi, \eta, \phi, \vartheta) \tilde{T}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \tilde{V}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] - \\
&\quad - [1 + \varepsilon_{V,V} g_{V,V}(\chi, T)] \tilde{V}_{001}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \tilde{T}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \tilde{T}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \tilde{T}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \tilde{T}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\
&\quad + \sqrt{\frac{D_{0I}}{D_{0V}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[g_I(\chi, T) \frac{\partial \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_I(\chi, T) \frac{\partial \tilde{T}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& + b_z \frac{\partial}{\partial \phi} \left[g_I(\chi, T) \frac{\partial \mathcal{I}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \Bigg\} - [1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] [\mathcal{I}_{100}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{000}(\chi, \eta, \phi, \vartheta) + \\
& + \mathcal{I}_{000}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{100}(\chi, \eta, \phi, \vartheta)]; \\
\frac{\partial \mathcal{V}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[b_x \frac{\partial^2 \mathcal{V}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \mathcal{V}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \mathcal{V}_{110}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \sqrt{\frac{D_{0V}}{D_{0I}}} \times \\
& \times \left\{ b_x \frac{\partial}{\partial \chi} \left[g_V(\chi, T) \frac{\partial \mathcal{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_V(\chi, T) \frac{\partial \mathcal{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + b_z \frac{\partial}{\partial \phi} \left[\frac{\partial \mathcal{V}_{010}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \times \right. \\
& \times g_V(\chi, T) \Bigg\} - [1 + \varepsilon_{V,V} g_{V,V}(\chi, T)] [\mathcal{I}_{100}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{000}(\chi, \eta, \phi, \vartheta) + \mathcal{I}_{000}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{100}(\chi, \eta, \phi, \vartheta)]; \\
\frac{\partial \mathcal{I}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \mathcal{I}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \mathcal{I}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \mathcal{I}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\
& + \sqrt{\frac{D_{0I}}{D_{0V}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[g_I(\chi, T) \frac{\partial \mathcal{I}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_I(\chi, T) \frac{\partial \mathcal{I}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right. \\
& + b_z \frac{\partial}{\partial \phi} \left[g_I(\chi, T) \frac{\partial \mathcal{I}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \Bigg\} - [1 + \varepsilon_I g_I(\chi, T)] \mathcal{I}_{100}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \mathcal{V}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[b_x \frac{\partial^2 \mathcal{V}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \mathcal{V}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \mathcal{V}_{101}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\
& + \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ b_x \frac{\partial}{\partial \chi} \left[g_V(\chi, T) \frac{\partial \mathcal{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right] + b_y \frac{\partial}{\partial \eta} \left[g_V(\chi, T) \frac{\partial \mathcal{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right] + \right. \\
& + b_z \frac{\partial}{\partial \phi} \left[g_V(\chi, T) \frac{\partial \mathcal{V}_{001}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right] \Bigg\} - [1 + \varepsilon_I g_I(\chi, T)] \mathcal{I}_{100}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \mathcal{I}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[b_x \frac{\partial^2 \mathcal{I}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \mathcal{I}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \mathcal{I}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] + \\
& - [1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] \mathcal{I}_{000}(\chi, \eta, \phi, \vartheta) \mathcal{I}_{010}(\chi, \eta, \phi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \mathcal{I}_{001}(\chi, \eta, \phi, \vartheta) \mathcal{V}_{000}(\chi, \eta, \phi, \vartheta); \\
\frac{\partial \mathcal{V}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[b_x \frac{\partial^2 \mathcal{V}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \chi^2} + b_y \frac{\partial^2 \mathcal{V}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \eta^2} + b_z \frac{\partial^2 \mathcal{V}_{011}(\chi, \eta, \phi, \vartheta)}{\partial \phi^2} \right] +
\end{aligned}$$

$$\begin{aligned}
& -[1 + \varepsilon_{I,I} g_{I,I}(\chi, T)] \tilde{I}_{000}(\chi, \eta, \phi, \vartheta) \tilde{I}_{010}(\chi, \eta, \phi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, T)] \tilde{I}_{001}(\chi, \eta, \phi, \vartheta) \tilde{V}_{000}(\chi, \eta, \phi, \vartheta); \\
& \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right|_{\chi=0} = 0, \quad \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right|_{\chi=1} = 0, \quad \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right|_{\eta=0} = 0, \quad \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right|_{\eta=1} = 0, \\
& \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right|_{\phi=0} = 0, \quad \left. \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right|_{\phi=1} = 0, \quad \left. \frac{\partial \tilde{T}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \chi} \right|_{\chi=\frac{x_1}{L_x}} = 0, \quad \left. \frac{\partial \tilde{T}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \eta} \right|_{\eta=\frac{y_1}{L_y}} = 0, \\
& \tilde{V}_{000} \left(\frac{x_1}{L_x} + \vartheta \frac{V_n}{L_x} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \frac{y_1}{L_y} + \vartheta \frac{V_n}{L_y} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \frac{z_1}{L_z} + \vartheta \frac{V_n}{L_z} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \vartheta \right) = 1 + \frac{2 \ell \omega}{k T \sqrt{x_1^2 + y_1^2 + z_1^2}}, \\
& \tilde{V}_{ijk} \left(\frac{x_1}{L_x} + \vartheta \frac{V_n}{L_x} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \frac{y_1}{L_y} + \vartheta \frac{V_n}{L_y} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \frac{z_1}{L_z} + \vartheta \frac{V_n}{L_z} \frac{L_x^2 + L_y^2 + L_z^2}{\sqrt{D_{0I} D_{0V}}}, \vartheta \right) = 0, \quad (i \geq 1, j \geq 1, k \geq 1); \\
& \left. \frac{\partial \tilde{T}_{ijk}(\chi, \eta, \phi, \vartheta)}{\partial \phi} \right|_{\phi=\frac{z_1}{L_z}} = 0, \quad (i \geq 1, j \geq 1, k \geq 1); \quad \tilde{\rho}_{000}(\chi, \eta, \phi, 0) = \frac{f_\rho(\chi, \eta, \phi)}{\rho^*}, \quad \tilde{\rho}_{ijk}(\chi, \eta, \phi, 0) = 0 \quad (i \geq 1, j \geq 1, k \geq 1).
\end{aligned}$$

Solutions of the above equations with account boundary and initial conditions are

$$\begin{aligned}
\tilde{I}_{000}(\chi, \eta, \phi, \vartheta) &= -\frac{2\pi b_x}{\sqrt{D_{0I} D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) [F_{nI} + \\
&+ \frac{D_{IS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial u}] d w d v d u d \tau - \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \times \\
&\times \int_0^1 \cos(\pi n w) \left[F_{nI} + \frac{D_{IS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial v} \right] d w d v d u d \tau - \frac{2\pi b_y}{\sqrt{D_{0I} D_{0V}}} - \frac{2\pi b_z}{\sqrt{D_{0I} D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) \times \\
&\times e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) \left[F_{nI} + \frac{D_{IS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial w} \right] d w d v d u d \tau; \\
\tilde{V}_{000}(\chi, \eta, \phi, \vartheta) &= -\frac{2\pi b_x}{\sqrt{D_{0I} D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) [F_{nV} + \\
&+ \frac{D_{VS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial u}] d w d v d u d \tau - \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \times \\
&\times \int_0^1 \cos(\pi n w) \left[F_{nV} + \frac{D_{VS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial v} \right] d w d v d u d \tau - \frac{2\pi b_y}{\sqrt{D_{0I} D_{0V}}} - \frac{2\pi b_z}{\sqrt{D_{0I} D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) \times
\end{aligned}$$

$$\times e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) \left[F_{nV} + \frac{D_{VS}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial w} \right] d w d v d u d \tau;$$

$$\text{where } C_n(\chi, \eta, \phi) = \cos(\pi n \chi) \cos(\pi n \eta) \cos(\pi n \phi); F_{n\rho} = \frac{1}{\rho^*} \int_0^1 \int_0^1 \int_0^1 C_n(u, v, w) f_{n\rho}(u, v, w) d w d v d u;$$

$$e_{nI}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0V}/D_{0I}}); e_{nV}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0I}/D_{0V}}), c_n(\chi) = \cos(\pi n \chi / L_\chi);$$

$$T_{i00}(\chi, \eta, \phi, \vartheta) = -2\pi b_x \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) g_I(u, T) \times$$

$$\times n \frac{\partial T_{i-100}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sum_{n=1}^{\infty} n \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) g_I(u, T) \frac{\partial T_{i-100}(u, v, w, \tau)}{\partial u} d w d v d u d \tau \times$$

$$\times b_y n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \sqrt{\frac{D_{0I}}{D_{0V}}} - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \int_0^1 g_I(u, T) \times$$

$$\times b_y \cos(\pi n w) \frac{\partial T_{i-100}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2 \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \times$$

$$\times \int_0^1 \sin(\pi n w) g_I(u, v, w, T) \frac{\partial T_{i-100}(u, v, w, \tau)}{\partial w} d w d v d u d \tau e_{nI}(\vartheta) \pi b_z, i \geq 1;$$

$$\tilde{V}_{i00}(\chi, \eta, \phi, \vartheta) = -2\pi b_x \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) g_V(u, T) \times$$

$$\times n \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi \sum_{n=1}^{\infty} n \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) g_V(u, T) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial u} d w d v d u d \tau \times$$

$$\times b_y C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \sqrt{\frac{D_{0V}}{D_{0I}}} - 2\pi b_y \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \int_0^1 g_V(u, T) \times$$

$$\times n \cos(\pi n w) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi b_z \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \times$$

$$\times \int_0^1 \sin(\pi n w) g_V(u, T) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial w} d w d v d u d \tau e_{nV}(\vartheta), i \geq 1;$$

$$\tilde{\rho}_{010}(\chi, \eta, \phi, \vartheta) = -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{n\rho}(\vartheta) \int_0^{\vartheta} e_{n\rho}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \times$$

$$\times T_{000}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau;$$

$$\begin{aligned}
\tilde{T}_{020}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nl}(\vartheta) \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \times \\
&\times \cos(\pi n w) \sqrt{\frac{D_{0I}}{D_{0V}}} [\tilde{T}_{010}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{T}_{000}(u, v, w, \tau) \tilde{V}_{010}(u, v, w, \tau)] d w d v d u d \tau ; \\
\tilde{V}_{020}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \times \\
&\times \cos(\pi n w) \sqrt{\frac{D_{0V}}{D_{0I}}} [\tilde{T}_{010}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{T}_{000}(u, v, w, \tau) \tilde{V}_{010}(u, v, w, \tau)] d w d v d u d \tau ; \\
\tilde{T}_{001}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nl}(\vartheta) \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{T}_{000}^2(u, v, w, \tau) \times \\
&\times [1 + \varepsilon_{I,I} g_{I,I}(u, T)] d w d v d u d \tau ; \\
\tilde{V}_{001}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{V}_{000}^2(u, v, w, \tau) \times \\
&\times [1 + \varepsilon_{V,V} g_{V,V}(u, T)] d w d v d u d \tau ; \\
\tilde{T}_{002}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nl}(\vartheta) \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{T}_{001}(u, v, w, \tau) \times \\
&\times [1 + \varepsilon_{I,I} g_{I,I}(u, T)] \tilde{T}_{000}(u, v, w, \tau) d w d v d u d \tau ; \\
\tilde{V}_{002}(\chi, \eta, \phi, \vartheta) &= -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{V}_{001}(u, v, w, \tau) \times \\
&\times [1 + \varepsilon_{V,V} g_{V,V}(u, T)] \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau ; \\
\tilde{T}_{110}(\chi, \eta, \phi, \vartheta) &= -2 \pi b_x \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nl}(\vartheta) \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \times \\
&\times g_I(u, T) \frac{\partial \tilde{T}_{010}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2 \pi b_y \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nl}(\vartheta) \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \cos(\pi n u) \times \\
&\times \int_0^1 \sin(\pi n v) \int_0^1 \cos(\pi n w) g_I(u, T) \frac{\partial \tilde{T}_{010}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2 \pi b_z \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} e_{nl}(\vartheta) C_n(\chi, \eta, \phi) \times \\
&\times n \int_0^{\vartheta} e_{nl}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) g_I(u, T) \frac{\partial \tilde{T}_{010}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - 2 \sum_{n=1}^{\infty} e_{nl}(\vartheta) \times
\end{aligned}$$

$$\begin{aligned}
& \times C_n(\chi, \eta, \phi) \int_0^{\vartheta} \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 [\tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{100}(u, v, w, \tau)] \times \\
& \quad \times e_{nl}(-\tau) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \cos(\pi n w) d w d v d u d \tau ; \\
\tilde{V}_{110}(\chi, \eta, \phi, \vartheta) = & -2\pi b_x \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \times \\
& \times g_V(u, T) \frac{\partial \tilde{V}_{010}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi b_y \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \times \\
& \times \int_0^1 \sin(\pi n v) \int_0^1 \cos(\pi n w) g_V(u, T) \frac{\partial \tilde{V}_{010}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi b_z \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} e_{nV}(\vartheta) C_n(\chi, \eta, \phi) \times \\
& \times n \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) g_V(u, T) \frac{\partial \tilde{V}_{010}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - 2 \sum_{n=1}^{\infty} e_{nV}(\vartheta) \times \\
& \times C_n(\chi, \eta, \phi) \int_0^{\vartheta} \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 [\tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{100}(u, v, w, \tau)] \times \\
& \quad \times e_{nV}(-\tau) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \cos(\pi n w) d w d v d u d \tau ; \\
\tilde{I}_{101}(\chi, \eta, \phi, \vartheta) = & -2\pi b_x \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) g_I(u, T) \times \\
& \times n \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi b_y \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \times \\
& \times \int_0^1 \cos(\pi n w) g_I(u, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi b_z \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} \int_0^1 \cos(\pi n u) \times \\
& \times \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) g_I(u, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial w} d w d v d u e_{nI}(-\tau) d \tau - 2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \times \\
& \times \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{I}_{100}(u, v, w, \tau) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau ; \\
\tilde{V}_{101}(\chi, \eta, \phi, \vartheta) = & -2\pi b_x \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \sin(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) g_V(u, T) \times \\
& \times n \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - 2\pi b_y \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \sin(\pi n v) \times \\
& \times \int_0^1 \cos(\pi n w) g_V(u, T) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2\pi b_z \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} \int_0^1 \cos(\pi n u) \times \\
& \times \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) g_V(u, T) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial w} d w d v d u e_{nV}(-\tau) d \tau - 2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \times \\
& \times \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \tilde{I}_{100}(u, v, w, \tau) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \tilde{V}_{000}(u, v, w, \tau) d w d v d u d \tau ;
\end{aligned}$$

$$\begin{aligned}
& \times \int_0^1 \cos(\pi n w) g_V(u, T) \frac{\partial \mathcal{V}_{001}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - 2 \pi b_z \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} \int_0^1 \cos(\pi n u) \times \\
& \times \int_0^1 \cos(\pi n v) \int_0^1 \sin(\pi n w) g_V(u, T) \frac{\partial \mathcal{V}_{001}(u, v, w, \tau)}{\partial w} d w d v d u e_{nV}(-\tau) d \tau - 2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \times \\
& \times \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \mathcal{I}_{000}(u, v, w, \tau) [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \mathcal{V}_{100}(u, v, w, \tau) d w d v d u d \tau ; \\
& \mathcal{I}_{011}(\chi, \eta, \phi, \vartheta) = -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nI}(\vartheta) \int_0^{\vartheta} e_{nI}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \{ \mathcal{I}_{000}(u, v, w, \tau) [1 + \\
& + \varepsilon_{I,I} g_{I,I}(u, v, w, T)] \mathcal{I}_{010}(u, v, w, \tau) + [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \mathcal{I}_{001}(u, v, w, \tau) \mathcal{V}_{000}(u, v, w, \tau) \} d w d v d u d \tau ; \\
& \mathcal{V}_{011}(\chi, \eta, \phi, \vartheta) = -2 \sum_{n=1}^{\infty} C_n(\chi, \eta, \phi) e_{nV}(\vartheta) \int_0^{\vartheta} e_{nV}(-\tau) \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) \{ \mathcal{V}_{000}(u, v, w, \tau) [1 + \\
& + \varepsilon_{V,V} g_{V,V}(u, v, w, T)] \mathcal{V}_{010}(u, v, w, \tau) + [1 + \varepsilon_{I,V} g_{I,V}(u, T)] \mathcal{I}_{000}(u, v, w, \tau) \mathcal{V}_{001}(u, v, w, \tau) \} d w d v d u d \tau .
\end{aligned}$$

Equations for functions $\Phi_{\rho i}(x, y, z, t)$ ($i \geq 0$) and boundary and initial conditions for them could written as

$$\begin{aligned}
\frac{\partial \Phi_{I0}(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[\frac{D_{\phi I S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{\phi I S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial y} \right] + \left[\frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial x^2} + \right. \\
&+ \frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial z^2} \left. \right] D_{0\phi I} + \frac{\partial}{\partial z} \left[\frac{D_{\phi I S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial z} \right] + k_{I,I}(x, T) I^2(x, y, z, t) - \\
&- k_I(x, T) I(x, y, z, t); \\
\frac{\partial \Phi_{V0}(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[\frac{D_{\phi V S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{\phi V S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial y} \right] + \left[\frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial x^2} + \right. \\
&+ \frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial z^2} \left. \right] D_{0\phi V} + \frac{\partial}{\partial z} \left[\frac{D_{\phi V S}(x, T)}{V k T} \frac{\partial \mu(x, y, z, t)}{\partial z} \right] + k_{V,V}(x, T) V^2(x, y, z, t) - \\
&- k_V(x, T) V(x, y, z, t); \\
\frac{\partial \Phi_{Ii}(x, y, z, t)}{\partial t} &= D_{0\phi I} \left\{ \frac{\partial}{\partial x} \left[g_{\phi I}(x, T) \frac{\partial \Phi_{Ii-1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[g_{\phi I}(x, T) \frac{\partial \Phi_{Ii-1}(x, y, z, t)}{\partial y} \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{\partial}{\partial z} \left[g_{\phi l}(x, T) \frac{\partial \Phi_{li-1}(x, y, z, t)}{\partial z} \right] + \frac{\partial^2 \Phi_{li}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{li}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{li}(x, y, z, t)}{\partial z^2} \Bigg\}, i \geq 1; \\
& \frac{\partial \Phi_{Vi}(x, y, z, t)}{\partial t} = D_{0\phi V} \left\{ \frac{\partial}{\partial x} \left[g_{\phi V}(x, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[g_{\phi V}(x, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial y} \right] + \right. \\
& \left. + \frac{\partial}{\partial z} \left[g_{\phi V}(x, T) \frac{\partial \Phi_{Vi-1}(x, y, z, t)}{\partial z} \right] + \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{Vi}(x, y, z, t)}{\partial z^2} \right\}, i \geq 1; \\
& \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial x} \Bigg|_{x=0} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial x} \Bigg|_{x=L_x} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial y} \Bigg|_{y=0} = 0, \\
& \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial y} \Bigg|_{y=L_y} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial z} \Bigg|_{z=0} = 0, \frac{\partial \Phi_{\rho i}(x, y, z, t)}{\partial z} \Bigg|_{z=L_z} = 0, i \geq 0; \\
& \Phi_{l0}(x, y, z, 0) = f_{\phi l}(x, y, z), \Phi_{li}(x, y, z, 0) = 0, \Phi_{V0}(x, y, z, 0) = f_{\phi V}(x, y, z), \Phi_{Vi}(x, y, z, 0) = 0, i \geq 1.
\end{aligned}$$

Solutions of the equations could be written as

$$\begin{aligned}
\Phi_{l0}(x, y, z, t) = & - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi ln}(t) \int_0^t e_{\phi ln}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} \int_0^{L_z} \left[F_{n\phi l} + \frac{D_{\phi l s}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial u} \right] \times \\
& \times n c_n(w) dw c_n(v) dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{\phi ln}(t) \int_0^t e_{\phi ln}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} \left[\frac{\partial \mu(u, v, w, \tau)}{\partial v} \times \right. \\
& \times \frac{D_{\phi l s}(u, T)}{V k T} + F_{n\phi l} \Bigg] c_n(w) dw dv du d\tau - \pi \frac{2L_z^2}{L_x L_y} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi ln}(t) \int_0^t e_{\phi ln}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} \left[F_{n\phi l} + \right. \\
& \left. + \frac{D_{\phi l s}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial w} \right] s_n(w) dw dv du d\tau n + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi ln}(t) \int_0^t e_{\phi ln}(-\tau) \int_0^{L_x} c_n(u) \times \\
& \times \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \left[k_{l, l}(u, T) I^2(u, v, w, \tau) - k_l(u, T) I(u, v, w, \tau) \right] dw dv du d\tau; \\
\Phi_{V0}(x, y, z, t) = & - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi Vn}(t) \int_0^t e_{\phi Vn}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} \int_0^{L_z} \left[F_{n\phi V} + \frac{D_{\phi V s}(u, T)}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial u} \right] \times \\
& \times n c_n(w) dw c_n(v) dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{\phi Vn}(t) \int_0^t e_{\phi Vn}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} \left[\frac{\partial \mu(u, v, w, \tau)}{\partial v} \times \right.
\end{aligned}$$

$$\begin{aligned} & \times \frac{D_{\phi_V S}(u, T)}{\bar{V} k T} + F_{n\phi_V} \left] c_n(w) d w d v d u d \tau - \pi \frac{2L_z^{-2}}{L_x L_y} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi V n}(t) \int_0^t e_{\phi V n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} [F_{n\phi_V} + \right. \\ & \left. + \frac{D_{\phi_V S}(u, T)}{\bar{V} k T} \frac{\partial \mu(u, v, w, \tau)}{\partial w} \right] s_n(w) d w d v d u d \tau n + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi V n}(t) \int_0^t e_{\phi V n}(-\tau) \int_0^{L_x} c_n(u) \times \\ & \times \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) [k_{V, V}(u, T) V^2(u, v, w, \tau) - k_V(u, T) V(u, v, w, \tau)] d w d v d u d \tau, \end{aligned}$$

$$\text{where } F_{n\phi_p} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_C(u, v, w) d w d v d u, \quad e_{\phi p n}(t) = \exp \left[-\pi^2 n^2 D_{0\phi p} t \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \right],$$

$$C_n(x, y, z) = c_n(x) c_n(y) c_n(z);$$

$$\begin{aligned} \Phi_{Ii}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n \int_0^t e_{\phi I n}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) g_{\phi I}(u, T) \frac{\partial \Phi_{Ii-1}(u, v, w, \tau)}{\partial u} d w d v d u d \tau \times \\ & \times C_n(x, y, z) e_{\phi I n}(t) - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi I n}(t) \int_0^t e_{\phi I n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} g_{\phi I}(u, T) \frac{\partial \Phi_{Ii-1}(u, v, w, \tau)}{\partial v} \times \\ & \times n c_n(w) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi I n}(t) \int_0^t e_{\phi I n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{\partial \Phi_{Ii-1}(u, v, w, \tau)}{\partial w} \times \\ & \times n g_{\phi I}(u, T) d w d v d u d \tau, \quad i \geq 1; \end{aligned}$$

$$\begin{aligned} \Phi_{Vi}(x, y, z, t) = & -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n \int_0^t e_{\phi V n}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) g_{\phi V}(u, T) \frac{\partial \Phi_{Vi-1}(u, v, w, \tau)}{\partial u} d w d v d u d \tau \times \\ & \times C_n(x, y, z) e_{\phi V n}(t) - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi V n}(t) \int_0^t e_{\phi V n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} g_{\phi V}(u, T) \frac{\partial \Phi_{Vi-1}(u, v, w, \tau)}{\partial v} \times \\ & \times n c_n(w) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} C_n(x, y, z) e_{\phi V n}(t) \int_0^t e_{\phi V n}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{\partial \Phi_{Vi-1}(u, v, w, \tau)}{\partial w} \times \\ & \times n g_{\phi V}(u, T) d w d v d u d \tau, \quad i \geq 1. \end{aligned}$$

Equations for the functions $C_{ij}(x, y, z, t)$ ($i \geq 0, j \geq 0$), boundary and initial conditions for them are

$$\begin{aligned} \frac{\partial C_{00}(x, y, z, t)}{\partial t} = & D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{00}(x, y, z, t)}{\partial z^2} + \\ & + \frac{\partial}{\partial x} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{CS}}{\bar{V} k T} \frac{\partial \mu(x, y, z, t)}{\partial z} \right]; \end{aligned}$$

$$\begin{aligned}
& \frac{\partial C_{i0}(x, y, z, t)}{\partial t} = D_{0L} \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial}{\partial x} \left[g_L(x, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial x} \right] + \\
& + D_{0L} \frac{\partial^2 C_{i0}(x, y, z, t)}{\partial z^2} + D_{0L} \frac{\partial}{\partial y} \left[g_L(x, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial y} \right] + D_{0L} \frac{\partial}{\partial z} \left[g_L(x, T) \frac{\partial C_{i-10}(x, y, z, t)}{\partial z} \right], i \geq 1; \\
& \frac{\partial C_{01}(x, y, z, t)}{\partial t} = D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial y^2} + D_{0L} \frac{\partial}{\partial x} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \\
& + D_{0L} \frac{\partial^2 C_{01}(x, y, z, t)}{\partial z^2} + D_{0L} \frac{\partial}{\partial y} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + D_{0L} \frac{\partial}{\partial z} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right]; \\
& \frac{\partial C_{02}(x, y, z, t)}{\partial t} = \left\{ \frac{\partial}{\partial x} \left[C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \times \right. \right. \\
& \times \left. \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[C_{01}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial x} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{01}(x, y, z, t)}{\partial x} \right] + \\
& + \frac{\partial}{\partial y} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{01}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{01}(x, y, z, t)}{\partial z} \right] + \frac{\partial^2 C_{02}(x, y, z, t)}{\partial x^2} + \\
& \left. + \frac{\partial^2 C_{02}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 C_{02}(x, y, z, t)}{\partial z^2} \right\} D_{0L}; \\
& \frac{\partial C_{11}(x, y, z, t)}{\partial t} = \left\{ \frac{\partial}{\partial x} \left[g_L(x, T) \frac{\partial C_{01}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[g_L(x, T) \frac{\partial C_{01}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[g_L(x, T) \times \right. \right. \\
& \times \left. \frac{\partial C_{01}(x, y, z, t)}{\partial z} \right] + \frac{\partial}{\partial x} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial y} \right] + \\
& + \frac{\partial}{\partial z} \left[\frac{C_{00}^\gamma(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial z} \right] + \frac{\partial}{\partial x} \left[C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial x} \right] + \\
& + \frac{\partial}{\partial y} \left[C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[C_{10}(x, y, z, t) \frac{C_{00}^{\gamma-1}(x, y, z, t)}{P^\gamma(x, T)} \frac{\partial C_{00}(x, y, z, t)}{\partial z} \right] + \\
& \left. + \frac{\partial^2 C_{11}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 C_{11}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 C_{11}(x, y, z, t)}{\partial z^2} \right\} D_{0L};
\end{aligned}$$

$$\begin{aligned} \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \right|_{z=0} &= 0, \left. \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, i \geq 0, j \geq 0; C_{00}(x, y, z, 0) = f_C(x, y, z), C_{ij}(x, y, z, 0) = 0, i \geq 1, j \geq 1. \\ C_{00}(x, y, z, t) &= -\frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} \left[F_{nC} + \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{D_{CS}}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial u} d w d v d u d \tau \right] \times \\ &\times n C_n(x, y, z) e_{nC}(t) - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} n \left[\int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{D_{CS}}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial v} d w d v d u d \tau + \right. \\ &+ F_{nC} \left. \right] C_n(x, y, z) e_{nC}(t) - \frac{2\pi}{L_x L_y L_z} \sum_{n=1}^{\infty} \left[\int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{D_{CS}}{V k T} \frac{\partial \mu(u, v, w, \tau)}{\partial w} d w d v d u d \tau + \right. \\ &\left. + F_{nC} \right] n C_n(x, y, z) e_{nC}(t), \end{aligned}$$

$$\text{where } F_{nC} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_C(u, v, w) d w d v d u, e_{nC}(t) = \exp \left[-\pi^2 n^2 D_{0L} \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \right];$$

$$\begin{aligned} C_{i0}(x, y, z, t) &= -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_L(u, T) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial u} \times \\ &\times c_n(w) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial v} \times \\ &\times F_{nC} g_L(u, T) d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial w} \times \\ &\times F_{nC} s_n(w) g_L(u, T) d w d v d u d \tau, i \geq 1; \end{aligned}$$

$$\begin{aligned} C_{01}(x, y, z, t) &= -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \times \\ &\times \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \times \\ &\times \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} d w d v d u d \tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \times \end{aligned}$$

$$\begin{aligned}
& \times \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dw dv du d\tau; \\
C_{02}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \times \\
& \times \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} dw dv du d\tau n F_{nC} - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \times \\
& \times C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} dw dv du d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \times \\
& \times n \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dw dv du d\tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} \times \\
& \times C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} dw dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \times \\
& \times \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} dw dv du d\tau - \\
& - 2 \sum_{n=1}^{\infty} \frac{\pi n F_{nC}}{L_x L_y L_z^2} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} dw dv du d\tau; \\
C_{11}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_L(u, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} c_n(w) dw dv du d\tau \times \\
& \times n F_{nC} C_n(x, y, z) - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} F_{nC} \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_L(u, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} dw dv du d\tau \times \\
& \times n C_n(x, y, z) e_{nC}(t) - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_L(u, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} \times \\
& \times n s_n(w) dw dv du d\tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{\partial C_{10}(u, v, w, \tau)}{\partial u} \times \\
& \times F_{nC} \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} dw dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{\partial C_{10}(u, v, w, \tau)}{\partial v} \times \\
& \times n F_{nC} \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} dw dv du d\tau C_n(x, y, z) - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \times
\end{aligned}$$

$$\begin{aligned}
& \times n \int_0^{L_z} s c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{10}(u, v, w, \tau)}{\partial w} d w d v d u d \tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \times \\
& \times \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n e_{nC}(t) \times \\
& \times C_n(x, y, z) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau \times \\
& \times F_{nC} - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} \\
& \times c_n(w) d w d v d u d \tau n F_{nC} - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n C_n(x, y, z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{10}(u, v, w, \tau) \times \\
& \times F_{nC} \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} d w d v d u d \tau .
\end{aligned}$$

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