Numerical Methods to Determine Convective Heating Rates on Aerodynamic Surfaces

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Abstract

The approach for the present work is to develop a one-dimensional heat conduction model to infer the surface heating rates from the temperature data. The temperature history is obtained from a nickel thin film sensor mounted on a quartz substrate during a supersonic flight test. Polynomial curve fitting with regression analysis and cubic spline methods are used to fit the temperature data. One-dimensional numerical schemes are developed to infer surface heating rates by using Duhamel's superposition integral. Since the temperature data are acquired for 10 s, the one-dimensional behavior of heat penetration might not be applicable for entire time scale. In order to include the lateral conduction of heat along the depth of substrate, finite-element analysis of a more realistic gauge-substrate system is carried out with commercial package ANSYS 11. With the inputs of surface heating rates predicted from Duhamel's superposition integral, the temperature history are then recovered at various depths of the substrate and on the surface. The surface temperature history recovered from FE analysis compares well with experimental temperature history up to a time scale of 4 s. Also, numerical results for the representative problems show that the surface heat flux can be predicted well by the cubic spline method.

Keywords: Thin film gauge, cubic spline, duhamel's superposition integral

1. INTRODUCTION

Aerodynamic heating is one of the fundamental issues in the design of hypersonic flight. In general, there is no direct method to determine the convective surface heating rates on the aerodynamic surfaces rather they are obtained from transient temperature data. They are usually obtained by various types of temperature sensors (such as thin film gauges, thermocouples etc) embedded at desired places on the aerodynamic surface. So, accurate prediction of surface heating is based on appropriate mathematical modeling of a realistic gauge-substrate system.

Several methods have been developed for interpretation of heat transfer rates obtained from temperature data. Cook and Felderman (1966) presented a concise numerical technique to calculate the transient heat flux by using piecewise linear function of the transient temperature history. Mehta et al. (1988) investigated the influence of normal and lateral conduction on the temperature distribution and heat transfer co-efficient on the surface of a typical sounding rocket with finite-element technique. Babinsky and Edwards (1996) developed a technique to measure surface heat transfer on models in hypersonic flow based on the colour response of encapsulated thermo-chromic liquid crystals. The local heat transfer from the liquid crystal response is obtained by identifying suitable colour and comparing them with the calibration curve. Yvonnet et al (2006) developed a simple inverse procedure to identify the heat flux from temperature history in a typical orthogonal cutting process.

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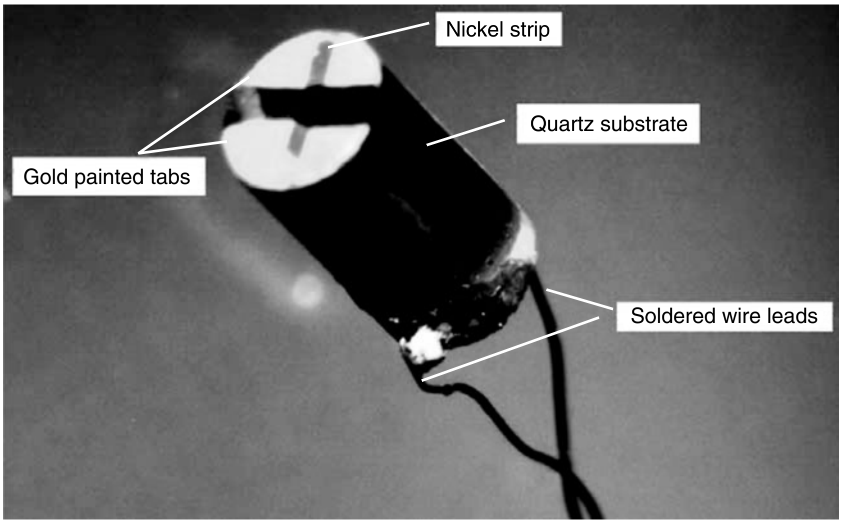

Figure 1. Main features of a thin film gauge (Sahoo 2008).

Most of the high speed flight experiments are performed in the shock tunnels where the test flow duration is in the range of 1ms. The transient temperature data are obtained by using thin film gauges during the test flow duration. These gauges are resistance temperature detectors (RTD) as shown in Fig. 1. The thin film (thickness in the order of few microns) is made out of temperature sensitive materials and is deposited on an insulated substrate (Lyons and Gai 1988; Sahoo 2003). In most of the cases, the sensing material is nickel/platinum and the substrate material is quartz/MACOR. It ensures the fact that during the experimental time scale of 1ms, there is a very least chance of heat penetration along the depth of the substrate. Thus, the temperature recorded by the sensor is same as that on the surface of the substrate. As shown in Fig. 1, a nickel thin film is mounted on a quartz rod (2 mm diameter and 4 mm length) and the fine flexible wires are used for electrical connections from the sensor through gold painting across the sides of the sensor. Thin film gauges are powered by a constant current source. The thin film resistance (R) change across a thin-film with respect to temperature (T) change can be expressed as (Miller 1981),

$$R = R_0\left(1 + \alpha\left\{T - T_0\right\}\right) \tag{1}$$

where α is the temperature coefficient of resistance which is obtained during calibration of the gauge; R_0 is the initial gauge resistance corresponding to initial temperature of T_0.

Basically, the thin-film gauge operates on the principle that penetration of heat pulse is very much small compared to the thickness of the substrate material within the time span of the measurement. Therefore, the medium can be considered semi-infinite for the very short time period and the surface heat transfer rates can be measured from temperature history with one-dimensional heat transfer modeling with semi-infinite substrate (Cook and Felderman 1966; Schultz and Jones 1973; Beck 1985; Monde 2000). However, the extent (in terms of time scale) to which the semi-infinite assumption holds good, is still unknown. In the present work, one of the temperature signal (Fig. 2) obtained for 10 s, from a nickel thin film sensor mounted on a quartz substrate in supersonic flight test, is considered (Sahoo 2008). Mathematical techniques such as polynomial fitting and cubic spline methods are used to obtain the closed-form solution of temperature data. With known thermal properties of nickel and quartz, surface heating rates are obtained from Duhamel's superposition integral. In order to check the effects of multi-dimensional effects of heat penetration along the depth of the substrate, the gauge-substrate system is modeled through FE simulation. With the inputs of transient heating rates from the analytical analysis, the temperature history is obtained at various depths of the substrate.

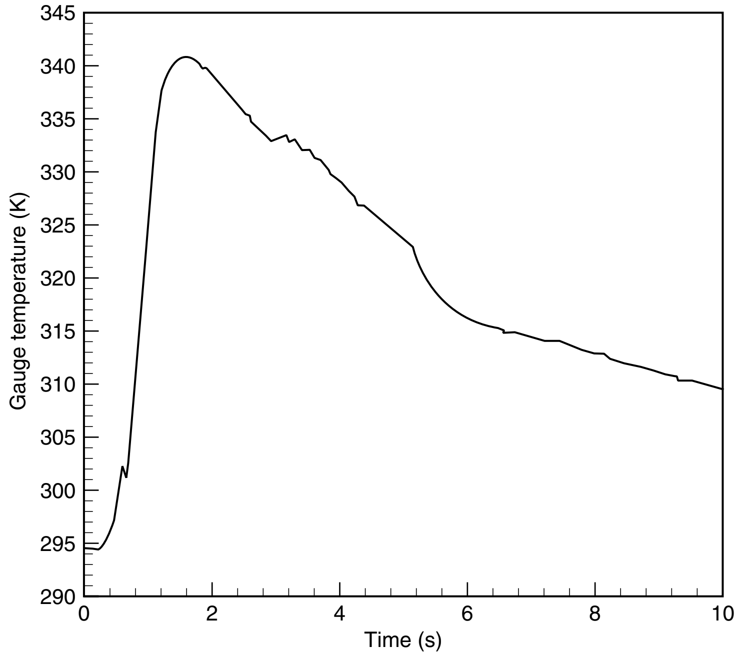

Figure 2. Absolute gauge temperature recorded from thin film gauge during supersonic flight test (Sahoo 2008).

2. ONE DIMENSIONAL HEAT CONDUCTION MODEL

Governing equation for surface heat transfer rate is based on one-dimensional, semi-infinite solid model (Fig. 3). The following assumptions are made in the development of the model; (i) temperature measured by sensing element is identical to the temperature of surface of the substrate; (ii) there is no lateral heat conduction through the substrate and heat is conducted only in the direction normal to the surface; (iii) thermal properties of the substrate are constant; (iv) the substrate is of infinite length and the temperature rise at infinity is zero.

Using these assumptions, the transient temperature distribution $T(t)$ along the depth of the substrate (x) can be written as;

$$\frac{\partial^2 T}{\partial x^2} = \left(\frac{\rho c}{k}\right)\frac{\partial T}{\partial t} \tag{2}$$

where ρ, c and k are the density, specific heat and thermal conductivity of the substrate material and for quartz, the values are 2200 kg/m^3, 670 J/kg.K and 1.4 W/m.K, respectively (Doorly and Oldfield 1986).

2.1. Duhamel's Superposition Integral

Most of the aerodynamic bodies flying at hypersonic speeds encounter a strong shock through which there is a sudden increase in the static pressure, temperature and density. In terms of heat transfer modeling, it can be treated as a step change in the surface temperature. Carslaw and Jaeger (1959) obtained the one-dimensional semi-infinite medium solution for a step change in the surface temperature with Duhamel's superposition integral and is given by the following equation,

$$q_s(t) = \sqrt{\frac{\rho c k}{\pi}} \int_0^t \frac{1}{\sqrt{(t-\tau)}} \frac{dy(\tau)}{d\tau} d\tau \tag{3}$$

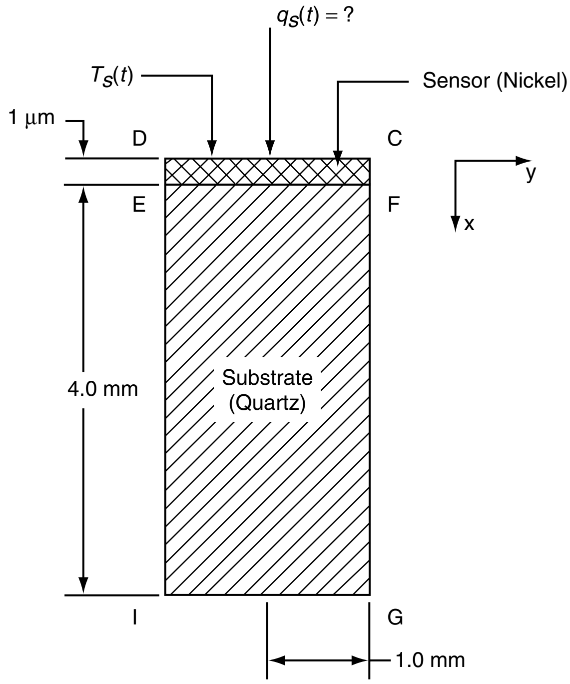

Figure 3. Schematic diagram of thin film gauge.

where $y(\tau)$ is closed form solution of measured temperature history. They can be obtained through appropriate curve fitting technique such as polynomial fitting and cubic-spline technique with regression analysis.

2.1.1. Least Square Curve Fitting

Polynomial regression data fitting technique can be applied to smooth the surface temperature time response (Jain *et al* 2003).

$$y(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + \ldots\ldots\ldots\ldots. + A_m t^m = \sum_{i=0}^{m} A_i t^i \tag{4}$$

Re-writing these $(i+1)$ equations and putting into matrix form,

$$\begin{bmatrix} n & \sum t_j & \sum t_j^2 & \ldots\ldots\ldots\sum t_j^i \\ \sum t_j & \sum t_j^2 & \sum t_j^3 & \ldots\ldots.\sum t_j^i \\ : & : & : & : \\ \sum t_j^i & \sum t_j^{i+1} & \sum t_j^{i+2} & \ldots\ldots\sum t_j^{i+i} \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ : \\ A_j \end{bmatrix} = \begin{bmatrix} \sum y_j \\ \sum(t_j y_j) \\ : \\ \sum(t_j^i y_j) \end{bmatrix} \tag{5}$$

where all summations above are over $j = 1,2\ldots\ldots\ldots m$ number of the data points. The data points (t_j, y_j) for $j = 1,2,3\ldots..m$, in the matrix are known from the experimental temperature history. So, the coefficients $A_0, A_1, \ldots..A_j$ can be determined by using matrix inversion method.

Substitution of the polynomial approximation (4) into equation (3) and its integration yields,

$$q_s(t) = 2\sqrt{\frac{\rho ck}{\pi}}\left\{\left(A_1\sqrt{t} + \sum_{i=2}^{m} iA_i t^{((2i-1)/2)}\right) \times (1+(i-1)!)\sum_{k=1}^{i-1}\frac{(-1)^k}{(2k+1)k!(i-1-k)!}\right\} \tag{6}$$

2.1.2. Cubic Spline Method

The other method to reproduce the temperature-time curve is by cubic-spline technique in which the small segments of the curve can be closely approximated. A spline is simply a piecewise polynomial in which the pieces are joined together at points called as, 'knots'. In particular, a cubic spline is a piecewise cubic polynomial, constructed in such a way that second derivative continuity is preserved at the knots (Reinsch 1967).

The experimental temperature is represented by a third-order spline in the form (Jan 1996);

$$\begin{aligned} &y_i(\tau) = a_{1,i} + a_{2,i}(\tau-\tau_i) + \frac{1}{2}a_{3,i}(\tau-\tau_i)^2 + \frac{1}{6}a_{4,i}(\tau-\tau_i)^3 \\ &\text{for } \ \tau_i \le \tau \le \tau_{i+1}, i = 1,2,.....,M \end{aligned} \tag{7}$$

where $\tau = s_t t$ is scaled time and s_t is the scaling factor. The four constants in the above equation can be obtained as;

$$\left.\begin{aligned} a_{1,i} &= y(\tau_i) \\ a_{2,i} &= y'(\tau_i) = \frac{dy_i(\tau_i)}{d\tau} = \frac{1}{s_t}\frac{dy_i(t_i)}{dt} \\ a_{3,i} &= y''(\tau_i) = \frac{d^2y_i(\tau_i)}{d\tau^2} = \frac{1}{s_t^2}\frac{d^2y_i(t_i)}{dt^2} \\ a_{4,i} &= y'''(\tau_i) = \frac{d^3y_i(\tau_i)}{d\tau^3} = \frac{1}{s_t^3}\frac{d^3y_i(t_i)}{dt^3} \end{aligned}\right\} \tag{8}$$

Starting with equation (3) and the assumption that the surface temperature response is approximated by cubic splines (7), the surface heat flux can be expressed as (Jan 1996);

$$\begin{aligned} q_s(\tau_{M+1}) &= \left[\sqrt{\frac{\rho ck}{\pi}}\sum_{i=1}^{M}\int_0^t \frac{1}{\sqrt{(t-\tau)}}\frac{dy(\tau)}{d\tau}d\tau\right]\sqrt{s_t} \\ &= \begin{bmatrix} 2\sqrt{\frac{\rho ck}{\pi}}\sum_{i=1}^{M-1}\left\{v_i\left(P_i^{1/2}-R_i^{1/2}\right) - \frac{W_i}{3}\left(P_i^{3/2}-R_i^{3/2}\right) + \frac{a_{4,i}}{10}\left(P_i^{5/2}-R_i^{5/2}\right)\right\} \\ +2\sqrt{\frac{\rho ck}{\pi}}\left(V_M P_M^{1/2} - \frac{W_M}{3}P_M^{3/2} + \frac{a_{4,M}}{10}P_M^{5/2}\right) \end{bmatrix}\sqrt{s_t} \\ &\text{for } M = 1,2,....(J-1) \end{aligned} \tag{9}$$

where,

$$P_i = \tau_{M+1} - \tau_i;\ R_i = \tau_{M+1} - \tau_{i+1};\ V_i = \frac{dy_i}{d\tau_{M+1}} = a_{2,i} + a_{2,i}P_i + \frac{a_{4,i}}{2}P_i^2;\ W_i = \frac{d^2y_i}{d\tau_{M+1}^2} = a_{3,i} + a_{4,i}P_i \tag{10}$$

2.1.3. Prediction of Surface Heating Rates

Temperature history obtained from the thin film sensor is fitted through least square regression analysis (polynomial fitting) and cubic spline method. Various degrees of polynomial are attempted and it is observed that 18-degree polynomial closely approximates the experimental temperature history. The major weakness of this technique is the low accuracy because it allows the calculation of high-order derivatives, especially when the time spread of the fitted data is large (Jan 1996). These restrictions can be avoided by using an alternate procedure based on cubic spline approximation of the temperature time data. Here, the smoothing spline is continuously differentiable and the second derivative also exits at the time interval under consideration. Fig. 4 shows the comparison of both the techniques with respect to experimental temperature history. With the closed-form temperature solution by both the techniques, the MATLAB code is developed using Eqs. (6) and (9) to infer the surface heating rates. The transient surface heating rates are then compared in Fig. 5. It is clear from the figure that the surface heating rates suddenly raises (after an initial delay of 0.3 s) to a peak value of 80 kW/m^2 within the time scale of 1.587 s. Similar, time scale is also observed from the temperature-time history (Fig. 4). Thus, it resembles closely to the nature of step rise in the heating rates at the surface of the sensor. However, the peak surface heat flux predicted by polynomial fitting is about 6% less compared to that of cubic-spline method. As discussed above in this section, the cubic spline method of fitting the temperature data is a better approximation while predicting the surface heating rates because it resembles the trend of experimental temperature data.

3. SIMULATION OF GAUGE-SUBSTRATE SYSTEM THROUGH FEM

The methods discussed in Sec. 2, mainly deals with one-dimensional semi-infinite medium solution for the governing Eq. (2). During the short time scale measurement (~1 ms), it is reasonable to assume piece-wise linear fitting of temperature data while inferring the surface heating rates (Cook and Felderman 1966; Schultz and Jones 1973; Sahoo 2003). When extended to larger time scale (say 10 s), the effects of

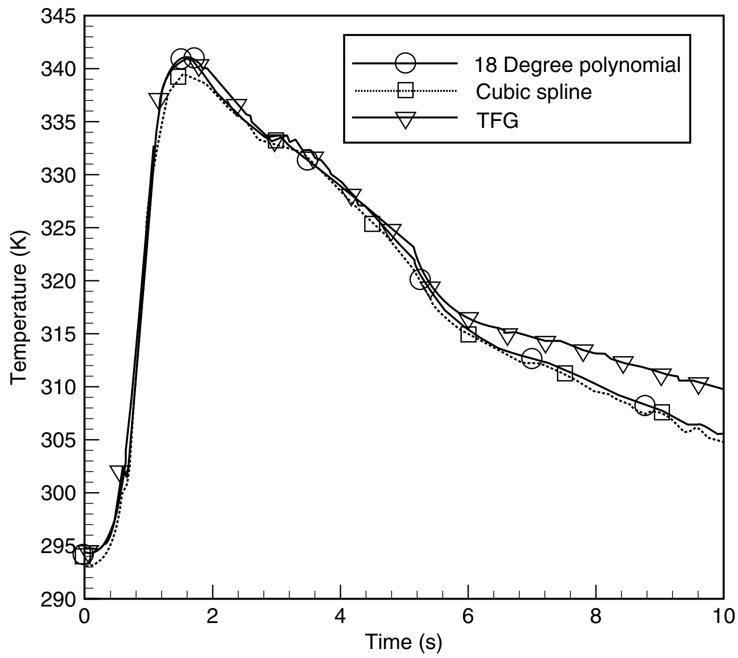

Figure 4. Curve fitting techniques of temperature-time data.

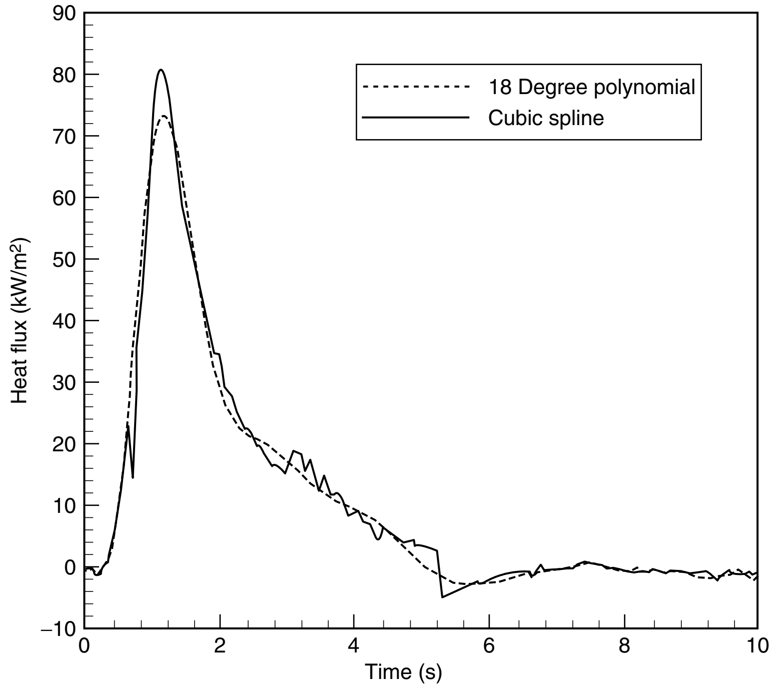

Figure 5. Prediction of convective surface heating rates from temperature history (one dimensional semi-infinite solution).

lateral heat conduction can be quite significant. However, the extent to which the one-dimensional assumption holds well is still unknown. One of the approaches is to develop a finite-element model of a realistic gauge-substrate system, apply the transient surface heat flux obtained through one-dimensional analysis (Eqs. 6 and 9), recover temperature history $\{T_S'(t)\}$ and then compare it with experimental signal. In this work, the commercial finite element analysis package, ANSYS 11.0, is used to simulate the phenomenon of heat transfer of a gauge-substrate (nickel-quartz) system. The flow chart for this procedure is shown in Fig. 6 where the recovered temperature history $\{T_S'(t)\}$ is compared with experimental time history of temperature $\{T_S(t)\}$ obtained for flight test.

3.1. Finite Element Analysis

Since thickness of the thin film gauge is very small (~1 μm), the thermal resistance offered by the film for the flow of heat is very small. Therefore only substrate is modeled for finite element analysis with appropriate boundary conditions as shown in Fig. 7. As shown in the figure, variable heat flux is applied on the upper surface EF while lower boundary GI is maintained at constant temperature of 294 K. The surfaces EI and FG are taken as adiabatic walls. The FE mesh (Fig. 8) is generated by quadratic 8-node element (PLANE 77) each with length of 0.0001mm. The element has one degree of freedom at each node. The 8-node elements have compatible temperature shapes and are well suited to model curved boundaries in transient studies. In order to capture the sudden temperature change, finer mesh is used at the upper boundary wall.

3.2. Analytical Results vs FE Analysis

With appropriate boundary conditions, the simulation is carried out with the surface heating rates (Eqs 6 and 9; Fig. 5) as an input applied at the upper surface (EF) in the FE model. In both the cases

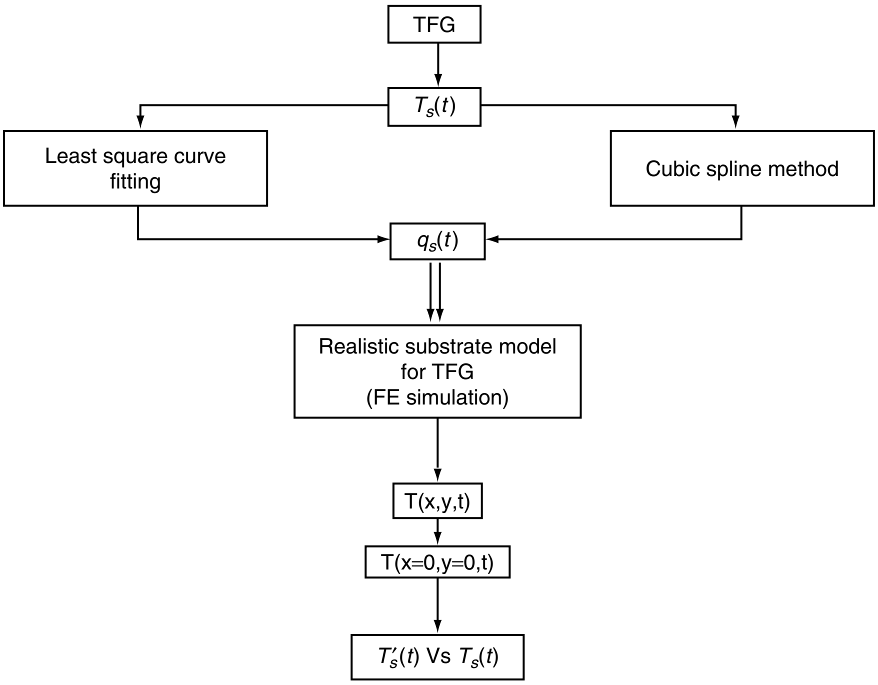

Figure 6. Flow chart for finite element simulation of gauge-substrate system.

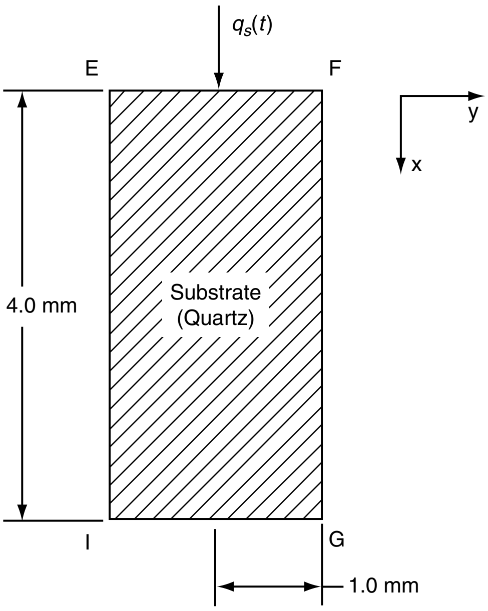

Figure 7. Quartz substrate used for finite element modeling in ANSYS 11.

of heat input, the transient temperature variation along the depth of the substrate is obtained (Fig. 9). From these two figures, it is observed that both the methods almost reproduce same surface temperature history compared to the experimental signal. At a closer look in experimental temperature history (Fig. 2), it is seen that there is a discontinuity in surface temperature appearing at time 0.7 s. Such discontinuity is only appearing in the recovered temperatures (Fig. 9-a) with FE simulation

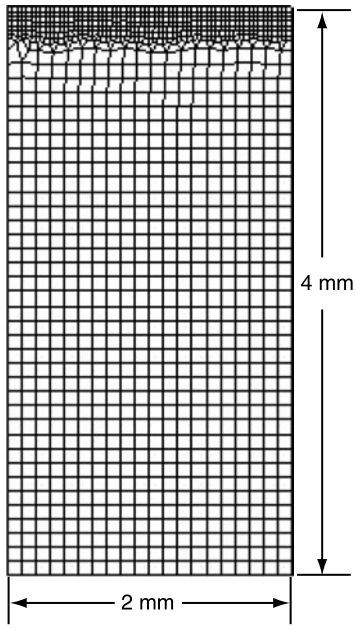

Figure 8. Finite element mesh for the substrate system in ANSYS 11.

almost exactly at same time when the heat input is applied corresponding to cubic-spline fitting of temperature data. Fig. 9-b does not show discontinuity in surface heating rates. So, as discussed in Sec. 2.2.3, cubic-spline method of fitting the temperature data is a better approximation in predicting surface heating rates.

Another close look in Figs. 9 reveals that the thermal penetration into the depth of the substrate is very less up to 2.5 s but it goes on increasing after 2.5 s. Also, for $x \geq 3.9$ *mm*, the temperature remains constant up to 4 s but it goes on increasing beyond 4 s. The picture becomes clearer in the temperature contours obtained through FE simulation at different time steps (Fig. 10). It is clear that till 2 s analysis of temperature data, the thermal penetration appears to be at slower rate such that the one-dimensional heat conduction modeling semi-infinite medium can still be valid for the present gauge-substrate system. The isotherm lines are almost parallel to the surface i.e. the heat flow can be considered as one dimensional. Till 4 s, the transient solution also supports one-dimensional approximations because the experimental temperature history is almost exactly recovered (Fig. 4). Beyond 4 s, it would be inappropriate to consider the heat modeling to be one-dimensional for a quartz substrate because the thermal penetration rate occurs at a faster rate. The limiting time for which the semi-infinite assumption can be considered, is given by the equation (Diller and Kidd 1997),

$$\frac{x}{\sqrt{\alpha t}} \leq 2 \tag{11}$$

where $\alpha = \dfrac{k}{\rho c}$, is the thermal diffusivity of the substrate. For nickel substrate with thermal penetration depth of 4 mm, the limiting time is obtained as,

$$t \leq 4.21\ s \tag{12}$$

This means that for any time longer than 4.21 s, the semi-infinite approximation will no longer be valid and thus supports the inferences obtained though FE analysis.

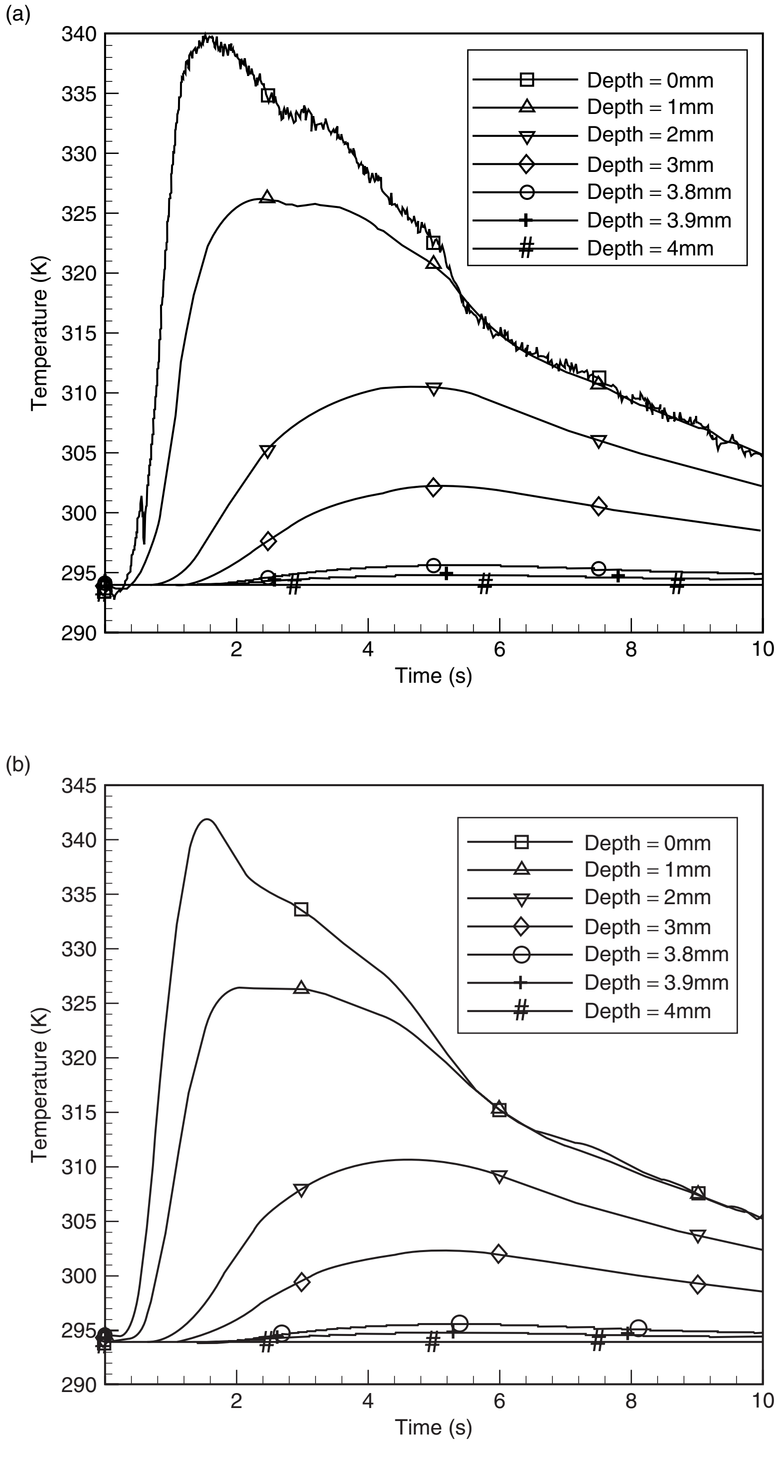

Figure 9. Transient temperature distribution along the depth of substrate through finite element simulation: (a) heat input with cubic-spline fitting of experimental temperature data; (b) heat input with polynomial fitting of experimental temperature data.

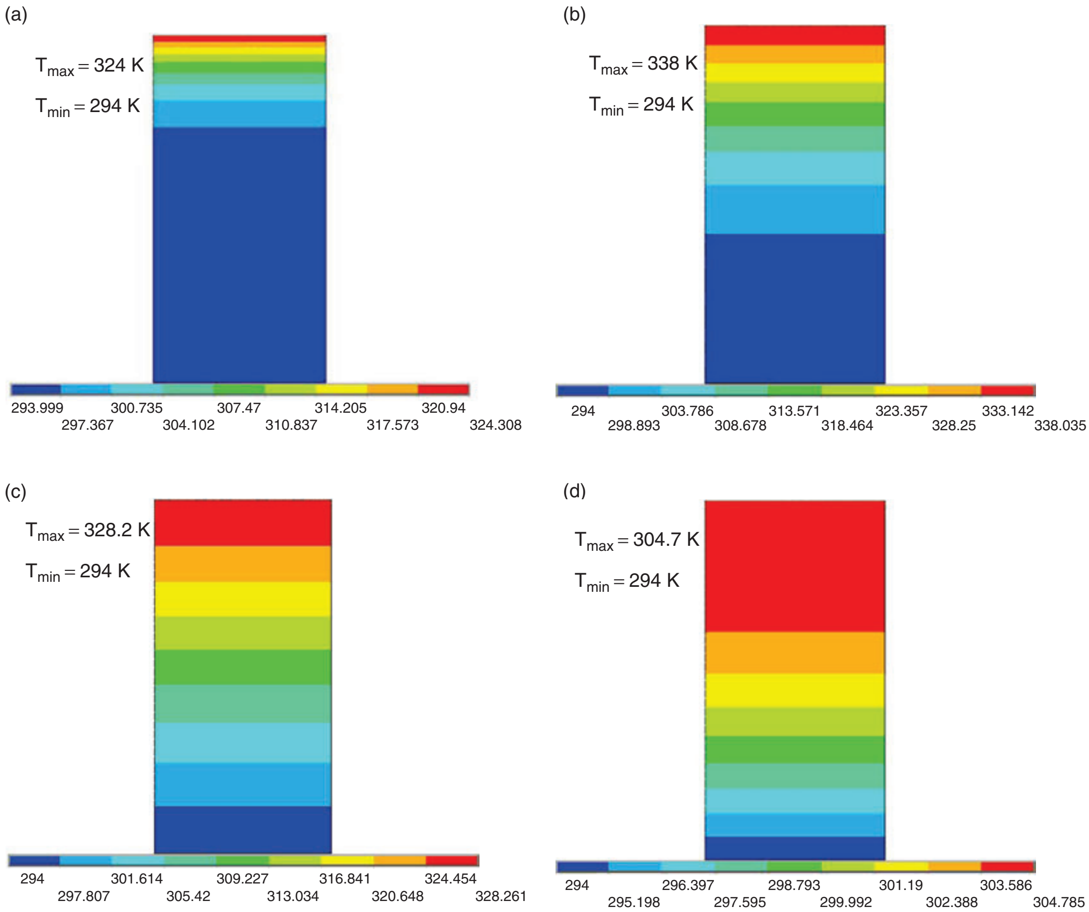

Figure 10. Temperature contours through finite element simulation at various time steps (heat input with cubic-spline fitting of experimental temperature data): (a) 1 s; (b) 2 s; (c) 4 s; (d) 10 s.

4. CONCLUSION

The transient temperature-time history is recorded during a supersonic flight test from a nickel thin film gauge mounted on a quartz substrate. The convective surface heating rates are obtained from the temperature data by one-dimensional heat conduction modeling. Since the substrate material (quartz) is an insulator, it is reasonable to assume semi-infinite medium in the analysis. The results are also supported by the FE simulation of a realistic gauge-substrate system. Although the temperature data is recorded for 10 s, the comparative results from analytical analysis and FE simulation show that the semi-infinite assumption is valid only up to 4 s. In this present work, it is an attempt to see the effect of lateral heat conduction with respect to time scale under consideration. Nevertheless, the model problem studied is taken as justification of the one-dimensional assumption in the calculation methods. The most advanced verification of the two-dimensional effects is currently ongoing with the realistic boundary conditions through FE simulation and will be reported in future publications.

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