# Micro Convective Heat Transfer of Gas Flow Subject to H and T Boundary Conditions

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**Abstract**
This paper focuses on experimental and numerical analysis of convective heat transfer characteristics of pressure-driven gaseous flows through microtubes, which is frequently encountered in practical application of microfluidic devices accommodating gas flow, heat transfer and/or chemical reactions at microscale. The present work has been carried out with the objectives to: (i) verify the applicability of conventional theory for the prediction of internal forced convection heat transfer coefficient for tubes having an inner diameter lower than 1 mm and (ii) check the performance of some specific correlations proposed for the analysis of forced micro convection with gases in the last decades. Single commercial stainless steel microtubes are tested with inner diameters ranging from 1 mm down to 0.17 mm. The most common thermal boundary conditions, namely uniform heat flux (H boundary condition) and uniform wall temperature (T boundary condition), have been implemented by applying Joule heating on external surface of microtubes (H b.c.) and by submerging microtube in water with constant temperature regulated by means of a thermostatic bath (T b.c.). The test section has been designed with care in order to ensure experiment reliability and to improve the accuracy of measurement at microscale. Experimental data are supplemented by numerical simulation, which demonstrates the local temperature distribution inside microtubes under various thermal boundary conditions. The values of Nusselt number are experimentally determined and compared with both conventional theory and the prediction of the specific correlations developed for microchannels.

## 1. INTRODUCTION

During the last decades there has been a rapid progress in the technology of miniaturization, with devices and systems being scaled to microscale or smaller dimensions. This trend was not only forced by extensive engineering and industrial applications with promising market potential but also stimulated by multidisciplinary research intersecting chemistry, physics, biology, life science, pharmaceutics and engineering, etc. The analysis of flow and heat transfer mechanisms at microscale level is an interesting topic not completely investigated up to now, as remarked in a recent review by Kandlikar *et al.* [1]. One of the main questions remaining unanswered is in which conditions the macroscale rules for single-phase flow heat transfer are still valid for microscale phenomena. For this reason, in the last years a large amount of experimental analyses has been addressed to the analysis of fluid-dynamics and heat transfer characteristics of single- and two-phase flows in microchannels [1]; however, the results are not always univocal as evidenced by Morini [2] and Hetsroni et al. [3].

As highlighted in [1], forced convection of single-phase liquid flows in microchannels has been extensively investigated in the past and now it is possible to conclude that the conventional theory developed at macroscale is able to predict the convective heat transfer coefficient for liquid flows in microchannels, with minor deviations caused by experimental errors [4] and/or by the presence of non-negligible scaling effects [5, 6, 7]. On the contrary, in the case of gas micro convection very few experiments support the theoretical models and significant efforts are still needed in this direction [1]; this fact has been also addressed by Colin [8] in a recent review focused on gas micro convection.

The first experimental investigation devoted to the analysis of gas micro convection can be traced back to the work by Wu and Little [9] in the 1980s, who explored the flow and heat transfer characteristics of N2, H2 and Ar through miniaturized rectangular channels with hydraulic diameter from 40 to 80 mm. The comparison of their experimental Nusselt numbers with the predictions of the classical correlations (i.e. Sieder and Tate [10], Hausen [11] and Dittus and Boelter [12]) evidenced a strong disagreement in laminar, transitional and turbulent regimes.

The experimental data published by Choi et al. [13] successively, obtained with nitrogen flow in microtubes with an inner diameter down to 3 µm, confirmed that the Nusselt number in turbulent regime was larger than the prediction of the Dittus-Boelter correlation [12] but their data were not in agreement with the correlation proposed previously by Wu and Little [9]. In laminar regime Choi et al. [13] obtained very low values of Nusselt number compared to the predictions of the conventional correlations. The authors gave no justification to this trend, both in laminar and turbulent regime, and proposed two new correlations for the prediction of the Nusselt number in microtubes by fitting their own experimental data.

Yu et al. [14] investigated the convective heat transfer of nitrogen flow in turbulent regime through microtubes with inner diameters between 19 µm and 102 µm. Also in this case, the Nusselt numbers obtained in turbulent regime were larger than those predicted by means of conventional correlations but the authors avoided any physical explanation of their results and proposed a new correlation for the prediction of the Nusselt number in microtubes, not in agreement with the previous correlations of Wu and Little [9] and Choi et al. [13].

Hara et al. [15] experimentally investigated the convective heat transfer of air flows through square minichannels with hydraulic diameters between 0.3 mm and 2 mm. Unlike the other researchers, they found that the deviation of Nusselt number from conventional theory may depend on the hydraulic diameter and length of the tested minichannels.

More recently, Chen et al. [16] and Yang et al. [17] conducted experimental research on forced convection of air and carbon dioxide through microtubes with an inner diameter from 86 µm to 920 µm. The trend of the Nusselt number in both laminar and turbulent regime was found in agreement with the classical correlation proposed by Gnielinski [18], which was validated for incompressible flow at macro scale.

It is possible to conclude that the results obtained recently by Chen et al. [16] and Yang et al. [17] are in disagreement with the results of the previous works by Wu and Little [9] and Choi et al. [13] Yu et al. [14] and Hara et al. [15], and these last works are not in agreement with each other.

For these reasons, the main objective of the present work is to verify the applicability of the conventional correlations for the prediction of Nusselt numbers in microtubes under both H boundary condition and T boundary condition. In addition, the experimental investigation is carried out to carefully check out whether the correlations proposed specifically for gas micro convection are necessary or not. The analysis is conducted by varying the inner diameter of the microtubes from 1 mm down to 0.17 mm and the Reynolds number between 400 up to 5000 in order to investigate the behavior of the gas flow in both laminar and transitional regimes.

## 2. EXPERIMENTAL AND NUMERICAL METHODS

### 2.1. Experimental setup

The lay-out of the test rig used for the experimental runs of this work is shown in Fig. 1. Nitrogen is used as working fluid; it is stored in a high-pressure flask (200 bar) (*1*) and it is expanded using a valve (*2*) to approximately 10 bar at ambient temperature. Then, the gas flow enters a 7 µm particle filter (*3, Hamlet*) which is used to prevent possible impurities from clogging the microtubes or the mass flow controllers (*4, Bronkhorst EL-Flow E7000*, *KOFLOC 3300 and KOFLOC 3100*). The mass flow controllers can impose an expected mass flow rate by means of a computer-steered valve which makes an indirect regulation of the pressure at the inlet of the microtube. Microtubes (*5*) of various lengths

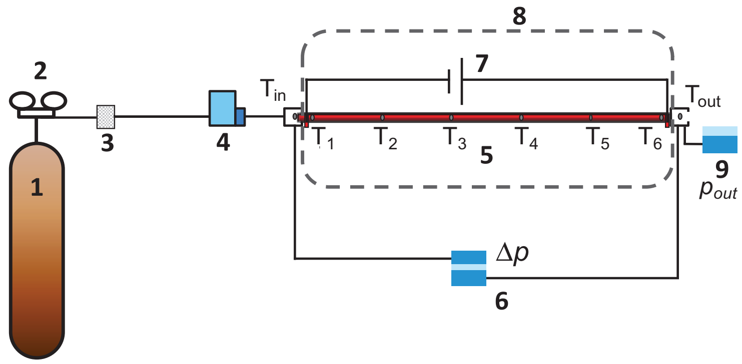



Figure 1 Schematic layout of experimental test rig (1- gas source, 2- valve, 3- filter, 4- mass flow controllers, 5- thermocouples, 6- differential pressure sensors, 7- DC heating, 8- thermostatic bath, 9- absolute pressure sensor).

(from 1 to 100 cm) and variable inner diameters can be tested. After exiting the microtube, the gas is vented to the atmosphere. The total pressure drop between the inlet and the outlet of the microtube is measured by a differential pressure transducer (*6, Validyne DP15*). To measure the temperature distribution along the external surface of the microtube, five calibrated K-type thermocouples are glued to the outer surface of the microtube using an electrically non-conductive epoxy resin. In addition, at the inlet and outlet of the channel two additional K-type thermocouples are carefully inserted into the manifolds. The microtube is heated by Joule effect by means of a programmable DC power supply (*7, HP6032A*) in order to impose a uniform heat flux along the walls of the tube (H boundary condition). Under H boundary condition, the microtube is thermally insulated at the external surface (Armaflex®, $k_{in}$=0.035 W/mK) and enclosed in an insulated box in order to decouple the test section from the thermal conditions existing in the room. In order to obtain a T boundary condition the microtube is submerged in a thermostatic bath (*8, EYELA NCB-1200*) which circulates flowing water at constant temperature. An absolute pressure sensor (*9, Validyne AP42*) is used to monitor the ambient pressure at the exit of the test rig.

Four stainless steel microtubes having different inner diameters have been selected for the experiment. The main geometrical characteristics of the tested microtubes and the applied thermal boundary conditions are listed in Table 1, where *d* is the inner diameter of microtube and *L* the length. Uniform heat flux boundary condition is applied to microtube #1 and #2, while microtube #3 and #4 are analyzed under uniform wall temperature.

Table 1 Geometrical characteristics and thermal boundary conditions of the tested microtubes.

| Microtube | *d* (μm) | *L* (cm) | *L/d* | *D* (μm) | Thermal boundary condition |
|---|---|---|---|---|---|
| #1<br>#2 | 750<br>170 | 50<br>10 | 667<br>588 | 1588<br>1588 | Uniform heat flux (H) |
| #3<br>#4 | 1000<br>500 | 5<br>5 | 100<br>50 | 1250<br>807 | Uniform wall temperature (T) |

## 2.2. Numerical model

The experimental setup allows to measure microtube wall temperature and fluid inlet/outlet temperature. However, it is generally challenging to measure fluid local temperature inside microtubes without disturbance of flow field. In order to gain insight of fluid local temperature development inside microtubes, a numerical calculation based on the Arbitrary-Lagrangian-Eulerian (ALE) method has

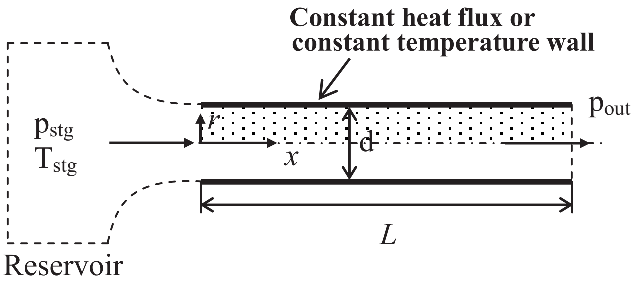


Figure 2. Computation domain and reservoir considered in numerical model.

been performed by solving two-dimensional compressible momentum and energy equations. The problem is modeled by considering microtubes subject to either a constant heat flux or constant temperature at walls as shown in Fig. 2. The governing equations are identical to those documented in our previous work [19] and will not be detailed here.

## 3. DATA REDUCTION

In order to determine the axial trend of the local convective heat transfer coefficient between gas flow and heated wall of the microtube, the bulk fluid temperature at each location along the microtube is needed. This quantity is not easy to measure in a tube having an inner diameter smaller than 1 mm. On the contrary, it is comparatively easier to deduce the mean value of the convective heat transfer coefficient along the whole microtube. The Nusselt number can be calculated by using the following equation:

$$Nu_m = \frac{hd}{k_f} \tag{1}$$

where $h$ is the average convective heat transfer coefficient, $d$ the hydraulic diameter of the microtube, and $k_f$ the fluid thermal conductivity calculated at the fluid average bulk temperature.

If the microtube is isothermal (T boundary condition), the average convective heat transfer coefficient can be calculated by:

$$h = \frac{\dot{m}c_p}{A}\ln\left(\frac{T_w - T_{b,in}}{T_w - T_{b,out}}\right) \tag{2}$$

where $\dot{m}$ is the mass flow rate, $c_p$ is the gas specific heat capacity at the average bulk temperature, $A$ is the heat transfer area which coincides with the microtube inner surface, $T_{b,in}$ and $T_{b,out}$ are the values of the gas temperature measured by thermocouples inserted in the inlet and outlet plenums of the test section, and $T_w$ is the average value of the inner wall temperature along the microtube.

If the flow is heated by uniform heat flux (H boundary condition) on the wall, the overall convective heat transfer coefficient is determined by the following parameters:

$$h = \frac{\dot{q}}{T_w - \overline{T}_b} \tag{3}$$

where $\overline{T}_b$ is the gas mean bulk temperature averaged from the inlet to outlet of microtube, and $\dot{q}$ is the heat flux on the inner wall.

The heat flux can be calculated by means of an energy balance between the inlet and the outlet of the microtube:

$$\dot{q} = \frac{\dot{m}c_p\left(T_{b,out} - T_{b,in}\right)}{A} \tag{4}$$

By considering the order of magnitude of the wall heat flux imposed by Joule heating on the external surface of the microtubes, the maximum temperature difference between the external and internal surfaces of the microtubes has been estimated to be of the order of 0.15 K for stainless steel ($k_w$=15 W/mK) by using the classical expression of the thermal resistance of an annulus. This temperature difference is smaller than the typical uncertainty of the thermocouples. Therefore, it is possible to replace the wall internal surface temperature by the external surface temperature with negligible influence on the Nusselt number. The average temperature of the wall external surface is directly determined by the measurement of a series of K-type thermocouples attached to the microtubes.

It has been demonstrated that due to strong axial conduction in the microtube wall under H boundary condition, the bulk temperature is not linearly distributed from the inlet to the outlet [20]. Therefore the actual mean bulk temperature is not simply estimated by algebraic average but obtained by a curve fitting method which is detailed in [20].

To assess the accuracy of the experimental Nusselt numbers presented in this paper, the uncertainty associated to each measurement device used in the test rig is reported in Table 2. The uncertainty on the inner diameter evaluated through SEM imaging is estimated to be of the order of ± 2%. The uncertainty on the microtube length is of the order of ±0.3%. By considering the typical uncertainty of each measured parameter, the estimated relative uncertainty of the experimental Nusselt numbers is 8%-53% under H thermal boundary condition and 1%-42% under T thermal boundary condition.

Table 2 Characteristics and accuracy of instrumentation.

| Instrument | Range (0-FS) | Accuracy |
|---|---|---|
| Flowmeter (Bronkhorst EL-Flow E7000) | 0-5000 (Nml/min) | ±0.6 % FS |
| | 0-500 (Nml/min)<br>0-50 (Nml/min) | ±0.5 % FS |
| Flowmeter (KOFLOC 3300) | 0-1000 (Nml/min)<br>0-5000 (Nml/min) | ±1.0 % FS |
| Flowmeter (KOFLOC 3100) | 0-5000 (Nml/min) | ±1.0 % FS |
| Pressure transducer (Validyne DP15) | 0-35 (kPa)<br>0-86 (kPa)<br>0-220 (kPa)<br>0-860 (kPa)<br>0-1440 (kPa) | ±0.5 % FS |
| Thermocouple (K type) | 0-200 (°C) | ±0.25 % FS |
| Thermostatic bath (EYELA NCB-1200) | -35 (°C) – 95 (°C) | ±0.1 °C |

## 4. RESULTS AND DISSCUSION

### 4.1 Main scaling and micro effects

The flow and heat transfer characteristics in the present tests may be influenced by the main scaling and micro effects, such as thermal entrance effects, gas compressibility, conjugated heat transfer between tube wall and fluid, and gas rarefaction effects. It is possible to verify whether these effects can be considered negligible or not in the range of operating conditions adopted during the experimental tests by recalling the definition of the Graetz number (*Gz*, thermal entrance effects), the outlet Mach number ($Ma_{out}$, compressibility effects), conduction parameter ($\lambda$, conjugate effects) and Knudsen number ($Kn$, rarefaction effects):

$$
\begin{aligned}
&Gz = \mathrm{RePr}_f \frac{d}{L} \\
&Ma_{out} = \frac{4\dot{m}}{\pi d^2 p_{out}} \sqrt{\frac{RT_{b,out}}{\gamma}} \\
&\lambda = \frac{\dot{q}_w}{\dot{q}_w} = \frac{k_w}{k_f}\left(\frac{D^2 - d^2}{dL}\right)\frac{1}{\mathrm{RePr}_f}\frac{T_{w,out} - T_{w,in}}{T_{f,out} - T_{f,in}} \\
&Kn_{out} = \frac{Ma_{out}}{\mathrm{Re}}\sqrt{\frac{\gamma\pi}{2}}
\end{aligned}
\tag{5}
$$

where $\mathrm{Pr}_f$ is the fluid Prandtl number and $\gamma$ is the gas specific heat ratio.

The values of these dimensionless parameters in each single test are calculated and listed in Table 3. If the experimental values are larger than the threshold values shown in Table 3, the relevant effect may influence the gas flow and heat transfer behavior in the present test.

For incompressible flow in laminar regime, the influence of flow entrance on convective heat transfer can be checked by the Graetz number. It can be noted from Table 3 that for laminar flows through microtube #3 and #4, the values of Graetz number exceed the threshold value. Therefore the gas flows through these two microtubes are not fully developed in most cases, and this fact may cause significant influence on the convective heat transfer coefficient.

For microtube #1 and #2, the Graetz number is always smaller than the threshold value (Gz =10) for all the test conditions. However, this does not ensure that the flow is fully developed due to the fact that the gas flow may accelerate noticeably if the outlet Mach number is larger than 0.3. As in the case of microtube #2 in laminar regime, although the flow becomes fully developed after flowing over a short length into the microtube, near the outlet the fully developed velocity profile is breached by the flow compressibility effects. For laminar flow through microtube #1, it can be concluded that the influence of flow entrance effect and compressibility effect on convective heat transfer can be neglected.

The compressibility effect induces flow acceleration inside the microtube and causes larger pressure drops. It has been demonstrated both experimentally [21] and numerically [22] that the influence of compressibility on the flow dynamics, in terms of pressure drop or friction factor, becomes significant when the outlet Mach number exceeds 0.3. Therefore in Table 3 the outlet Mach number of each flow is calculated. The influence on flow dynamics caused by the compressibility effect may further influence the thermal aspect of the flow, due to the conversion of internal energy into kinetic energy during the flow acceleration process. However, it is unknown whether the influence on convective heat transfer becomes noticeable only when the outlet Mach is larger than the same threshold as for pressure drop (0.3), or the thermal influence of compressibility effect is at different scale from its hydrodynamical influence and a threshold of larger/smaller value is needed in order to fully take it into account. From Table 3 it can be noted that the outlet Mach number of flow through microtube #2 is generally large compared with that of the rest microtubes. Especially in transitional regime, the outlet Mach number is always larger than 0.3. It will be examined in the paper under such condition whether the Nusselt number is higher than that of incompressible flow or not.

The wall-fluid conjugate heat transfer is caused by axial heat conduction in the solid wall and can be measured by the conduction parameter. It is clear in Table 3 that due to the thick walls of microtube, the axial conduction is very strong especially in microtube #2. This leads to non-linear distribution of fluid temperature under H thermal boundary condition. In order to avoid such strong axial conduction in the solid wall, in the present study T thermal boundary conditions are also applied and tested. Under T boundary condition there is no temperature difference along the microtube wall and no wall axial conduction is activated.

In gas microflow the rarefaction effect may be involved if the Knudsen number is larger than 0.001. In all the present tests the values of Knudsen number remains smaller than this threshold, which means in the present study the influence of the gas rarefaction on the convective heat transfer coefficient can be neglected.

Table 3 Typical ranges of the non-dimensional parameters linked to the main effects for the tested microtubes.

| Microtube | Re | Gz (<10) | $Ma_{out}$ (<0.3) | $\lambda$ (<0.01) | $Kn_{out}$*1000 (<1) |
|---|---|---|---|---|---|
| #1 | 492-1824 (laminar) | 0.5-2.0 | 0.03-0.11 | 0.012-0.002 | 0.42-0.19 |
| | 2366-4001 (transitional) | - | 0.14-0.23 | 0.008-0.001 | 0.04-0.02 |
| #2 | 461-2238 (laminar) | 0.6-2.7 | 0.14-0.53 | 0.24-0.05 | 0.45-0.35 |
| | 2907-3349 (transitional) | - | 0.62-0.67 | 0.04-0.03 | 0.30-0.32 |
| #3 | 282-2284 (laminar) | 4.1-32.9 | 0.01-0.10 | 0 | 0.06-0.05 |
| | 2444-5371 (transitional) | - | 0.11-0.24 | 0 | 0.07-0.07 |
| #4 | 483-2247 (laminar) | 3.5-16.2 | 0.05-0.21 | 0 | 0.15-0.14 |
| | 2360-5794 (transitional) | - | 0.22-0.54 | 0 | 0.14-0.14 |

## 4.2. Thermal entrance effect

As discussed before, microtube #1 and #2 have relatively small Graetz number which indicates fully developed flow along most length of the microtube. However, the larger values of Graetz number in the case of microtube #3 and #4 suggest that the entrance region can take up a major part of the microtube total length.

In the entrance region the convective heat transfer coefficient is larger than that in the fully developed region. In laminar regime the value assumed by the convective heat transfer coefficient along the fully developed region is constant and depends only on the geometry of the channel cross-section. Therefore, the effect of thermal entrance region cannot be neglected if the length of this region is large compared to the total length of the microtube. For incompressible flow in laminar regime the thermal entry length can be calculated as [23]:

$$L_{thermal} = 0.034\,\mathrm{Re}\,\mathrm{Pr}_f\, d \tag{6}$$

where Re is the Reynolds number, $\mathrm{Pr}_f$ the Prandtl number of the fluid and $d$ the microtube inner diameter.

The thermal entry length non-dimensionalized by microtube total length has been calculated for nitrogen flows through the four microtubes listed in Table 1. As Eq. (6) is only valid for laminar and incompressible flows, microtube #2 listed in Table 3 is only considered for Reynolds number smaller than 1020, beyond which the outlet Mach number exceed 0.3 and the flow acceleration becomes significant. The results are shown in Fig. 3, from which it can be noted that faster flows lead to longer thermal entrance length. For microtube #1 and #2, the thermal entrance length is generally smaller than 10% of the tube total length. However, in the case of microtube #3 and #4, the thermal entrance length exceeds 10% of the tube length even when the Reynolds number is small. Especially for microtube #3, the thermal entrance length can be longer than the tube total length when the flow Reynolds number is larger than 2000. This indicates that the flow is thermally developing from the inlet through to the outlet of the microtube, and the thermally developed flows can never be reached under such circumstances. As a result, the effects of thermal development should be carefully considered in the calculation of the average value of the convective heat transfer coefficient.

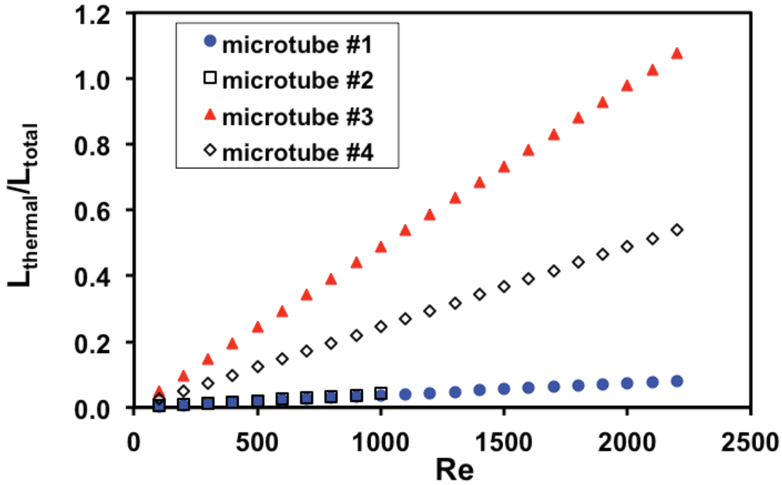
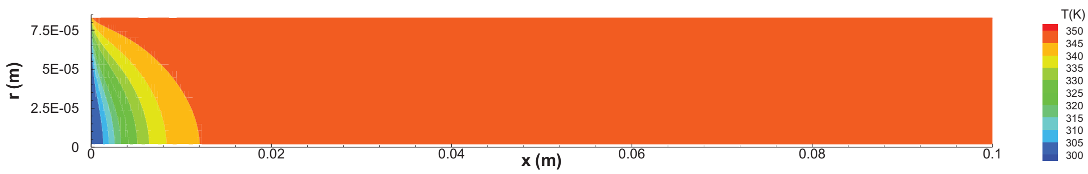
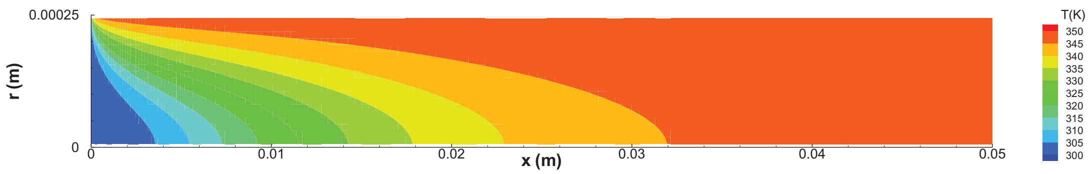



Figure 3 Dimensionless thermal entrance length of the four microtubes as a function of Reynolds number.

This issue can become critical as vast majority application of microdevices employ relatively short microchannels to avoid large pumping pressure. Fig. 4 shows how the fluid temperature develops along microtube #2 and #4 which have constant wall temperature. For both microtubes the wall temperature is fixed at 350 K and the Reynolds number is set to be 490. It can be noted that the fluid temperature in microtube #2 becomes fully developed after a short flow length; while in microtube #4 the fluid temperature is still developing when the flow passes half the tube length. This trend is in agreement with the data presented in Fig. 3. It should be clarified that the prediction by Eq. (6) only considers the fluid thermal entry length where the velocity profile is assumed fully developed at the inlet. In our simulation model the flow experiences simultaneous development of both temperature and velocity after it enters microtube, which is exactly the situation of experimental tests.




(a)




(b)

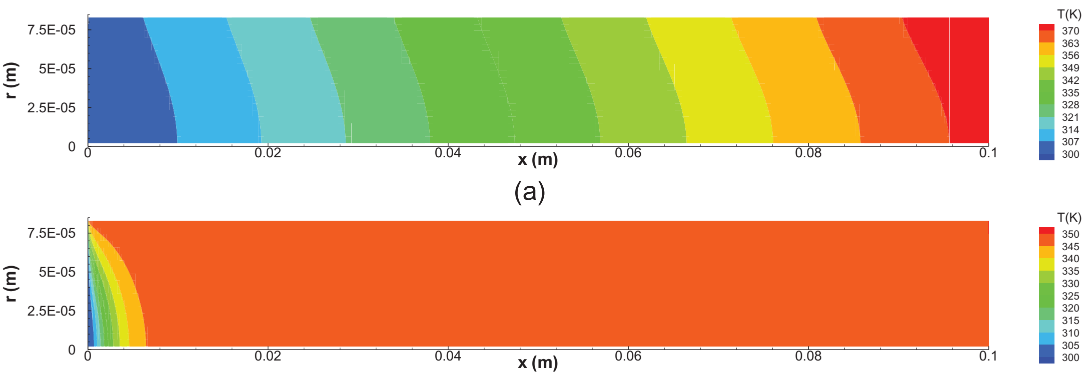
Figure 4 Fluid local temperature development through (a) microtube #2 and (b) microtube #4.

### 4.3. Micro convection temperature contour

Further understanding of the fluid temperature distribution inside microtubes may shed light on how to perform experimental measurement at microscale while ensuring measurement accuracy. Hence we numerically applied both H and T thermal boundary conditions to the same microtube (microtube #2). In the numerical model the flow Reynolds number is kept at 290 and fluid inlet temperature is set to be 300 K. In the case of H boundary condition a uniform heat flux of 1000 W/m$^2$ is imposed on microtube inner surface; while under T boundary condition the temperature of microtube inner surface is set at 350 K.

Fig. 5 shows the fluid temperature contour in both cases. It is noticeable that a simple change in microtube thermal boundary condition leads to striking difference in fluid temperature distribution. More specifically, under H boundary condition the flow is continuously heated by microtube wall as long as it proceeds along the tube. However, by switching to T boundary condition the fluid temperature increases substantially when it enters microtube. As a result, the fluid bulk temperature reaches microtube wall temperature merely after a short flow length. This implicates that the major length of microtube is actually ineffective in heat transfer, as the temperature gradient across microtube wall becomes zero.

If the large area of non-heat exchange zone is included in the total heat transfer area and used for experimental determination of Nusselt number, then the value of Nusselt number can be greatly underestimated. This probably explains why “anomalously low values” of Nusselt number are experimentally obtained by Demsis et al. [24] and Choi et al. [13], who applied T thermal boundary conditions to gas forced convection. The experimental underestimation of Nusselt number under T boundary condition is more serious when the flow Reynolds number is smaller. Due to this reason, in our experimental test of forced convection subject to T boundary condition, microtubes of smaller length or larger diameter are used in order to ensure there is still temperature difference between fluid and wall at microtube outlet. By satisfying this condition, the total area of tube inner surface is effective in terms of heat exchange.




(b)

Figure 5 Temperature contour of flow through microtube #2 heated by uniform heat flux (a) and uniform wall temperature (b).

### 4.4. Nusselt number under H boundary condition

The experimental values of Nusselt number as a function of Reynolds number are plotted in Fig. 6 for microtube #1 and #2, which are heated by a uniform heating power of 0.6 W and 1.2 W, respectively. In Fig. 6 the Nusselt number is calculated by careful consideration of the effect of non-linear axial bulk temperature distribution caused by wall-fluid conjugate heat transfer, as discussed in [20]. Fig. 6 also shows the correlations developed by Gnielinski [18] for macro-tubes, and the correlations of Wu and

Little [9], Choi *et al.* [13] proposed specifically for microtubes. It is interesting to note that the trend of the Nusselt number versus Reynolds number is not in agreement with any of the latter, neither in laminar regime nor in transitional regime. However, Gnielinski's correlation obtained from tests of convectional convection tends to predict the present experimental data with fairly satisfying accuracy in both laminar and transitional regimes, considering the large error bars of experimental data at very low mass flow rate (small Reynolds number). Choi's correlation proposed for the prediction of the Nusselt numbers in laminar regime for microtubes strongly underestimates the Nusselt number and, for this reason, it is not recommended.

By comparison between the data obtained from microtube #1 (d= 0.75 mm) and #2 (d= 0.17 mm), it can be noted that the reduction in the tube inner diameter does not influence the non-dimensional heat transfer coefficient of internal forced convection. This means that the convective heat transfer is not expected to be either enhanced or degraded in the wide spreading application of microfluidics devices. However, it should be highlighted that this conclusion is drawn under the condition that the experiment is performed by careful consideration of measurement uncertainties, which can be very challenging for measurements accomplished at micro scale. Fig. 6 shows the experimental uncertainty of Nusselt numbers. The relative uncertainty of Nusselt number can reach 34% for microtube #1 and 53% for microtube #2 at low Reynolds numbers. This observation clarifies how the increase of the experimental Nusselt number evidenced by microtube #1 for Re<600 is mainly due to the uncertainty on the experimental data. The large experimental uncertainty also makes it impossible to tell from Fig. 6 whether the convective heat transfer is improved or not by flow compressibility effect, compared with incompressible flow as predicted by Gnielinski's correlation. Or it can be concluded that, the influence of compressibility effect (especially in the case of microtube #2 which is characterized by large outlet Mach number) on heat transfer is less predominant than its influence on pressure drops.

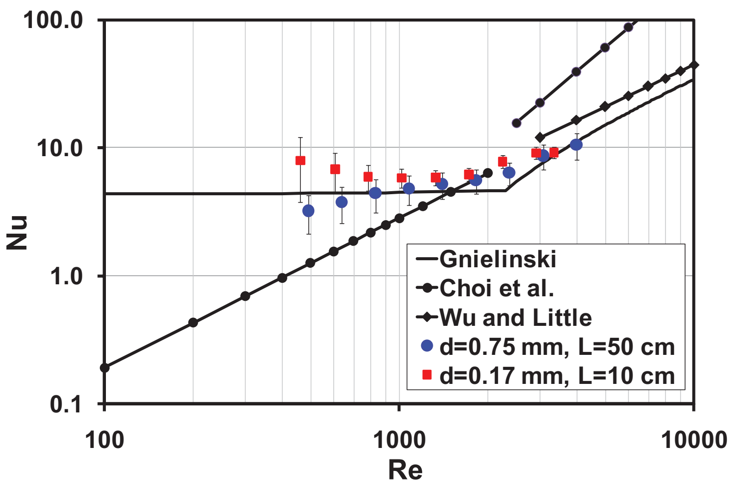



Figure 6 Experimental Nusselt number of nitrogen (Pr= 0.72) convection through microtube #1 and #2 under H boundary condition.

### 4.5. Nusselt number under T boundary condition

As H and T thermal boundary conditions are frequently encountered in heat transfer process, in the present study nitrogen convection through microtubes with constant wall temperature is also tested by using commercial stainless steel microtubes. The microtubes are submerged in the running water from the thermostatic bath which regulates the temperature at 5 °C, 40 °C, 60 °C or 80 °C. The temperature difference between the internal surface and the external surface of the microtube is estimated to be smaller than 0.15 °C. Therefore, the temperature measured by thermocouples attached to the external surface of microtube is assumed to be equal to that of the internal surface.

Fig. 7 shows the experimental values of Nusselt number for nitrogen convection through microtube #3. The test has been repeated by fixing the wall temperature at three distinctive values, and no dependence of Nusselt number on the wall temperature has been observed. The correlations proposed by Wu and Little [9], and Choi et al. [13] for micro convection again fail to predict the experimental trend and values of Nusselt number. On the contrary, the experimental data are in agreement with the conventional theory as proposed by Gnielinski [18], especially in the transitional regime.

In laminar regime the experimental value of Nusselt number is not constant at 3.66 but increases with the increase of Reynolds number. This is due to the large thermal entrance length in the case of microtube #3. As indicated in Fig. 3, microtube #3 has the largest ratio of thermal entrance length to the total length among the four microtubes for the Reynolds numbers. In the thermal entrance region the heat transfer is stronger than that in the thermally developed region. As the Reynolds number increases, the thermal entrance region takes up larger percentage of the microtube total length, which means the enhanced heat transfer occurs in a relatively larger part of the flow path. Therefore, under such circumstances the Nusselt number becomes larger as the Reynolds number increases.

The correlations from Wu and Little [9], and Choi et al. [13] only consider thermally fully developed flows. However, Gnielinski's correlation takes into account the effect of thermal entrance region and is declared applicable to the situation in which the $L/d$ value can be as low as 1. In the case of microtube #3 the $L/d$ value is equal to 50, and the influence of thermal entrance effect is clearly indicated by the ascending curve of Gnielinski correlation in Fig. 7. By comparing the three sets of experimental data against the conventional theory, it turns out that the theory tends to slightly overestimate the effect of thermal entrance region in laminar regime (Re<1600), although the agreement is quite good in transitional regime (Re>2000).

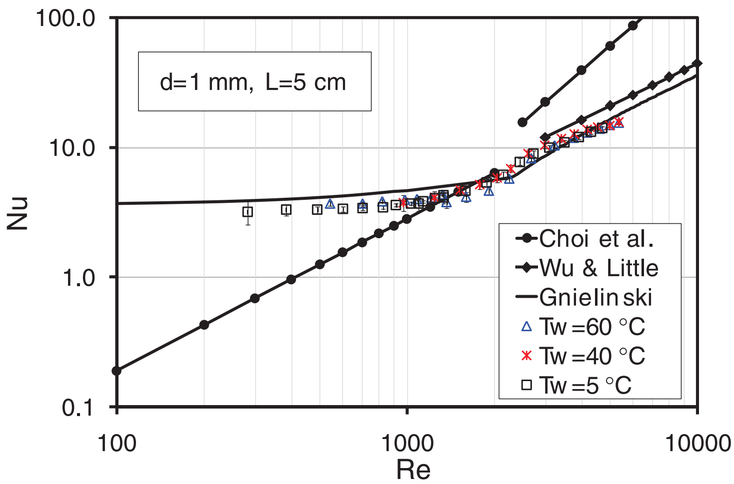



Figure 7 Experimental Nusselt number of nitrogen (Pr= 0.72) convection through microtube #3 under T boundary condition.

The experimental results obtained from microtube #4 are plotted in Fig. 8. The data show similar observation to microtube #3 that the wall temperature does not influence the convective heat transfer. The experimental data are in very good agreement with the prediction of Gnielinski correlation in both laminar and transitional regimes. By comparing the results presented in Fig. 7 and Fig. 8 it shows how the microtube geometry ($L/d$) can influence the convective heat transfer characteristics. The comparison seems to confirm that in laminar regime the Gnielinski correlation works better for flows that are more thermally developed. In transitional regime the length of thermal entrance region is greatly reduced due to existence of vortexes which enhance the development of bulk temperature profile. Therefore, in transitional regime there is smaller difference between the experimental results from microtube #3 and the ones from microtube #4.

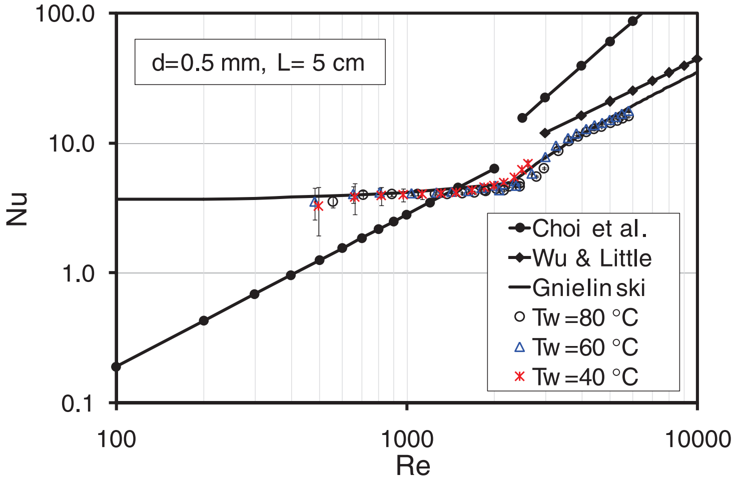



Figure 8 Experimental Nusselt number of nitrogen (Pr= 0.72) convection through microtube #4 under T boundary condition.

## 5. CONCLUSIONS

Convective heat transfer of nitrogen flows in laminar and transitional regimes has been experimentally and numerically investigated in this work by applying both H and T thermal boundary conditions to stainless steel microtubes. The main conclusions of this work are summarized as follows:

- In the case of short microtubes ($L/d$<50) thermal entrance length can be comparable to tube length, especially at large values of Reynolds number. In the case of long microtubes ($L/d$>500) the application of T boundary condition may lead to ineffective heat transfer area in which fluid temperature becomes equal to wall temperature.
- The conventional theory on convective heat transfer, as represented by Gnielinski's correlation, has been experimentally verified to be able to well predict the Nusselt number of gas micro convection under both H and T boundary conditions.
- Besides fully developed flows, Gnielinski's correlation can also be well applied to thermally developing micro flows. However, for laminar flows that are far from being thermally developed ($L/d$<50), Gnielinski's correlation tends to slightly overestimate the heat transfer coefficient of micro convection.
- The correlations proposed by Wu and Little [9], and Choi et al. [13] on the prediction of gas micro convection, may be applicable to their specific cases decades ago but are not able to characterize the main features of gas micro convection under either H boundary condition or T boundary condition.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## NOMENCLATURE

| | | |
|---|---|---|
| $A$ | Heat transfer area | ($m^2$) |
| $c_p$ | Specific heat capacity | (J/kgK) |
| $d$ | Inner diameter | (m) |
| $D$ | External diameter | (m) |
| $h$ | Heat transfer coefficient | ($W/m^2K$) |
| $k$ | Thermal conductivity | (W/mK) |
| $L$ | Microtube total length | (m) |

| | | |
|---|---|---|
| $L_{th}$ | Thermal entrance length | (m) |
| $\dot{m}$ | Mass flow rate | (kg/s) |
| Nu | Nusselt number | (-) |
| $Pr$ | Prandtl number | (-) |
| $\dot{q}$ | Heat flux | (W/m$^2$) |
| Re | Reynolds number | (-) |
| $T$ | Temperature | (K) |

**Greek symbols**

| | | |
|---|---|---|
| $\gamma$ | Gas specific heat ratio | (-) |

**Subscript**

| | |
|---|---|
| b | bulk fluid |
| in | inlet value |
| m | mean value |
| out | outlet value |
| w | wall |

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